

CHAPTER - IX

CHAPTER IX

APPLICATION OF MODIFIED FUZZY ANALYTIC HIERARCHY PROCESS METHOD

In this chapter the author of this thesis presented another mathematical model for the same problem “Selecting the Best School for the Children” considered in Chapter VIII by applying Modified Fuzzy Analytic Hierarchy Process and by using the same data.

Modified Fuzzy AHP Method

Consider a decision making problem, of selecting the best school for the children. The hierarchical structure of this selection problem is shown in Fig 8, where C_1, C_2, C_3, C_4, C_5 are five major criteria for developing a model for the selection of the best school for the children, each involve some sub criteria ($C_{11}, C_{12}, C_{21}, C_{22}, C_{31}, C_{32}, C_{33}, C_{41}, C_{42}, C_{43}, C_{51}, C_{52}, C_{53}$) and A_1 (P.M.G Higher Secondary School, College Road, Palakkad), A_2 (Bharath Matha Higher Secondary School, Chandranagar, Palakkad) and A_3 (Vyasa Vidya Peethom School, Kallekad, Palakkad) are the three alternatives chosen after initial screening.

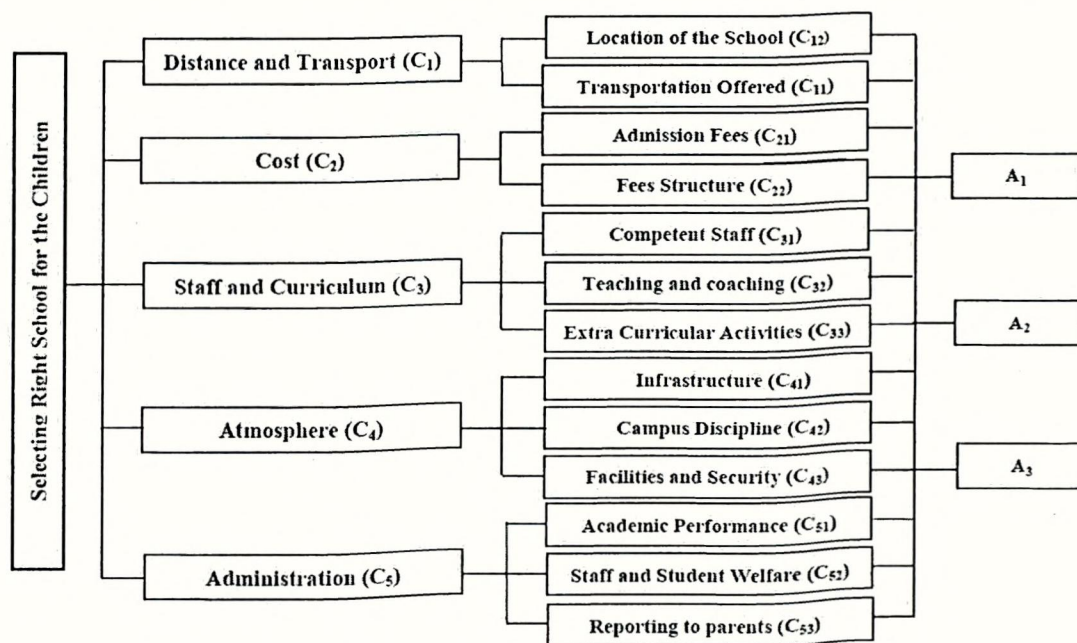


Fig 8. The hierarchy of selecting the best school for the children

After building the hierarchy, the pairwise comparison of the importance of one criterion over the others, one sub criterion over the others and one alternative over the others were estimated with the help of the same data used in previous chapter. In this Modified Fuzzy AHP method, five point scaling technique (Equally Important, Weakly More Important, Strongly More Important, Very Strongly More Important, and Absolutely More Important) was adopted to assess the degree of importance of one criterion, sub criterion, or alternative over another. Using the Triangular Fuzzy Conversion Scale given in Table 4, the pairwise comparison matrices (Triangular Fuzzy Number Matrices $A_i, i = 1, \dots, 100$) for 100 respondents are constructed. By using the definition of addition of Triangular Fuzzy Number Matrices a new matrix $D = A_1 + A_2 + \dots + A_{100}$ is constructed. Here in this method we assign same weights for all the 100 decision makers, i.e. $e_i = 0.01, i = 1, \dots, 100$. The aggregated fuzzy comparison matrix with respect to the goal (Table 22) is constructed by applying the fuzzy weighted triangular averaging operator given in equation (13).

