Summary and Conclusion

SUMMARY AND CONCLUSION

The linear preserver problem is an active area of research in matrix theory. The linear algebra over semirings is a subject of intensive research because of its numerous applications. There have been numerous investigations into the theory of matrices over algebraic structures and there are many research articles on linear operators that preserve the rank of matrices over several semirings and semifields.

This thesis is devoted to the study of

(1) Linear operators preserving zero-term rank of real matrices.

(2) Linear operators preserving factor rank of matrices over semirings.

(3) Linear operators preserving term rank and zero-term rank of matrices over semirings.

(4) Linear operators preserving rank and perimeter of rank-1 matrices over semirings.

(5) Linear operators preserving rank and perimeter of rank-1 matrices over semifields.

(6) Linear operators preserving pairs of Hermitian matrices on which the rank is additive.

The first chapter deals with preliminary definitions and notations.

Chapter II deals with the linear operators preserving zero-term rank of real matrices.

In this chapter, the linear operators preserving the zero-term rank of $m \times n$ real matrices is characterized. Combinatorial equivalent condition for the zero-term rank of a real matrix is also obtained.

Linear operators preserving factor rank of matrices over semirings are studied in chapter III.

In this chapter, the linear preservers for the sets of matrix ordered tuples which satisfy extremal properties with respect to factor rank are characterized and

the characterizations of these linear preservers are given in theorems 3.18, 3.21, 3.24, 3.27, 3.29, 3.33, 3.36 and 3.39.

In chapter IV, the linear operators on matrices over semirings that preserve the extremal cases in the bounds on term ranks and zero-term ranks of sums and products of matrices are characterized and the characterizations of these linear preservers are given in theorems 4.2, 4.4, 4.5, 4.8, 4.11, 4.12, 4.13, 4.14 and 4.15.

In chapter V, the set of linear operators preserving the rank and perimeter of every rank-1 matrix over any chain semiring is characterized.

Chapter VI deals with linear operators preserving rank and perimeter of rank-1 matrices over semifields. It is proved here that a linear operator T preserves the rank and perimeter of rank-1 matrices over semifields if and only if it has the form $T(A) = UAV$, or $T(A) = UA^tV$ with some invertible matrices U and V (A^t denotes transpose of A).

Linear operators preserving pairs of Hermitian matrices on which the rank is additive is studied in chapter VII.

The linear maps from the set of all $n \times n$ Hermitian matrices into itself which preserve the set of rank-additive pairs is characterized and is given in Theorem 7.2.