

CHAPTER - IX

CHAPTER IX

FUZZY SEMI-PRE-GENERALIZED IRRESOLUTE FUNCTIONS

Definition: 9.1

A fuzzy set α of (X, τ) is called **fuzzy semi-pre-generalized closed** (briefly, **Fspg-closed**) if $\text{spcl}(\alpha) \leq \lambda$, whenever $\alpha \leq \lambda$ and λ is Fs-open in X .

By FSPGC (X, τ) , we denote the family of all fuzzy semi-pre-generalized closed sets of the fts X .

Definition: 9.2

A fuzzy set α of (X, τ) is called **fuzzy semi-pre-generalized open** (briefly, **Fspg-open**) iff $(1 - \alpha)$ is Fspg - closed in X . That is, α is Fspg - open iff $\delta \leq \text{spint}(\alpha)$ whenever $\delta \leq \alpha$ and δ is a Fs-closed set in X .

By FSPGO (X, τ) , we denote the family of all fuzzy semi-pre-generalized open sets of the fts X .

Definition: 9.3

A fuzzy set α of (X, τ) is called **fuzzy generalized semi-pre-closed** (briefly, **Fgsp-closed**) if $\text{spcl}(\alpha) \leq \lambda$, whenever $\alpha \leq \lambda$ and λ is a Fuzzy open set in X .

Definition: 9.4

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **fuzzy semi-pre-generalized continuous** (briefly, **Fspg-continuous**) if $f^{-1}(v)$ is Fspg-closed in (X, τ) for every fuzzy closed set v of (Y, σ) .

Definition: 9.5

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **fuzzy semi-pre-generalized irresolute (briefly, Fspg-irresolute)** if $f^{-1}(v)$ is Fspg-closed in (X, τ) for every Fspg-closed set v of (Y, σ) .

Definition: 9.6

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **fuzzy generalized semi-pre-continuous (briefly, Fgsp-continuous)** if $f^{-1}(v)$ is Fgsp-closed in X for every fuzzy closed set v in Y .

Theorem: 9.7

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be Fsg-continuous. Then f is Fspg-continuous.

Proof:

Let v be a fuzzy closed set of Y . Since f is Fsg-continuous, then $f^{-1}(v)$ is Fsg-closed in X . Since every Fsg-closed set is Fspg-closed, then $f^{-1}(v)$ is Fspg-closed. Thus, f is Fspg-continuous.

Remark: 9.8

Following example shows that the converse of the above theorem is not true in general.

Example: 9.9

Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$. Fuzzy sets α and β are defined as:

$$\alpha(a) = 0.1, \alpha(b) = 0.2, \alpha(c) = 0.7;$$

$$\beta(x) = 0.1, \beta(y) = 0.8, \beta(z) = 0.5.$$

Let $\tau = \{0, \alpha, 1\}$ and $\sigma = \{0, \beta, 1\}$. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$ is Fspg-continuous but not Fsg-continuous.

Theorem: 9.10

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be Fspg-irresolute, then f is Fspg-continuous.

Proof:

Proof is immediate as every fuzzy closed set is Fspg-closed and f is Fspg-irresolute map.

Remark: 9.11

The converse of the above theorem is not true in general as it can be seen from the following example.

Example: 9.12

Let $X = \{a, b\}$, $Y = \{x, y\}$. The fuzzy set α is defined as:

$\alpha(a) = 0.3$, $\alpha(b) = 0.7$. Let $\tau = \{0, \alpha, 1\}$ and $\sigma = \{0, 1\}$. Then the mapping

$f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is Fspg-continuous but not Fspg-irresolute.

Theorem: 9.13

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be Fspg-continuous. Then f is Fgsp-continuous but not conversely.

Proof:

Let ν be a fuzzy closed set of Y . Since f is Fspg-continuous, then $f^{-1}(\nu)$ is a Fspg-closed set of X . Since every Fspg-closed set is Fgsp-closed, $f^{-1}(\nu)$ is also a Fgsp-closed set of X . Thus, f is Fgsp-continuous.

Following example shows that the converse is not true in general:

Example: 9.14

Let $X = \{a, b\}$, $Y = \{x, y\}$.

Fuzzy sets α and β are defined as:

$$\alpha(a) = 0.3, \alpha(b) = 0.7; \beta(x) = 0.3, \beta(y) = 0.4.$$

Let $\tau = \{0, \alpha, 1\}$ and $\sigma = \{0, \beta, 1\}$. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is F_{gsp}-continuous but not F_{spg}-continuous.

