Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test II –April 2025

II Semester

Class : I PG **Time : 2 Hours**

Major : Mathematics **Max.Marks : 60**

23MMAC08 – Real Analysis II

Course Outcomes:

CO1: Distinguish between the Lebesgue and Riemann integrals.

CO2: Apply the concept of Lebesgue integral to broader class of functions

CO3: Test the convergence using Riemann's localization theorem.

CO4: Solve problems in a closed form using Fourier integrals.

CO5: Evaluate the multiple integrals using iterated integration.

Part A **6 x 1 = 6**

Choose the Correct Answer

1. If S has measure zero, then $\chi_S \in L(R)$ and $\int_R \chi_S$ is CO3K1
   a. 1 b. ∞
   c. 0 d. None of these

2. An orthonormal system of complex-valued functions on every interval of length 2π is given by CO3K1
   a. $\frac{e^{inx}}{\sqrt{2\pi}}$ b. $\frac{e^{-inx}}{\sqrt{2\pi}}$ c. $\frac{e^{inx}}{\sqrt{\pi}}$ d. $\frac{e^{-inx}}{\sqrt{\pi}}$

3. The Dirichlet integral is CO3K1
   a. $\int_0^\delta g(t)dt$ b. $\int_0^\delta \frac{sin\alpha t}{t}dt$ c. $\int_0^\delta g(t)\frac{sin\alpha t}{t}dt$ d. $\int_0^\delta \frac{g(t)}{t}dt$

4. If f is of bounded variation on the compact interval $[\ -\delta, x+\delta\ ]$ for some $\delta < \pi$,then the limit $s(x)$ exists and the Fourier series generated by f CO4K2
   a. diverges b. converges to $s(x)$ c. constant d. oscillates

5. For sets in R^3, c(S) is called CO5K2
   a. volume of S b. area of S c. circumference of S d. height of S

6. If f is defined and bounded on a compact interval I in R^n, then $f \in R$ on S if, and only if, the discontinuities of f in I has CO5K1
   a. R -measure b. f-measure c. n- measure zero. d. measure zero.

Part B **3 x 6 = 18**

Answer ALL questions

7.a. State and Prove Riemann-Lebesgue Lemma. CO3K3

(or)

7.b. State and Prove Dini Theorem. CO3K3

8. a. State and Prove Dini's Test. CO4K3

(or)

8. b. Obtain the Exponential form of the Fourier Integral Theorem. CO4K3

9. a. If S is a bounded set in R^n and ∂S, its boundary, then prove that

$\bar{c}(\partial S) = \bar{c}(S) - \underline{c}(S)$. CO5K3

(or)

9. b. If $f \in R$ and $g \in R$ on a Jordan measurable set S in R^n. If $f(x) \leq g(x)$ for each x in S, then Show that $\int_S f(x)dx \leq \int_S g(x)dx$ CO5K3

Part C **3 x 12 = 36**

Answer ALL questions

10. a. State and Prove Jordan Theorem. CO3K4

(or)

10. b. State and prove Riemann Localisation Theorem. CO3K4

11. a. State and Prove Fejer's Theorem. CO4K4

(or)

11. b. State and Prove Weierstrass Approximation Theorem. CO4K4

12. a. State and establish the Mean -Value Theorem for multiple integrals. CO5K4

(or)

12. b. If f is defined and bounded on a compact interval I in Q = [a, b] × [c, d] in R^n. Show that the following statements are equivalent.

(i) $f \in R$ on I.

(ii) f satisfies Riemann's condition on I.

(iii) $\underline{\int}_I f dx = \overline{\int}_I f dx$ CO5K4

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