Chapter V

CHAPTER-V

Supra b-Open Soft Sets in Soft Topological Spaces

Section 5.1

Supra b-Open Soft Sets and Supra b-closed Soft Sets

Definition:5.1.1

Let (X,μ,E) be a supra topological space and F ∈ S_E (X). Then, F is called a **supra b-open soft set** if $F \tilde{\subseteq} cl^s(\text{int}^s(F)) \tilde{\cup} \text{int}^s(cl^s(F))$. The complement of a supra b-open soft set is a **supra b-closed soft set**. The set of all supra b-open soft sets is denoted by supra-BOS(X,μ,E), or supra-BOS(X) and the set of all supra b-closed sets is denoted by supra-BCS(X,μ,E), or supra-BCS(X).

Theorem:5.1.2

Let (X,μ,E) be a supra soft topological space. Then,

(1) Arbitrary soft union of supra b-open soft sets in supra b-open soft.

(2) Arbitrary soft intersection of supra b-closed soft sets in supra b-closed soft.

Proof:

(1) Let {F_j : j ∈ J} ⊆ supra-BOS(X). Then, $F_j \tilde{\subseteq} \text{int}^s(cl^s(F_j)) \tilde{\cup} cl^s(\text{int}^s(F_j))$.

For all j∈J.

It follows that, $\bigcup_{j\in J} F_j \tilde{\subseteq} \bigcup_{j\in J}[\text{int}^s(cl^s(F_j)) \tilde{\cup} cl^s(\text{int}^s(F_j))]$

$$= (\bigcup_{j\in J} \text{int}^s(cl^s(F_j))) \tilde{\cup} (\bigcup_{j\in J} cl^s(\text{int}^s(F_j)))$$

$$\tilde{\subseteq} \text{int}^s(cl^s(\bigcup_{j\in J} F_j))) \tilde{\cup} (cl^s(\text{int}^s(\bigcup_{j\in J} F_j)))$$

Hence, $\bigcup_{j\in J} F_j$ ∈ supra –BOS(X).

Proof of (2) can also be proved in the similar way.

Example:5.1.3

Suppose that there are three phones in the universe X given by $X = \{h_1, h_2, h_3\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "expensive" and "beautiful" respectively. Let $F_1, F_2, F_3 \in S_E(X)$ which describe the composition of the computers, such that

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$,

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_1, h_2\}$,

$F_3(e_1) = \{h_2, h_3\}$, $F_3(e_2) = \{h_2, h_3\}$.

Then, defines a supra soft topology on X. Hence, the soft sets G and H which defined as follows:

$G(e_1) = \{h_1, h_3\}$, $G(e_2) = \{h_1, h_3\}$,

$H(e_1) = \{h_2, h_3\}$, $H(e_2) = \{h_2, h_3\}$.

Are supra b-open soft sets of (X, μ, E), but their soft intersection $G \tilde{\cap} H = M$, where $M(e_1) = \{h_3\}$, $M(e_2) = \{h_3\}$ is not supra b-open soft.

Remark:5.1.4

The family of all supra b-open soft sets on a supra soft topological space (X,μ,E) forms a supra soft topology, which is collection of soft sets contains $\tilde{X}, \tilde{\Phi}$ and closed under arbitrary soft union.

Definition:5.1.5

Let (X,μ,E) be a supra soft topological space, $F \in S_E(X)$ and $e_H \in S_E(X)$. Then,

(1) e_H is called a **supra b - interior soft point** of F if there exists G ∈ supra-BOS(X) such that $e_H \tilde{\in} G \tilde{\subseteq} F$, the set of all supra b - interior soft point of F is called the supra b - interior of F and is denoted by $bint^s(F)$, consequently,

$bint^s(F) = \bigcup\{G : G \tilde{\subseteq} F, G \in supra-BOS(X)\}$.

