CHAPTER - VIII

CHAPTER VIII

APPLICATION OF EXTENT ANALYSIS METHOD ON FUZZY ANALYTIC HIERARCHY PROCESS

Decision Making is the act of choosing between two or more courses of action. Decision-making can also be regarded as a problem-solving activity terminated by a solution deemed to be satisfactory among several alternative possibilities. It is, therefore, a reasoning or emotional process which can be rational or irrational and can be based on explicit assumptions or implicit assumption.

People often find it hard to make decisions in a complex, subjective situation with more than a few realistic options. So we need is a systematic, organized way to evaluate our choices and figure out which one offers the best solution to our problem. There are processes and techniques to improve decision-making and the quality of decisions.

Nowadays, decision making is a problem of every common man to take right decision on many routine affairs like education for children, food, transportation, purchase of durables, healthcare, shelter and so on. Here in this thesis the author consider the problem of selecting the best school for the children.

Parents have a growing array of options in choosing a school, though the extent of the options varies from place to place. Parents can exercise choice in many ways. Parents can choose from neighborhood schools, or other public schools of choice, or transfer their child to another public school (in or out of district). They can also select a private school (religious or secular) or teach their child at home. *Choosing a School for Your Child* offers step-by-step procedure on how to choose among the schools available to your child. Generally parents consider various factors to select the best school for their children. The author of this thesis identified many such important factors and used the same to develop a mathematical model for decision making regarding the selection of right school for the children using Extent Analysis Method on Fuzzy Analytic Hierarchy Process. The details of this model is illustrated in this chapter.

Based on the pilot study, the author of this thesis identified five major criteria for developing a model for the selection of the best school by the parents for their children. Further, care was taken to enlist possible sub criteria for each major criteria, which are considered by them as vital for achieving the objective. The details are presented below.

1) **C_1 : Distance and Transport**

   Sub Criteria

   C_{11} : Transportation Offered

   C_{12} : Location of the School

2) **C_2 : Cost**

   Sub Criteria

   C_{21} : Admission Fees

   C_{22} : Fees Structure

3) **C_3 : Staff and Curriculum**

   Sub Criteria

   C_{31} : Competent Staff

   C_{32} : Teaching and coaching

   C_{33} : Extra Curricular Activities

4) **C_4 : Atmosphere**

   Sub Criteria

   C_{41} : Infrastructure

   C_{42} : Campus Discipline

   C_{43} : Facilities and Security

5) **C_5 : Administration**

   Sub Criteria

   C_{51} : Academic Performance

   C_{52} : Staff and Student Welfare

   C_{53} : Reporting to parents

After the initial screening, three schools listed below were considered as alternatives and an attempt has been made by the author of this thesis to develop a model to select the best one based on the above criteria.

1) A_1 : P.M.G HIGHER SECONDARY SCHOOL, COLLEGE ROAD, PALAKKAD

1) A_2 : BHARATH MATHA HIGHER SECONDARY SCHOOL, CHANDRANAGAR, PALAKKAD

2) A_3 : VYASA VIDYA PEETHOM SCHOOL, KALLEKAD, PALAKKAD.

The selection hierarchy of the best school for the children is shown in Fig.7. In this four level hierarchy, the overall objective is placed in level 1, the five criteria in level 2, thirteen sub criteria in level 3, and three alternatives in level 4.

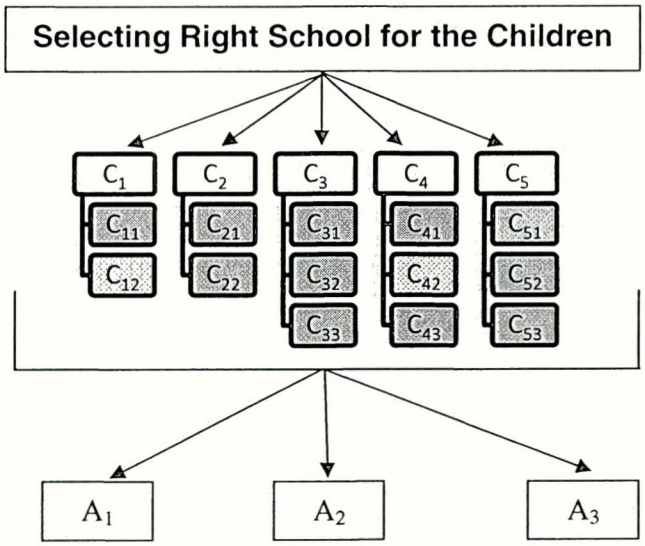

Fig 7. The hierarchy of selecting the right school for the children.

