CHAPTER – VIII

CHAPTER VIII

FUZZY SP- IRRESOLUTE FUNCTIONS

Definition: 8.1

A fuzzy set α of an fts X is called **fuzzy pre-open** if $\alpha \leq \text{int}(\text{cl } \alpha)$.

Definition: 8.2

A fuzzy set α of an fts X is called **fuzzy pre-closed** if $\alpha \geq \text{cl } (\text{int } \alpha)$.

Definition: 8.3

The intersection of all fuzzy pre-closed sets containing α is called the **pre-closure** of α and is denoted by **pcl** (α). The **pre-interior** of α, denoted by **pint** (α) is defined by the union of all fuzzy pre-open sets contained in α.

Definition: 8.4

A fuzzy set α of an fts X is called **fuzzy strongly pre-open** if $\alpha \leq \text{int } (\text{pcl } \alpha)$.

Definition: 8.5

A fuzzy set α of an fts X is called **fuzzy strongly pre-closed** if $\alpha \geq \text{cl } (\text{pint } \alpha)$.

Notation: 8.6

The family of all **fuzzy strongly pre-open (fuzzy strongly pre-closed)** sets of an fts (X,τ) will be denoted by **FSPO(τ) (FSPC(τ))**.

Definition: 8.7

A mapping f : $(X,\tau_1) \longrightarrow (Y, \tau_2)$ from an fts (X,τ_1) into an fts (Y,τ_2) is called **fuzzy SP-irresolute continuous** if $f^{-1}(\beta) \in \text{FSPO}(\tau_1)$, for each $\beta \in \text{FSPO}(\tau_2)$.

Example: 8.8

Let X = {a, b, c} and α, β, γ be fuzzy sets of X defined as follows:

α (a) = 0.3 , α (b) = 0.2 , α (c) = 0.7;

β (a) = 0.8 , β (b) = 0.8 , β (c) = 0.4;

γ (a) = 0.8 , γ (b) = 0.7 , γ (c) = 0.6.

We put $\tau_1 = \{0, \alpha, \beta, \alpha \wedge \beta, \alpha \vee \beta, 1\}$,

$\tau_2 = \{0, \alpha, 1\}$

and f = id : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ .Then f is fuzzy SP-irresolute continuous.

Theorem: 8.9

Let f : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). Then the following statements are equivalent:

(i) f is fuzzy SP-irresolute continuous;

(ii) $f^{-1}(\beta) \in FSPC(\tau_1)$, for each $\beta \in FSPC(\tau_2)$;

(iii) f(spcl α) $\leq$ spcl f(α), for each fuzzy set α of X;

(iv) spcl $f^{-1}(\beta) \leq f^{-1}$ (spcl β), for each fuzzy set β of Y ;

(v) f^{-1} (spint β) $\leq$ spint $f^{-1}(\beta)$, for each fuzzy set β of Y .

Proof:

(i)⟹(ii) :

It follows from the definitions.

(ii)⟹(iii) :

Let α be a fuzzy set of X. Then spcl f (α) $\in FSPC(\tau_2)$.

According to the assumption, f^{-1}(spcl f (α)) ∈ FSPC (τ_1).

Hence, spcl $\alpha \leq$ spcl f^{-1} f (α) $\leq$ spcl f^{-1}(spcl f (α))

= f^{-1}(spcl f (α)).

Thus, f (spcl α) $\leq$ spcl f (α).

(iii)⟹(iv):

Let β be a fuzzy set of Y. According to the assumption,

f(spcl f^{-1} (β)) $\leq$ spcl f f^{-1} (β) $\leq$ spcl β.

Thus, spcl f^{-1}(β) $\leq f^{-1}$ f (spcl f^{-1} (β)) $\leq f^{-1}$ (spcl β).

(iv)⟹(v):

It follows from the definitions.

(v)⟹(i):

Let β ∈ FSPO (τ_2). Then β = spint β. From (v) we obtain

f^{-1}(β) = f^{-1}(spint β) $\leq$ spint f^{-1}(β) $\leq$ f^{-1}(β).

Thus, f^{-1}(β) = spint f^{-1}(β).

Hence, f is a fuzzy SP-irresolute continuous mapping.