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(3.6,5.28,6.7)	(1.01,2.42,4.42)	(1.21,2.7,4.7)	(3.31,5.06,6.78)
C_2	(-6.7, -5.28, -3.6)	(1,1,1)	(1.49,3.22,5.18)	(1.97,3.66,5.5)	(3.76,5.6,7.16)
C_3	(-4.42, -2.42, -1.01)	(-5.18, -3.22, -1.49)	(1,1,1)	(1.99,3.52,5.44)	(4.62,6.5,8.02)
C_4	(-4.7, -2.7, -1.21)	(-5.5, -3.66, -1.97)	(-5.44, -3.52, -1.99)	(1,1,1)	(4.92,6.86,8.28)
C_5	(-6.78, -5.06, -3.31)	(-7.16, -5.6, -3.76)	(-8.02, -6.5, -4.62)	(-8.28, -6.86, -4.92)	(1,1,1)

Table 22. Aggregated Fuzzy Comparison Matrix with respect to the goal

Then the comparison matrix which include negative fuzzy numbers were converted to positive fuzzy numbers by using equation (14). Table 22 was converted as follows:

$$\widetilde{a}_{12}^* = e^{(\widetilde{a}_{12}/4)} = e^{((3.6,5.28,6.7)/4)} = (2.46,3.743,5.339)$$

Other \widetilde{a}_{ij}^* values of Table 22 were given in Table 23.

	C ₁	C ₂	C ₃	C ₄	C ₅
C ₁	(1,1,1)	(2.46,3.743,5.339)	(1.287,1.831,3.019)	(1.353,1.964,3.238)	(2.288,3.543,5.447)
C ₂	(0.187,0.267,0.407)	(1,1,1)	(1.451,2.237,3.651)	(1.636,2.497,3.955)	(2.56,4.055,5.989)
C ₃	(0.331,0.546,0.777)	(0.274,0.447,0.689)	(1,1,1)	(1.645,2.411,3.896)	(3.174,5.078,7.426)
C ₄	(0.309,0.509,0.739)	(0.253,0.401,0.611)	(0.257,0.415,0.608)	(1,1,1)	(3.421,5.557,7.925)
C ₅	(0.184,0.282,0.437)	(0.167,0.247,0.391)	(0.135,0.197,0.315)	(0.126,0.18,0.292)	(1,1,1)

Table 23. Fuzzy Evaluation Matrix with respect to the goal

This converted matrix was normalized by using equation (15) as follows:

$$a_{12} = (2.46,3.743,5.339)/5.339 = (0.461,0.701,1)$$

Similarly other normalized a_{ij} values of Table 23 were also calculated (Table 24).

	C ₁	C ₂	C ₃	C ₄	C ₅
C ₁	(1,1,1)	(0.461,0.701,1)	(0.426,0.607,1)	(0.418,0.607,1)	(0.42,0.651,1)
C ₂	(0.461,0.657,1)	(1,1,1)	(0.398,0.613,1)	(0.414,0.631,1)	(0.427,0.677,1)
C ₃	(0.426,0.703,1)	(0.398,0.649,1)	(1,1,1)	(0.422,0.619,1)	(0.427,0.684,1)
C ₄	(0.418,0.689,1)	(0.414,0.655,1)	(0.422,0.682,1)	(1,1,1)	(0.432,0.701,1)
C ₅	(0.42,0.646,1)	(0.427,0.631,1)	(0.427,0.625,1)	(0.432,0.616,1)	(1,1,1)