Theorem: 9.15

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is F_{spg}-continuous iff inverse image of each fuzzy open set of Y is F_{spg}-open in X .

Proof:

It is obvious because $f^{-1}(1-\lambda) = 1-f^{-1}(\lambda)$ for each fuzzy open set λ of Y .

Theorem: 9.16

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is F_{spg}-continuous then for each fuzzy point x_p of X and each $\alpha \in \sigma$ such that $f(x_p) \in \alpha$, there exists a F_{spg}-open set β of X such that $x_p \in \beta$ and $f(\beta) \leq \alpha$.

Proof:

Let x_p be a fuzzy point of X and $\alpha \in \sigma$ such that $f(x_p) \in \alpha$. Put $\beta = f^{-1}(\alpha)$. Then by hypothesis β is a F_{spg}-open set of X such that $x_p \in \beta$ and $f(\beta) = f(f^{-1}(\alpha)) \leq \alpha$.

Theorem: 9.17

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is F_{spg}-continuous, then for each fuzzy point x_p of X and

each $\alpha \in \sigma$ such that $f(x_p) \in \alpha$, there exists a Fspg-open set β of X such that $x_p \in \beta$ and $f(\beta) \leq \alpha$.

Proof:

Let $x_p \in X$ and $\alpha \in \sigma$ such that $f(x_p) \in \alpha$. Put $\beta = f^{-1}(\alpha)$. Then by hypothesis β is a Fspg-open set of X such that $x_p \in \beta$ and $f(\beta) = f(f^{-1}(\alpha)) \leq \alpha$.

Definition: 9.18

A fuzzy set α of (X, τ) is called **fuzzy generalized closed (briefly, Fg- closed)** if $\text{cl } \alpha \leq \lambda$, whenever $\alpha \leq \lambda$ and λ is a fuzzy open set in X .

Definition: 9.19

A fuzzy topological space (X, τ) is **fuzzy $T_{1/2}$ - space** if every Fg-closed set in X is fuzzy closed.

Definition: 9.20

If every Fspg-closed set in X is Fsp-closed in X , then the space X is called

Fsp $T_{1/2}$ - space.

Definition: 9.21

A fuzzy set α of (X, τ) is called **fuzzy semi-pre-open (briefly, Fsp-open)** if $\alpha \leq \text{cl int}(\text{cl}(\alpha))$ and a **fuzzy semi-pre-closed (briefly, Fsp-closed)** if

$$\text{Int}(\text{cl}(\text{Int}(\alpha))) \leq \alpha .$$

Definition: 9.22

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be **fuzzy M-semi-pre-continuous** if $f^{-1}(v)$ is Fsp-open in X , for each Fsp-open set v in Y .

Theorem: 9.23

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be any two functions. Then,

- (i) $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is Fspg-continuous, if g is fuzzy continuous and f is Fspg-continuous.
- (ii) $g \circ f$ is Fspg-irresolute, if f and g both are Fspg-irresolute.
- (iii) $g \circ f$ is Fspg-continuous, if g is Fspg-continuous and f is Fspg-irresolute.
- (iv) Let Y be a Fsp $T_{1/2}$ -space. Then, $g \circ f$ is Fspg-continuous, if g is Fspg-continuous and f is fuzzy M-semi-pre-continuous.

Definition: 9.24

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be **fuzzy M-semi-pre-closed** if $f(\nu)$ is Fspg-closed in Y , for every Fspg-closed set ν in X .

Theorem: 9.25

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be fuzzy irresolute and fuzzy M-semi-pre-closed. Then for every Fspg-closed set α of X , $f(\alpha)$ is a Fspg-closed in Y .

Proof:

Let α be a Fspg-closed set of X . Let ν be a fuzzy semi-open set of Y containing $f(\alpha)$. Since f is fuzzy irresolute, $f^{-1}(\nu)$ is a fuzzy semi-open set of X . As $\alpha \leq f^{-1}(\nu)$ and α is a Fspg-closed in X , then $\text{spcl}(\alpha) \leq f^{-1}(\nu)$ implies that $f(\text{spcl}(\alpha)) \leq \nu$. Since f is fuzzy M-semi-pre-closed, then $f(\text{spcl}(\alpha)) = \text{spcl}(f(\text{spcl}(\alpha)))$. Then, $\text{spcl}(f(\alpha)) \leq \text{spcl}(f(\text{spcl}(\alpha))) = f(\text{spcl}(\alpha)) \leq \nu$.