Theorem: 8.10

Let f : (X,τ_1) ⟶ (Y,τ_2) be a bijective mapping from an fts (X,τ_1) into an fts (Y,τ_2). The mapping f is fuzzy SP-irresolute continuous iff spint f (α) $\leq$ f (spint α), for each fuzzy set α of X.

Proof:

Let f be fuzzy SP-irresolute continuous. Then f^{-1} (spint f (α)) $\in$ FSPO (τ_1), for any fuzzy set α of X. Since f is injective, from Theorem 8.9 we obtain that ,

f^{-1} (spint f(α)) $\leq$ spint f^{-1}(f(α)) = spint α.Again, since f is surjective , we have

spint f(α) = f(f^{-1} (spint f(α)) $\leq$ f(spint α).

Conversely, let $\beta \in$ FSPO (τ_2). Then spint $\beta = \beta$. Since f is surjective, from the assumption we obtain that , f(spint f^{-1} (β)) $\geq$ spint f f^{-1} (β) = spint $\beta = \beta$.

This implies that f^{-1}(f(spint f^{-1} (β))) $\geq$ f^{-1} (β). Since f is injective, we have spint f^{-1} (β) = f^{-1}f(spint f^{-1} (β)) $\geq$ f^{-1} (β). Thus, spint f^{-1} (β) = f^{-1} (β).

Hence, f is fuzzy SP-irresolute continuous.

Definition: 8.11

Let α be a fuzzy set of an fts X.

(1)The union of all fuzzy strongly pre-open sets contained in α is called a **fuzzy strong pre-interior** of α, denoted by **spint α.**

(2)The intersection of all fuzzy strongly pre-closed sets containing α is called a **fuzzy strong pre-closure** of α, denoted by **spcl α**.

Theorem: 8.12

Let f : (X,τ_1) $\longrightarrow$ (Y,τ_2) be a mapping from an fts (X,τ_1) intoan fts (Y,τ_2). Then the following statements are equivalent:

(i) f is fuzzy SP-irresolute continuous;

(ii) cl (pint $f^{-1}(\beta)$) $\leq$ f^{-1}(spcl β), for each fuzzy set β of Y ;

(iii) f^{-1}(spint β) $\leq$ int (pcl $f^{-1}(\beta)$), for each fuzzy set β of Y ;

(iv) f (cl (pint α)) $\leq$ spcl f(α), for each fuzzy set α of X.

Proof:

(i)$\Longrightarrow$(ii):

Let β be a fuzzy set of Y. According to the assumption f^{-1}(spcl β)$\in$ FSPC (τ_1)).

Hence, f^{-1}(spcl β) $\geq$ cl (pint f^{-1}(spcl β)) $\geq$ cl (pint $f^{-1}(\beta)$).

(ii)$\Longrightarrow$(iii) :

It follows from the definitions.

(iii)$\Longrightarrow$(iv):

Let α be a fuzzy set of X. Let us put β = f(α), then $\alpha \leq f^{-1}(\beta)$. According to the assumption, (int (pcl α^c)) ^{c} $\leq$ (int (pcl $f^{-1}(\beta^c)$)) ^{c} $\leq$ (f^{-1} (spintβ^c)) ^{c}.

Thus, cl (pintα) $\leq$ cl (pint $f^{-1}(\beta)$) $\leq$ f^{-1}(spcl β).

Hence, f (cl(pint α)) $\leq$ f f^{-1} (spcl β) $\leq$ spcl β = spcl f(α).

(iv)$\Longrightarrow$(i):

Let $\beta \in$ FSPC (τ_2). According to the assumption,

f(cl(pint $f^{-1}(\beta)$)) $\leq$ spcl f $f^{-1}(\beta)$ $\leq$ spcl $\beta = \beta$.

Then cl(pint $f^{-1}(\beta)$) $\leq$ f^{-1} f(cl(pint $f^{-1}(\beta)$)) $\leq$ $f^{-1}(\beta)$. Thus $f^{-1}(\beta) \leq$ FSPC (τ_1), hence f is fuzzy SP-irresolute continuous.

Theorem: 8.13

Let f : (X,τ_1) $\longrightarrow$ (Y,τ_2) be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). If f is SP-irresolute continuous, then $f^{-1}(\beta) \leq$ spint f^{-1}(int (pcl β)), for each $\beta \in$ FSPO (τ_2).

