Maniu

Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – June / July 2021
II Semester

Class : I UG — **Time : 3 Hours**
Major : Special Education and Mathematics — **Max. Marks : 100**

18BSMC03 Calculus

Part A **10 x 1 = 10**
Choose the Correct Answer

1. The first derivative of $y = \sin^2 x$ is — CO1K1
   a. $2\sin x$ b. $2\sin^2 x$ c. $2\, sinx \cos x$ d. $2\cos x$

2. $D^n(\, e^{2x})$ — CO1K4
   a. $2n\, e^{2x}$ b. $2n\, e^{2nx}$ c. $2\, e^{2x}$ d. $2^n\, e^{2x}$

3. If $\tan u = \frac{x^3+y^3}{x-y}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is — CO2K4
   a. $\sin 2u$ b. $\sin u$ c. $\cos u$ d. $cos2u$

4. If $V = (x^2+y^2+z^2)^{-\frac{1}{2}}$ then $\frac{\partial^2 V}{\partial x^2}$ is — CO2K1
   a. $\frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$ b. $\frac{2x^2-y-z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$ c. $\frac{2x}{(x^2+y^2+z^2)^{\frac{5}{2}}}$ d. 0

5. Cartesian formula for the radius of curvature is — CO3K1
   a. $\frac{[1+y_1^2]^2}{y_2}$ b. $\frac{[1+y_1^2]^2}{y_2^2}$ c. $\frac{[1+y_1^2]^{\frac{3}{2}}}{y_2}$ d. $\frac{[1+y_1^2]^{\frac{3}{2}}}{y^2}$

6. The co-ordinates of the centre of curvature of the curve $xy = c^2$ at the point (c, c) is — C03K2
   a. $(2c,\ 2c)$ b. $(2c,\ 2c^2)$ c. $(0,\ 2c)$ d. $(c,\ c)$

7. $\int_0^a \int_0^x (x^2+y^2)dydx =$ — CO4K4
   a. $\frac{a^8}{3}$ b. $\frac{a^4}{3}$ c. $\frac{a^4}{8}$ d. $\frac{a^4}{4}$

8. Y- coordinate of centre of gravity is given by — CO4K4
   a. $\int\int x\, dx\, dy$ b. $\frac{\int\int x\, dy\, dx}{\int\int dx\, dy}$ c. $\frac{\int\int x\, dx\, dy}{\int\int y\, dx\, dy}$ d. $\frac{\int\int y\, dx\, dy}{\int\int dx\, dy}$

9. If $x = u+v,\ y = u-v$ then $\frac{\partial(x,y)}{\partial(u,v)}$ is — CO5K2
   a. $\frac{1}{2}$ b. $-\frac{1}{2}$ c. -2 d. 0

10. If $x = u(1-v),\ y = uv(1-w),\ z = uvw$ then $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ is — CO5K2
    a. u^2v b. v^2u c. uv d. u^2

Part B 5 x 6 = 30

Answer ALL questions
Each answer should not exceed 400 words or two pages

11.a. Find y_n when $y = \frac{3}{(x+1)(2x-1)}$. CO1K3

(or)

11.b. Find n^{th} differential coefficient of $\sin^3 x$. CO1K3

12.a If $V = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that $\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} = \frac{3}{x+y+z}$. CO2K3

(or)

12.b. If $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$, evaluate $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2}$. CO2K4

13.a. Show that the radius of curvature at any point of the catenary $y = c \cosh \frac{x}{c}$ is equal to the length of the portion of the normal intercepted between the curve and the axis of x. CO3K5

(or)

13.b. Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at $(2, 1)$. CO3K3

14.a. By changing the order of integration, evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dx \, dy$. CO4K4

(or)

14.b. Evaluate $\int\int r\sqrt{a^2 - r^2} \, dr \, d\theta$ over the upper half of the circle $r = a \cos\theta$. CO4K5

15.a. Transform $dx \, dy \, dz$ into spherical polar co-ordinates as $-r^2 \sin\theta \, dr \, d\phi \, d\theta$. CO5K6

(or)

15.b. Prove that $\int\int_D e^{-x^2-y^2} dx \, dy = \frac{1}{4} \pi \left(1 - e^{-R^2}\right)$ where D is the region $x \geq 0, y \geq 0$ and $x^2 + y^2 \leq R^2$ CO5K6

Part C 5 x 12 = 60

Answer ALL questions
Each answer should not exceed 800 words or four pages

16.a. Find the n^{th} differential coefficient of $\cos^5 \theta \sin^7 \theta$. CO1K5

(or)

16.b. If $y = \sin(m \sin^{-1} x)$, prove that
$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. CO1K5

17.a. i. If $f(x, y)$ is a homogeneous function of degree n, then prove that
$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = nf$. CO2K6

ii. Prove that $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = 3f$, if $f = x^3 + y^3 + z^3 + 3xyz$. CO2K6

(or)

17.b. If $z = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that CO2K6

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

18.a. Discuss the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. CO3K5

(or)

18.b. Show that the evolute of the cycloid $x=a(\theta-sin\theta); y=a(1-cos\theta)$ is another cycloid. CO3K5

19.a. Evaluate $\int\int\int xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$. CO4K5

(or)

19.b. Find the area the cardiod $r=a(1+cos\theta)$. CO4K6

20.a. i. Prove that $\frac{\partial(u,v)}{\partial(x,y)}.\frac{\partial(x,y)}{\partial(\chi,\eta)}=\frac{\partial(u,v)}{\partial(\chi,\ \eta)}$, where u,v are functions of x,y and x,y are themselves functions of $\chi,\ \eta$.

ii. Prove that $\frac{\partial(u,v)}{\partial(x,y)}.\frac{\partial(x,y)}{\partial(u,v)}=1$. CO5K5

(or)

20.b. Evaluate $\int\int_R xy\ dx\,dy$, where R is the region in the first quadrant bounded by hyperbolas $x^2-y^2=a^2$ and $x^2-y^2=b^2$ and the circles $x^2+y^2=c^2$ and $x^2+y^2=a^2(0<a<b<c<d)$. CO5K5

****