Therefore, $f(\alpha)$ is a Fspg-closed set in Y .

Theorem: 9.26

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be onto Fspg- irresolute and fuzzy M-semi-pre-closed. If X is Fsp $T_{1/2}$ -space, then (Y, σ) is also Fsp $T_{1/2}$ - space.

Proof:

Let α be a Fspg-closed set of Y . Since f is Fspg- irresolute, then $f^{-1}(\alpha)$ is Fspg-closed set in X . As X is a Fsp $T_{1/2}$ - space and hence $f^{-1}(\alpha)$ is Fsp-closed in X . Again, f is a fuzzy M-semi-pre-closed map, $f(f^{-1}(\alpha))$ is a Fsp-closed set in Y . Since f is onto, $f(f^{-1}(\alpha)) = \alpha$. Thus, α is a Fsp-closed set in Y or equivalently, (Y, σ) is Fsp $T_{1/2}$ - space.

Theorem: 9.27

If the bijective mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy pre-semi-open and fuzzy M-semi-pre-continuous, then f is Fspg-irresolute.

Proof:

Let ν be a Fspg-closed set in Y and let $f^{-1}(\nu) \leq \lambda$ where λ is a fuzzy semi - open set in X . Clearly, $\nu \leq f(\lambda)$. Since f is a fuzzy pre-semi-open map, $f(\lambda)$ is a fuzzy semi-open set in Y and ν is a Fspg-closed set in Y then $\text{spcl}(\nu) \leq f(\lambda)$ and thus $f^{-1}(\text{spcl}(\nu)) \leq \lambda$. Again, f is a fuzzy M-semi-pre-continuous, and $\text{spcl}(\nu)$ is Fsp-closed set, then $f^{-1}(\text{spcl}(\nu))$ is a Fsp-closed set in X . Thus, $\text{spcl}(f^{-1}(\nu)) \leq \text{spcl}(f^{-1}(\text{spcl}(\nu))) = f^{-1}(\text{spcl}(\nu)) \leq \lambda$. So $f^{-1}(\nu)$ is a Fspg-closed set in X . Hence, f is Fspg-irresolute map.

SUMMARY AND CONCLUSION

Continuous functions and irresolute functions play a vital role in the study of topological spaces and fuzzy topological spaces. The main aim of this thesis is to study some interesting generalizations of fuzzy irresolute functions in fuzzy topological spaces.

The first chapter deals with preliminary definitions and notations.

Chapter II deals with fuzzy irresolute functions. In this chapter, the concept of fuzzy irresolute functions which is stronger than the concept of semi-continuous functions are introduced and studied. An interesting characterization of fuzzy irresolute functions is established.

Chapter III deals with fuzzy semi-irresolute and strongly irresolute functions. Several characterizations of these functions along with their relationships with certain other types of functions are investigated. Mutual correlations among fuzzy irresolute, fuzzy strongly irresolute, fuzzy continuous and fuzzy semi-continuous functions are discussed with examples.

In chapter IV, two new classes of functions, called fuzzy completely irresolute and fuzzy weakly completely irresolute functions are introduced and their characterizations, examples, compositions and their relationships with other functions are studied.

In chapter V, interesting characterizations of fuzzy δ -semi irresolute functions are given.

Fuzzy semi-generalized irresolute functions and fuzzy generalized semi-irresolute functions are studied in chapter VI. Relationship with other functions, composition theorems are established.

Chapter VII deals with fuzzy approximately irresolute and contra fuzzy irresolute functions. Some of their basic properties are studied. The conditions under which maps and inverse maps preserve fuzzy semi-generalized closed sets are obtained. Perfectly contra irresolute maps are also introduced.

A characterization of fuzzy semi - $T_{1/2}$ spaces using the concepts of fuzzy approximately irresolute maps and fuzzy approximately semi closed maps is also established.

Chapter VIII deals with fuzzy SP- irresolute functions. The concepts of fuzzy SP-irresolute continuous, fuzzy SP- irresolute open and fuzzy SP-irresolute closed mappings are introduced and studied. Several characteristic properties of these functions are proved.

Finally in chapter IX, fuzzy semi-pre-generalized irresolute functions are studied with interesting characterizations, examples, composition theorems. Their relationships with other functions are also studied in detail.

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