Proof:

Let $\beta \in$ FSPO (τ_2). Then, $f^{-1}(\beta) \leq f^{-1}$(int(pcl β)). Since $f^{-1}(\beta) \in$ FSPO (τ_1), we have $f^{-1}(\beta) \leq$ spint f^{-1}(int(pcl β)).

Theorem: 8.14

Let f : (X,τ_1) $\longrightarrow$ (Y,τ_2) be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). Then the following statements are equivalent:

(i) f is fuzzy SP-irresolute continuous;

(ii) for any fuzzy point x_α of X and any $\beta \in$ FSPO (τ_2) containing f (x_α), there exists $\alpha \in$ FSPO (τ_1) containing x_α such that $\alpha \leq f^{-1}(\beta)$;

(iii) for any fuzzy point x_α of X and any $\beta \in$ FSPO (τ_2) containing f (x_α),there exists $\alpha \in$ FSPO (τ_1) containing x_α such that f (α) $\leq \beta$.

Proof :

(i) =) (ii):

Let f be fuzzy SP-irresolute continuous. Let x_α be a fuzzy point of X and let $\beta \in$ FSPO (τ_2) containing f (x_α).

Then $x_\alpha \in f^{-1}(\beta)$ = spint $f^{-1}(\beta)$.

The result follows for α = spint $f^{-1}(\beta)$.

(ii) =) (iii):

It follows from the relation f (α) $\leq$ f ($f^{-1}(\beta)$) $\leq \beta$.

(iii) =) (i):

Let $\beta \in$ FSPO (τ_2) and let x_α be a fuzzy point of X such that $x_\alpha \in f^{-1}(\beta)$. Then f $(x_\alpha) \in \beta$. According to the assumption, there exists $\alpha \in$ FSPO (τ_1) containing x_α such that f $(\alpha) \leq \beta$. Then $x_\alpha \in \alpha \leq f^{-1}$(f $(\alpha)) \leq f^{-1}(\alpha)$ and $x_\alpha \in \alpha$ = spint $\alpha \leq$ spint $f^{-1}(\beta)$. Since x_α is an arbitrary fuzzy point and $f^{-1}(\beta)$ is the union of all fuzzy points which belong in $f^{-1}(\beta)$, it follows that $f^{-1}(\beta) \leq$ spint $f^{-1}(\beta)$. Hence, f is fuzzy SP-irresolute continuous.

Theorem: 8.15

Let f : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ be a bijective mapping from an fts (X,τ_1) into an fts (Y,τ_2). The mapping f is fuzzy SP-irresolute continuous if and only if for any fuzzy point x_α of X and any $\beta \in$ FSPO (τ_2) containing f(x_α), pcl $f^{-1}(\beta)$ is a fuzzy neighbourhood of x_α.

Proof:

Let f be fuzzy SP-irresolute continuous. Let x_α be a fuzzy point of X and $\beta \in$ FSPO (τ_2) containing f (x_α).Then $x_\alpha \in f^{-1}(\beta) \leq$ int (pcl $f^{-1}(\beta)) \leq$ pcl $f^{-1}(\beta)$. Hence pcl $f^{-1}(\beta)$ is a fuzzy neighbourhood of x_α .

Conversely, let $\beta \in$ FSPO (τ_2) and let $x_\alpha \in f^{-1}(\beta)$.

According to the assumption, pcl $f^{-1}(\beta)$ is a fuzzy neighbourhood of x_α.

Thus, $x_\alpha \in$ int(pcl $f^{-1}(\beta)$).

Hence, $f^{-1}(\beta) \leq$ int (pcl $f^{-1}(\beta)$),

i.e. f is fuzzy SP-irresolute continuous.

Definition: 8.16

Let f : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ be a mapping from an fts (X,τ_1) intoan fts (Y,τ_2). The mapping f is called **fuzzy weakly continuous** if $f^{-1}(\beta) \leq \text{int } f^{-1}(\text{cl } \beta)$, for each $\beta \in \tau_2$.

Lemma: 8.17

Let α be a fuzzy set of an fts X .