After building the hierarchy, the pairwise comparison of the importance of one criterion over the others, one sub criterion over the others and one alternative over the others were estimated with the help of a pre-tested questionnaire. A sample of 100 parents in Palakkad city was selected by adopting convenient sampling technique and the questionnaire was administered to the parents. The data collected were edited and tabulated for further analysis. Five point scaling technique (Equally Important, Weakly More Important, Strongly More Important, Very Strongly More Important, Absolutely More Important) was adopted to assess the degree of importance of one criterion, sub criterion, or alternative over another.

Extent Analysis Method on Fuzzy AHP

In this Extent analysis method, by using the Triangular Fuzzy Conversion Scale given in Table 7, the pairwise comparison matrices (Triangular Fuzzy Number Matrices $A_i, i = 1, \dots 100$) for 100 respondents are constructed. By using the definition of addition of Triangular Fuzzy Number Matrices a new matrix $D = A_1 + A_2 + \cdots + A_{100}$ is arrived. Then by taking average of each element in the matrix D (the average is based only on the number of responses) the aggregated fuzzy evaluation matrix is constructed (Table 9).

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(1.08,1.58,2.08)	(0.86,1.36,1.86)	(0.93,1.43,1.93)	(1.38,1.87,2.36)
C_2	(0.48,0.63,0.93)	(1,1,1)	(1.06,1.56,2.06)	(1.17,1.67,2.17)	(1.65,2.15,2.65)
C_3	(0.54,0.74,1.16)	(0.49,0.64,0.94)	(1,1,1)	(1.13,1.63,2.13)	(1.88,2.38,2.88)
C_4	(0.52,0.7,1.08)	(0.46,0.6,0.85)	(0.47,0.61,0.88)	(1,1,1)	(1.97,2.47,2.97)
C_5	(0.42,0.53,0.72)	(0.38,0.47,0.61)	(0.35,0.42,0.53)	(0.34,0.4,0.51)	(1,1,1)

Table 9. Fuzzy Evaluation Matrix with respect to the goal

Then applying formula (24),

$$S_1 = (5.25,7.24,9.23) \otimes \left(\frac{1}{36.3}, \frac{1}{28.84}, \frac{1}{22.56}\right) = (0.14,0.25,0.41)$$

$$S_2 = (5.36,7.01,8.81) \otimes \left(\frac{1}{36.3}, \frac{1}{28.84}, \frac{1}{22.56}\right) = (0.147,0.24,0.39)$$

$$S_3 = (5.04,6.39,8.11) \otimes \left(\frac{1}{36.3}, \frac{1}{28.84}, \frac{1}{22.56}\right) = (0.139,0.22,0.36)$$

$$S_4 = (4.42,5.38,6.78) \otimes \left(\frac{1}{36.3}, \frac{1}{28.84}, \frac{1}{22.56}\right) = (0.12,0.19,0.3)$$

$$S_5 = (2.49,2.82,3.37) \otimes \left(\frac{1}{36.3}, \frac{1}{28.84}, \frac{1}{22.56}\right) = (0.07,0.1,0.15)$$

Using formula (26),

$$V(S_1 \geq S_2) = V(S_1 \geq S_3) = V(S_1 \geq S_4) = V(S_1 \geq S_5) = 1$$

$$V(S_2 \geq S_1) = \frac{0.14 - 0.39}{(0.24 - 0.39) - (0.25 - 0.14)} = 0.96$$

$$V(S_2 \geq S_3) = V(S_2 \geq S_4) = V(S_2 \geq S_5) = 1$$

$$V(S_3 \geq S_1) = \frac{0.14 - 0.36}{(0.22 - 0.36) - (0.25 - 0.14)} = 0.88$$

$$V(S_3 \geq S_2) = \frac{0.147 - 0.36}{(0.22 - 0.36) - (0.24 - 0.147)} = 0.91$$

$$V(S_3 \geq S_4) = V(S_3 \geq S_5) = 1$$

$$V(S_4 \geq S_1) = \frac{0.14 - 0.3}{(0.19 - 0.3) - (0.25 - 0.14)} = 0.73$$

$$V(S_4 \geq S_2) = \frac{0.147 - 0.3}{(0.19 - 0.3) - (0.24 - 0.14)} = 0.75$$

$$V(S_4 \geq S_3) = \frac{0.139 - 0.3}{(0.19 - 0.3) - (0.22 - 0.139)} = 0.84$$

$$V(S_4 \geq S_5) = 1$$

$$V(S_5 \geq S_1) = \frac{0.14 - 0.15}{(0.1 - 0.15) - (0.25 - 0.14)} = 0.063$$

$$V(S_5 \geq S_2) = \frac{0.147 - 0.15}{(0.1 - 0.15) - (0.24 - 0.147)} = 0.021$$

$$V(S_5 \geq S_3) = \frac{0.139 - 0.15}{(0.1 - 0.15) - (0.22 - 0.139)} = 0.084$$

$$V(S_5 \geq S_4) = \frac{0.12 - 0.15}{(0.1 - 0.15) - (0.19 - 0.12)} = 0.25$$

Thus,

$$d'(\mathrm{C}_1) = \min V(S_1 \geq S_2, S_3, S_4, S_5) = \min(1,1,1,1) = 1$$

$$d'(\mathrm{C}_2) = \min V(S_2 \geq S_1, S_3, S_4, S_5) = \min(0.96,1,1,1) = 0.96$$