Table 24. Normalized Fuzzy Evaluation Matrix with respect to the goal

The important weights of each criteria were obtained by using equation (16). Thus

the weight of the criteria C₁ is calculated as $W_1 = (0.109,0.198,0.371)$

the weight of the criteria C₂ is calculated as $W_2 = (0.108,0.198,0.371)$

the weight of the criteria C₃ is calculated as $W_3 = (0.107,0.203,0.371)$

the weight of the criteria C₄ is calculated as $W_4 = (0.107,0.207,0.371)$

the weight of the criteria C₅ is calculated as $W_5 = (0.108,0.195,0.371)$

In a similar manner the normalized comparison matrices for sub criteria with respect to each criteria are constructed, as shown below

$$\begin{array}{cc} & \begin{array}{c} C_{11} \\ C_{12} \end{array} \\ \begin{array}{c} C_{11} \\ C_{12} \end{array} & \left[\begin{array}{cc} (1,1,1) & (0.421,0.631,1) \\ (0.421,0.667,1) & (1,1,1) \end{array} \right] \end{array}$$

Table 25.Sub-criteria matrix with respect to C_1

The important weights of the sub criteria with respect to the criteria C_1 are as follows:

the weight of the sub criteria C_{11} is calculated as $W_{11} = (0.355,0.495,0.704)$

the weight of the sub criteria C_{12} is calculated as $W_{12} = (0.355,0.505,0.704)$

$$\begin{array}{cc} & \begin{array}{c} C_{21} \\ C_{22} \end{array} \\ \begin{array}{c} C_{21} \\ C_{22} \end{array} & \left[\begin{array}{cc} (1,1,1) & (0.433,0.622,1) \\ (0.433,0.696,1) & (1,1,1) \end{array} \right] \end{array}$$

Table 26.Sub-criteria matrix with respect to C_2

The important weights of the sub criteria with respect to the criteria C_2 are as follows:

the weight of the sub criteria C_{21} is calculated as $W_{21} = (0.358,0.489,0.698)$

the weight of the sub criteria C_{22} is calculated as $W_{22} = (0.358,0.511,0.698)$

$$\begin{array}{ccc} & \begin{array}{c} C_{31} \\ C_{32} \\ C_{33} \end{array} \\ \begin{array}{c} C_{31} \\ C_{32} \\ C_{33} \end{array} & \left[\begin{array}{ccc} (1,1,1) & (0.458,0.619,1) & (0.427,0.634,1) \\ (0.458,0.741,1) & (1,1,1) & (0.484,0.791,1) \\ (0.427,0.674,1) & (0.484,0.613,1) & (1,1,1) \end{array} \right] \end{array}$$

Table 27.Sub-criteria matrix with respect to C_3

The important weights of the sub criteria with respect to the criteria C_3 are as follows:

the weight of the sub criteria C_{31} is calculated as $W_{31} = (0.209,0.319,0.523)$

the weight of the sub criteria C_{32} is calculated as $W_{32} = (0.216,0.358,0.523)$

the weight of the sub criteria C_{33} is calculated as $W_{33} = (0.212,0.323,0.523)$

	C ₄₁	C ₄₂	C ₄₃
C ₄₁	(1,1,1)	(0.458,0.619,1)	(0.461,0.607,1)
C ₄₂	(0.458,0.741,1)	(1,1,1)	(0.468,0.616,1)
C ₄₃	(0.472,0.779,1)	(0.465,0.76,1)	(1,1,1)

Table 28.Sub-criteria matrix with respect to C₄

The important weights of the sub criteria with respect to the criteria C₄ are as follows:

the weight of the sub criteria C₄₁ is calculated as $W_{41} = (0.213,0.313,0.519)$

the weight of the sub criteria C₄₂ is calculated as $W_{42} = (0.214,0.331,0.519)$

the weight of the sub criteria C₄₃ is calculated as $W_{43} = (0.215,0.357,0.519)$

	C ₅₁	C ₅₂	C ₅₃
C ₅₁	(1,1,1)	(0.452,0.613,1)	(0.411,0.607,1)
C ₅₂	(0.452,0.737,1)	(1,1,1)	(0.418,0.616,1)
C ₅₃	(0.411,0.677,1)	(0.418,0.679,1)	(1,1,1)

Table 29.Sub-criteria matrix with respect to C₅

The important weights of the sub criteria with respect to the criteria C₅ are as follows:

the weight of the sub criteria C₅₁ is calculated as $W_{51} = (0.207,0.433,0.539)$

the weight of the sub criteria C₅₂ is calculated as $W_{52} = (0.208,0.433,0.539)$

the weight of the sub criteria C₅₃ is calculated as $W_{53} = (0.203,0.433,0.539)$

In the next step of the decision procedure, the alternatives under each sub criteria are compared and the normalized comparison matrices for the alternatives under each sub criteria are constructed. Some of them are shown below.