1) If α is fuzzy strongly pre-open, then pcl α = spcl α = cl α.
2) If α is fuzzy strongly pre-closed, then pint α = spint α = int α.

Lemma: 8.18

Let α be a fuzzy set of an fts X . Then

1) spint (cl α) = int (cl (α)) ;
2) spcl (int α) = cl (int (α)).

Theorem: 8.19

Every fuzzy SP-irresolute continuous mapping is a fuzzy weakly continuous mapping.

Proof:

Let f : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ be fuzzy SP-irresolute continuous and $\beta \in FO(\tau_2)$. Then, from Lemma 8.17, Lemma 8.18 and Theorem 8.9 it follows that

$f^{-1}(\beta)$ = spint $f^{-1}(\beta) \leq$ spint(cl $f^{-1}(\beta)$) = int (cl $f^{-1}(\beta)$) = int(spcl $f^{-1}(\beta)$) $\leq$ int f^{-1} (spcl β) $\leq$ int f^{-1} (cl β). Hence, f is fuzzy weakly continuous.

Definition: 8.20

Let f : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y, τ_2) . The mapping f is called SP-irresolute open (closed) if f $(\alpha) \in$ FSPO (τ_2) (f $(\alpha) \in$ FSPC (τ_2)), for each $\alpha \in$ FSPO (τ_1) ($\alpha \in$ FSPC (τ_1)).

Example: 8.21

Let (X,τ_1) and (X,τ_2) be the fts's from Example 8.8. Then the mapping f = id : $(X,\tau_2) \longrightarrow (X,\tau_1)$ is fuzzy SP-irresolute open (closed) .

Theorem: 8.22

Let f : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). Then the following statements are equivalent:

(i) f is fuzzy SP-irresolute open;

(ii) f(spint α) $\leq$ spint f (α), for each fuzzy set α of X;

(iii) spint f^{-1} $(\beta) \leq f^{-1}$ (spint β), for each fuzzy set β of Y ;

(iv) f^{-1}(spcl β) $\leq$ spcl f^{-1} (β), for each fuzzy set β of Y .

Proof:

(i) =) (ii):

Let α be a fuzzy set of X.

Then f (spint α) = spint f (spint α) $\leq$ spint f (α).

(ii) =) (iii):

Let β be a fuzzy set of Y. According to the assumption,

f (spint $f^{-1}(\beta)$) $\leq$ spint f $f^{-1}(\beta) \leq$ spint β.

Hence spint $f^{-1}(\beta) \leq f^{-1}$ f (spint($f^{-1}(\beta)$)) $\leq f^{-1}$(spint β).

(iii) =) (iv):

It follows from the definitions.

(iv) =) (i):

Let $\alpha \in$ FSPO (τ_1). Then α = spint α. According to the assumption,

α = spint $\alpha \leq$ pint f^{-1} f (α) = (spcl f^{-1} ($f(\alpha)^c$))$^c \leq f^{-1}$(spcl f (α)^c)^c =

f^{-1}(spint f(α)), hence f(α) $\leq$ f f^{-1}(spint f (α)) $\leq$ spint f (α).

Thus f (α) = spint f (α), i.e. f is fuzzy SP-irresolute open.

Theorem: 8.23

Let f : (X,τ_1) ⟶ (Y,τ_2) be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). The mapping f is fuzzy SP-irresolute closed if and only if spcl f (α) $\leq$ f (spcl α), for each fuzzy set α of X.

Proof:

Let f be fuzzy SP-irresolute closed and let α be any fuzzy set of X. Then f (spcl α) $\in$ FSPC (τ_2). From f (α) $\leq$ f (spcl α) it follows that spcl f(α) $\leq$ f(spcl α).

Conversely, let $\alpha \in$ FSPC (τ_1). From f (α) = f(spcl α) $\geq$ spcl f (α), we obtain spcl f (α) = f (α), hence f is fuzzy SP-irresolute closed.

Theorem: 8.24

Let f : (X,τ_1) ⟶ (Y,τ_2) be a mapping from an fts (X,τ_1) into an fts (Y,τ_2).

1) The mapping f is fuzzy SP-irresolute open if and only if f (spint α) $\leq$int (pcl f (α)), for each fuzzy set α of X.