$$d'(\mathrm{C}_3) = \min V(S_3 \geq S_1, S_2, S_4, S_5) = \min(0.88,0.91,1,1) = 0.88$$

$$d'(\mathrm{C}_4) = \min V(S_4 \geq S_1, S_2, S_3, S_5) = \min(0.73,0.75,0.84,1) = 0.73$$

$$d'(\mathrm{C}_5) = \min V(S_5 \geq S_1, S_2, S_3, S_4) = \min(0.063,0.021,0.084,0.25) = 0.021$$

Therefore,

$$W' = (1,0.96,0.88,0.73,0.021)^T$$

Via normalization, the weight vectors with respect to the decision criteria C_1, C_2, C_3, C_4, C_5 are obtained as

$$W = (0.28,0.27,0.25,0.2,0.006)^T$$

In a similar manner the aggregated pairwise comparison matrices for sub criteria with respect to each criteria are constructed, which are shown in Tables 10-14.

	C_{11}	C_{12}
C_{11}	(1,1,1)	(1.27,1.77,2.27)
C_{12}	(0.44,0.56,0.79)	(1,1,1)

Table 10. Sub-criteria matrix with respect to C_1

The weight vector from Table 10 is calculated as $W_1 = (0.91,0.088)^T$

	C_{21}	C_{22}
C_{21}	(1,1,1)	(1.03,1.53,2.03)
C_{22}	(0.49,0.65,0.97)	(1,1,1)

Table 11. Sub-criteria matrix with respect to C_2

The weight vector from Table 11 is calculated as $W_2 = (0.71,0.29)^T$

	C_{31}	C_{32}	C_{33}
C_{31}	(1,1,1)	(0.69,1.18,1.68)	(1.38,1.88,2.38)
C_{32}	(0.6,0.85,1.45)	(1,1,1)	(2.18,2.68,3.18)
C_{33}	(0.42,0.53,0.72)	(0.31,0.37,0.46)	(1,1,1)

Table 12. Sub-criteria matrix with respect to C_3

The weight vector from Table 12 is calculated as $W_3 = (0.22,0.24,0.54)^T$

	C_{41}	C_{42}	C_{43}
C_{41}	(1,1,1)	(0.74,1.24,1.74)	(0.58,1.08,1.58)
C_{42}	(0.57,0.81,1.35)	(1,1,1)	(0.62,1.12,1.62)
C_{43}	(0.63,0.93,1.72)	(0.62,0.89,1.61)	(1,1,1)

Table 13. Sub-criteria matrix with respect to C_4

The weight vector from Table 13 is calculated as $W_4 = (0.36,0.32,0.317)^T$

	C_{51}	C_{52}	C_{53}
C_{51}	(1,1,1)	(0.7,1.2,1.7)	(1.07,1.57,2.07)
C_{52}	(0.59,0.83,1.4)	(1,1,1)	(1.15,1.65,2.15)
C_{53}	(0.48,0.64,0.93)	(0.47,0.61,0.87)	(1,1,1)

Table 14. Sub-criteria matrix with respect to C_5

The weight vector from Table 14 is calculated as $W_5 = (0.41,0.39,0.204)^T$

In the next step of the decision procedure, the alternatives under each sub criteria are compared. These results in the matrices are shown below.

	A_1	A_2	A_3
A_1	(1,1,1)	(0.99,1.49,1.99)	(0.97,1.47,1.97)
A_2	(0.5,0.67,1.01)	(1,1,1)	(1.21,1.71,2.21)
A_3	(0.51,0.68,1.03)	(0.45,0.58,0.83)	(1,1,1)

Table 15. Alternative matrix with respect to C_{11}

The weight vector from Table 15 is calculated as $W_{11} = (0.45,0.37,0.18)^T$

	A_1	A_2	A_3
A_1	(1,1,1)	(1.25,1.75,2.25)	(1.24,1.74,2.24)
A_2	(0.44,0.57,0.8)	(1,1,1)	(1.18,1.68,2.18)
A_3	(0.45,0.574,0.81)	(0.46,0.6,0.85)	(1,1,1)

Table 16. Alternative matrix with respect to C_{12}

The weight vector from Table 16 is calculated as $W_{12} = (0.47,0.3,0.23)^T$

	A_1	A_2	A_3
A_1	(1,1,1)	(1.6,2.1,2.6)	(1.07,1.54,2.07)
A_2	(0.38,0.47,0.63)	(1,1,1)	(1.22,1.72,2.22)
A_3	(0.48,0.64,0.93)	(0.45,0.58,0.82)	(1,1,1)