	A ₁	A ₂	A ₃
A ₁	(1,1,1)	(0.4232,0.6771,1)	(0.3791,0.6219,1)
A ₂	(0.4232,0.625,1)	(1,1,1)	(0.3734,0.6126,1)
A ₃	(0.3791,0.6096,1)	(0.3734,0.6096,1)	(1,1,1)

Table 30.Alternative matrix with respect to C₁₁

The important weights of the alternative with respect to the sub criteria C_{11} are as follows:

the weight of the alternative A_1 is calculated as $W_{A_1,11} = (0.2003, 0.3403, 0.5606)$

the weight of the alternative A_2 is calculated as $W_{A_2,11} = (0.1996, 0.3312, 0.5606)$

the weight of the alternative A_3 is calculated as $W_{A_3,11} = (0.1947, 0.3285, 0.5606)$

	A_1	A_2	A_3
A_1	(1,1,1)	(0.42,0.6686,1)	(0.4015,0.6489,1)
A_2	(0.42,0.6281,1)	(1,1,1)	(0.4015,0.6424,1)
A_3	(0.4015,0.6188,1)	(0.4015,0.625,1)	(1,1,1)

Table 31. Alternative matrix with respect to C_{12}

The important weights of the alternative with respect to the sub criteria C_{12} are as follows:

the weight of the alternative A_1 is calculated as $W_{A_1,12} = (0.2024, 0.3392, 0.5509)$

the weight of the alternative A_2 is calculated as $W_{A_2,12} = (0.2024, 0.3323, 0.5509)$

the weight of the alternative A_3 is calculated as $W_{A_3,12} = (0.2003, 0.3284, 0.5509)$

	A_1	A_2	A_3
A_1	(1,1,1)	(0.3975,0.6521,1)	(0.4107,0.6126,1)
A_2	(0.3975,0.6096,1)	(1,1,1)	(0.3867,0.6065,1)
A_3	(0.4107,0.6703,1)	(0.387,0.6376,1)	(1,1,1)

Table 32. Alternative matrix with respect to C_{21}

The important weights of the alternative with respect to the sub criteria C_{21} are as follows:

the weight of the alternative A_1 is calculated as $W_{A_1,21} = (0.2009, 0.3336, 0.5566)$

the weight of the alternative A_2 is calculated as $W_{A_2,21} = (0.1982, 0.3264, 0.5566)$

the weight of the alternative A_3 is calculated as $W_{A_3,21} = (0.1997, 0.34, 0.5566)$

Using similar calculations, the weight vectors of the alternatives with respect to each sub criteria are also calculated and synthesized. The results are shown in Table 33.