2) The mapping f is fuzzy SP-irresolute closed if and only if cl (pint f (α)) $\leq$ f(spcl α), for each fuzzy set α of X.

Proof:

(1). Let α be a fuzzy set of X. Then f(spint α) $\in$ FSPO (τ_2), hence f(spint α) $\leq$ int(pcl f(spint α)) $\leq$ int (pcl f(α)).

Conversely, let $\alpha \in$ FSPO (τ_1), From f(α) = f(spint α) $\leq$ int (pcl f(α)) it follows that f(α) $\in$ FSPO (τ_2), hence f is fuzzy SP-irresolute open.

(2) can be proved in a similar manner.

Theorem: 8.25

Let f : (X,τ_1) $\longrightarrow$ (Y,τ_2) be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). Then the following statements hold:

1) If f (int(pcl α)) $\leq$ int (pcl f(α)), for each $\alpha \in$ FSPO (τ_1), then f is fuzzy SP-irresolute open;

2) If f (cl(pint α)) $\geq$ cl (pint f(α)), for each $\alpha \in$ FSPC (τ_1), then f is fuzzy SP-irresolute closed.

Proof:

(1). Let $\alpha \in$ FSPO (τ_1). Then $\alpha \leq$ int (pcl α). According to the assumption, f (α) $\leq$ f (int(pcl α)) $\leq$ int (pcl f(α)), hence f(α) $\in$ FSPO (τ_2).

i.e. f is SP-irresolute open.

(2) can be proved in a similar manner.

Theorem: 8.26

Let f : X ⟶ Y be a bijective mapping from an fts X into an fts Y. Then the following statements hold:

1) f is fuzzy SP-irresolute open if and only if it is fuzzy SP-irresolute closed;

2) f is fuzzy SP-irresolute open (closed) if and only if f^{-1} is fuzzy SP-irresolute continuous.

Proof:

1) follows from the definitions.

2) follows from the relation $(f^{-1})^{-1}(\alpha)$ = f(α), for each fuzzy set α of X.

Corollary: 8.27

Let f : X ⟶Y be a bijective mapping from an fts X into an fts Y . Then the following statements are equivalent:

(i) f is fuzzy SP-irresolute closed;

(ii) f(spint α) $\leq$ spint f(α), for each fuzzy set α of X;

(iii) spint $f^{-1}(\beta) \leq f^{-1}$(spint β), for each fuzzy set β of Y ;

(iv) f^{-1}(spcl β) $\leq$ spcl $f^{-1}(\beta)$, for each fuzzy set β of Y .

Theorem: 8.28

Let f : (X,τ_1) ⟶ (Y,τ_2) be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). The mapping f is fuzzy SP-irresolute open if and only if for each fuzzy set β of Y and each $\alpha \in$ FSPC (τ_1), $f^{-1}(\beta) \leq \alpha$, there exists $\gamma \in$ FSPC (τ_2) such that $\beta \leq \gamma$ and $f^{-1}(\gamma) \leq \alpha$.

Proof :

Let β be a fuzzy set of Y and let $\alpha \in$ FSPC (τ_1) such that $f^{-1}(\beta) \le \alpha$.Then $\alpha^c \ge f^{-1}(\beta^c)$, hence f $(\alpha^c) \le$ f $f^{-1}(\beta^c) \le \beta^c$. Since $\alpha^c \in$ FSPO (τ_1), we obtain that f $(\alpha^c) \in$ FSPO (τ_2), hence f $(\alpha^c) \le$ spint β^c. Thus $\alpha^c \le f^{-1}$ f$(\alpha^c) \le f^{-1}$ (spint β^c). It follows that $\alpha \ge f^{-1}$ (spint $\beta^c)^c = f^{-1}$ (spcl β). The result follows for γ = spcl β.