Table 17. Alternative matrix with respect to C_{21}

The weight vector from Table 17 is calculated as $W_{21} = (0.58,0.32,0.098)^T$

	A_1	A_2	A_3
A_1	(1,1,1)	(1.15,1.65,2.15)	(1.43,1.93,2.43)
A_2	(0.47,0.6,0.87)	(1,1,1)	(1.22,1.72,2.22)
A_3	(0.41,0.52,0.69)	(0.45,0.56,0.82)	(1,1,1)

Table 18. Alternative matrix with respect to C_{22}

The weight vector from Table 18 is calculated as $W_{22} = (0.59,0.37,0.044)^T$

	A_1	A_2	A_3
A_1	(1,1,1)	(1.5,2,2.5)	(1.21,1.71,2.21)
A_2	(0.4,0.5,0.67)	(1,1,1)	(1.32,1.82,2.32)
A_3	(0.45,0.58,0.83)	(0.43,0.55,0.76)	(1,1,1)

Table 19. Alternative matrix with respect to C_{31}

The weight vector from Table 19 is calculated as $W_{31} = (0.62,0.36,0.025)^T$

	A_1	A_2	A_3
A_1	(1,1,1)	(1.02,1.52,2.02)	(1.1,1.6,2.1)
A_2	(0.5,0.66,0.98)	(1,1,1)	(1.67,2.17,2.67)
A_3	(0.48,0.63,0.91)	(0.37,0.46,0.6)	(1,1,1)

Table 20. Alternative matrix with respect to C_{32}

The weight vector from Table 20 is calculated as $W_{32} = (0.47,0.43,0.11)^T$

Using similar calculations, the weight vectors of the alternatives with respect to

the sub-criteria C_{33} is calculated as $W_{33} = (0.52,0.46,0.022)^T$

the sub-criteria C_{41} is calculated as $W_{41} = (0.46,0.39,0.15)^T$

the sub-criteria C_{42} is calculated as $W_{42} = (0.46,0.38,0.16)^T$

the sub-criteria C_{43} is calculated as $W_{43} = (0.48,0.4,0.12)^T$

the sub-criteria C_{51} is calculated as $W_{51} = (0.54,0.44,0.022)^T$

the sub-criteria C_{52} is calculated as $W_{52} = (0.52,0.4,0.08)^T$

the sub-criteria C_{53} is calculated as $W_{53} = (0.47,0.39,0.14)^T$

Sub-criterias of C_1						
	C_{11}		C_{12}			Alternative Priority Weight
Weight	0.91		0.088			
Alternative						
A_1	0.45		0.47			0.45
A_2	0.37		0.3			0.36
A_3	0.18		0.23			0.18
Sub-criterias of C_2						
	C_{21}		C_{22}			Alternative Priority Weight
Weight	0.71		0.29			
Alternative						
A_1	0.58		0.59			0.58
A_2	0.32		0.37			0.33
A_3	0.098		0.044			0.08
Sub-criterias of C_3						
	C_{31}	C_{32}	C_{33}			Alternative Priority Weight
Weight	0.22	0.24	0.54			
Alternative						
A_1	0.62	0.47	0.52			0.53
A_2	0.36	0.43	0.46			0.43
A_3	0.025	0.11	0.022			0.04
Sub-criterias of C_4						
	C_{41}	C_{42}	C_{43}			Alternative Priority Weight
Weight	0.36	0.32	0.317			
Alternative						
A_1	0.46	0.46	0.48			0.46
A_2	0.39	0.38	0.4			0.39
A_3	0.15	0.16	0.12			0.14
Sub-criterias of C_5						
	C_{51}	C_{52}	C_{53}			Alternative Priority Weight
Weight	0.41	0.39	0.204			
Alternative						
A_1	0.54	0.52	0.47			0.52
A_2	0.44	0.4	0.39			0.42
A_3	0.022	0.08	0.14			0.07
Main Criteria of the Goal						
	C_1	C_2	C_3	C_4	C_5	Alternative Priority Weight
Weight	0.28	0.27	0.25	0.2	0.006	
Alternative						
A_1	0.45	0.58	0.53	0.46	0.52	**0.51**
A_2	0.36	0.33	0.43	0.39	0.42	0.38
A_3	0.18	0.08	0.04	0.14	0.7	0.114

Table 21. Obtained Results

The combination of priority weights for criteria, sub criteria and alternatives to determine the priority weight for the best school are shown in Table 21. Based on this result alternative 1(P.M.G Higher Secondary School, College Road, Palakkad) which has the highest alternative priority weight 0.51 is found to be the best school.