Criteria C ₁					
Weight	(0.109,0.198,0.371)				
Sub criteria	C ₁₁		C ₁₂		
Weight	(0.355,0.495,0.704)		(0.355,0.505,0.704)		
Alternative	Alternative weight		Local weight	Alternative weight with respect to C ₁	
A ₁	(0.1947,0.3285,0.5606)		(0.2003,0.3284,0.5509)	(0.143, 0.3397, 0.7825)	(0.0156, 0.0673,0.2903)
A ₂	(0.1996,0.3312,0.5606)		(0.2024,0.3323,0.5509)	(0.1427, 0.3318, 0.7825)	(0.016, 0.0657, 0.2903)
A ₃	(0.2003,0.3403,0.5606)		(0.2024,0.3392,0.5509)	(0.1402, 0.3284, 0.7825)	(0.0153, 0.065, 0.2903)
Criteria C ₂					
Weight	(0.108,0.198,0.371)				
Sub criteria	C ₂₁		C ₂₂		
Weight	(0.358,0.489,0.698)		(0.358,0.511,0.698)		
Alternative	Alternative weight		Local weight	Alternative weight with respect to C ₂	
A ₁	(0.2009,0.3336,0.5566)		(0.2009,0.3313,0.5542)	(0.1438,0.3324,0.7753)	(0.0155, 0.0658, 0.2877)
A ₂	(0.1982,0.3264,0.5566)		(0.1996,0.3259,0.5542)	(0.1424, 0.3261, 0.7753)	(0.015, 0.0646, 0.2877)
A ₃	(0.1997,0.34,0.5566)		(0.201,0.3428,0.5542)	(0.1435, 0.3414, 0.7753)	(0.0155, 0.0676, 0.2877)
Criteria C ₃					
Weight	(0.107,0.203,0.371)				
Sub criteria	C ₃₁	C ₃₂	C ₃₃		
Weight	(0.209,0.319,0.523)	(0.216,0.358,0.523)	(0.212,0.323,0.523)		
Alternative	Alternative weight		Local weight	Alternative weight with respect to C ₃	
A ₁	(0.1997,0.3365,0.5597)	(0.1963,0.3345,0.5689)	(0.2035,0.329,0.552)	(0.1273, 0.3334, 0.879)	(0.0136, 0.0677, 0.3261)
A ₂	(0.1982,0.3299,0.5597)	(0.1959,0.3248,0.5689)	(0.2027,0.3427,0.552)	(0.1267, 0.3358, 0.879)	(0.014, 0.0682, 0.3261)
A ₃	(0.1977,0.3336,0.5597)	(0.1937,0.3306,0.5689)	(0.1977,0.3283,0.552)	(0.1251, 0.3308, 0.879)	(0.0134, 0.0672, 0.3261)
Criteria C ₄					
Weight	(0.107,0.207,0.371)				
Sub criteria	C ₄₁	C ₄₂	C ₄₃		
Weight	(0.213,0.313,0.519)	(0.214,0.331,0.519)	(0.215,0.357,0.519)		
Alternative	Alternative weight		Local weight	Alternative weight with respect to C ₄	
A ₁	(0.1978,0.3375,0.5661)	(0.2,0.3385,0.5622)	(0.1978,0.3372,0.5648)	(0.1275, 0.3381, 0.8787)	(0.0136, 0.07, 0.326)
A ₂	(0.1973,0.3328,0.5661)	(0.1989,0.3324,0.5622)	(0.1975,0.3337,0.5648)	(0.1271, 0.3333, 0.8787)	(0.014, 0.069, 0.326)
A ₃	(0.1938,0.3296,0.5661)	(0.194,0.3291,0.5622)	(0.1948,0.3291,0.5648)	(0.1247, 0.3296, 0.8787)	(0.0133, 0.0682, 0.326)
Criteria C ₅					
Weight	(0.108,0.195,0.371)				
Sub criteria	C ₅₁	C ₅₂	C ₅₃		
Weight	(0.203,0.433,0.539)	(0.208,0.433,0.539)	(0.207,0.433,0.539)		
Alternative	Alternative weight		Local weight	Alternative weight with respect to C ₅	
A ₁	(0.2015,0.3322,0.5582)	(0.2009,0.3358,0.5578)	(0.2008,0.3386,0.5554)	(0.1243, 0.4359, 0.9009)	(0.0134, 0.085, 0.3342)
A ₂	(0.1992,0.338,0.5582)	(0.2007,0.3377,0.5578)	(0.2006,0.3322,0.5554)	(0.1237, 0.4364, 0.9009)	(0.013, 0.0851, 0.3342)
A ₃	(0.1964,0.3299,0.5582)	(0.196,0.3266,0.5578)	(0.1988,0.3292,0.5554)	(0.1218,0.4268,0.9009)	(0.0132, 0.0832, 0.3342)
Global Weight					
	A ₁		A ₂		A ₃
	(0.0718, 0.3557, 1.5643)		(0.071, 0.3525, 1.5643)		(0.0707, 0.3512, 1.5643)

Table 33. Obtained Results

Alternatives	$x_0(\tilde{A})$	$y_0(\tilde{A})$	$R(\tilde{A})$
A ₁	0.6639	0.3333	0.743
A ₂	0.663	0.3333	0.742
A ₃	0.6621	0.3333	0.741

Table 34. Ranking of the Alternatives

After obtaining Fuzzy important weights, the last step was performed, and the fuzzy weights are ranked using centroid-based distance method. The obtained results are shown in Table 34. Based on this method alternative 1(P.M.G Higher Secondary School, College Road, Palakkad) is found to be the best school.