Conversely, let $\lambda \in$ FSPO (τ_1). We claim that f $(\lambda) \in$ FSPO (τ_2). From $\lambda \le f^{-1}$ f (λ) it follows that $\lambda^c \ge f^{-1}$ f$(\lambda)^c$, where $\lambda^c \in$FSPC (τ_1). Hence there is $\beta \in$ FSPC (τ_2) such that $\beta \ge$ f$(\lambda)^c$and $f^{-1}(\beta) \le \lambda^c$. Since $\beta \ge$f $(\lambda)^c$, it follows that $\beta \ge$ spcl f $(\lambda)^c$ or $\beta^c \le$ (spcl f$(\lambda)^c)^c$ = spint f(λ). From $f^{-1}(\beta) \le \lambda^c$ we obtain $f^{-1}(\beta^c) \ge \lambda$ or $\beta^c \ge$ f $f^{-1}(\beta^c) \ge$ f (λ). Since f $(\lambda) \le \beta^c \le$ spint f (λ), we have f (λ) = spint f (λ).Thus, f$(\lambda) \in$ FSPO (τ_2), hence f is fuzzy SP-irresolute open.

Corollary: 8.29

Let f : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). If f is fuzzy SP-irresolute open then:

1) f^{-1} (cl(pint β)) $\le$ spcl $f^{-1}(\beta)$, for each fuzzy set β of Y ;

2) f^{-1} (cl β) $\le$ spcl $f^{-1}(\beta)$, for each $\beta \in$ FPO (τ_2).

Proof:

1) Let β be a fuzzy set of Y. Then spcl $f^{-1}(\beta) \in$ FSPC (τ_1).

From Theorem 8.17 it follows that there exists $\gamma \in$ FSPC (τ_2) such that $\beta \le \gamma$ and $f^{-1}(\gamma) \le$ spcl $f^{-1}(\beta)$. Thus f^{-1}(cl (pint β)) $\le f^{-1}$(cl(pint γ)) $\le f^{-1}(\gamma) \le$ spcl $f^{-1}(\beta)$.

2) It follows immediately from 1).

Theorem: 8.30

Let f : $(X,\tau_1) \longrightarrow (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). The mapping f is fuzzy SP-irresolute closed if and only if for each fuzzy set β of Y and each $\alpha \in$ FSPO (τ_1), $f^{-1}(\beta) \leq \alpha$, there exists $\gamma \in$ FSPO (τ_2) such that $\beta \leq \gamma$ and $f^{-1}(\gamma) \leq \alpha$.

Proof:

Let β be a fuzzy set of Y and let $\alpha \in$ FSPO (τ_1) such that $f^{-1}(\beta) \leq \alpha$. Then f $(\alpha^c) \in$ FSPC (τ_2). We put $\gamma = f(\alpha^c)^c$. Then $\gamma \in$ FSPO (τ_2), $\beta \leq \gamma$ and

$f^{-1}(\gamma) = f^{-1}(f(\alpha^c)^c) \leq f^{-1} f(\alpha) \leq \alpha$.

Conversely, let $\alpha \in$ FSPC (τ_1). Then $\alpha^c \in$ FSPO (τ_1) and $\alpha^c \geq f^{-1}(f(\alpha)^c)$. According to the assumption there exists $\gamma \in$ FSPO (τ_2) such that $f(\alpha)^c \leq \gamma$ and

$f^{-1}(\gamma) \leq \alpha^c$. Hence, $f(\alpha) = \gamma^c \in$ FSPC (τ_2).

Theorem: 8.31

Let f : X $\longrightarrow$ Y be a mapping from an fts X into an fts Y .The mapping f is fuzzy SP-irresolute closed and fuzzy SP-irresolute continuous if and only if

f (spcl α) = spcl f(α), for each fuzzy set α of X.

Proof:

It follows from Theorem 8.9 and Theorem 8.23.

Theorem: 8.32

Let f : X $\longrightarrow$ Y be a mapping from an fts X into an fts Y. The mapping f is fuzzy SP-irresolute open and fuzzy SP-irresolute continuous if and only if

f^{-1}(spcl β) = spcl $f^{-1}(\beta)$, for each fuzzy set β of Y.

roof:

It follows from Theorem 8.9 and Theorem 8.22.

Theorem 8.33:

Let f : X ⟶Y be a mapping from an fts X into an fts Y .The mapping f is fuzzy SP-irresolute open and fuzzy SP-irresolute continuous if and only if

f^{-1} (spint β) = spint $f^{-1}(\beta)$, for each fuzzy set β of Y.

Proof:

It follows from Theorem 8.9 and Theorem 8.22.