(2) e_H is called a **supra b - closure soft point** of F if $F \tilde{\cap} M \neq \tilde{\Phi}$ for all M ∈ supra-BOS(X) such that $e_H \tilde{\in}$ M. The set of all supra b-closure soft points of F is called supra b-soft closure of F and is denoted by $bcl^s(F)$, consequently,

$bcl^s(F) = \tilde{\bigcap}\{H : H \in supra-BCS(X), F \tilde{\subseteq} H\}$.

Theorem:5.1.6

Let (X,μ,E) be a supra soft topological space and F,G ∈ $S_E(X)$. Then, the following properties are satisfied.

(1) $b\,\text{int}^s(\tilde{X}) = \tilde{X}$ *and* $b\,\text{int}^s(\tilde{\Phi}) = \tilde{\Phi}$.

(2) $b\,\text{int}^s(F) \tilde{\subseteq} F$

(3) $b\,\text{int}^s(F)$ is the largest supra b-open soft set contained in F.

(4) If $F \tilde{\subseteq} G$, then $b\,\text{int}^s(F) \tilde{\subseteq} b\,\text{int}^s(G)$

(5) $b\,\text{int}^s(b\,\text{int}^s(F)) = b\,\text{int}^s(F)$

(6) $b\,\text{int}^s(F) \tilde{\cup} b\,\text{int}^s(G) \tilde{\subseteq} b\,\text{int}^s(F \tilde{\cup} G)$

(7) $b\,\text{int}^s(F \tilde{\cap} G) \tilde{\subseteq} b\,\text{int}^s(F) \tilde{\cap} b\,\text{int}^s(G)$

Theorem:5.1.7

Let (X,μ,E) be a supra soft topological space and F,G ∈ $S_E(X)$. Then, the following properties are satisfied.

(1) $bcl^s(\tilde{X}) = \tilde{X}$ *and* $bcl^s(\tilde{\Phi}) = \tilde{\Phi}$

(2) $F \tilde{\subseteq} bcl^s(F)$

(3) $bcl^s(F)$ is the smallest supra b-closed soft set contains F.

(4) If $F \tilde{\subseteq} G$, then $bcl^s(F) \tilde{\subseteq} bcl^s(G)$

(5) $bcl^s(bcl^s(F)) = bcl^s(F)$

(6) $bcl^{\backprime}(F)\tilde{\cup}bcl^{\backprime}(G)\tilde{\subseteq}bcl^{\backprime}(F\tilde{\cup}G)$

(7) $bcl^{\backprime}(F\tilde{\cap}G)\tilde{\subseteq}bcl^{\backprime}(F)\tilde{\cap}bcl^{\backprime}(G)$

Theorem:5.1.8

Let (X, τ, E) be a soft topological space and $F \in S_E(X)$. Then,

(1) F ∈ supra – SOS(X) if and only if $scl^s(F) = cl^s(int^s(F))$.

(2) If G ∈ supra – OS(X), then $G\tilde{\cap}cl^{\backprime}(F)\tilde{\subseteq}cl^{\backprime}(G\tilde{\cap}F)$.

(3) If H ∈ supra – CS(X), then $\text{int}^{\backprime}(G\tilde{\cup}H)\tilde{\subseteq}\text{int}^{\backprime}(G)\tilde{\cup}\text{int}^{\backprime}(H)$.

Section 5.2

Relations Between Supra b-Open Soft Sets and Other Subsets of Supra Soft Topological Spaces

Theorem:5.2.1

In a supra soft topological space (X,μ,E), the following statements hold,

(1) Every supra-open (supra-closed) soft set is supra b-open (supra b-closed) soft.

(2) Every supra-pre-open (supra-pre-closed) soft set is supra b-open (supra b-closed) soft.

(3) Every supra semi open (supra semi closed) soft set is supra b-open(supra b-closed) soft.

(4) Every supra b - open (supra b - closed) soft set is supra β - open(supra β - closed) soft.

(5) Every supra α - open (supra α - closed) soft set is supra b - open(supra b - closed) soft.

Proof:

We prove the assertion in the case of supra b – open soft set, the other cases are clear.

Let F ∈ supra – BOS(X). Then,

$$F \tilde{\subseteq} cl^{\backprime}(\operatorname{int}^{\backprime}(F)) \tilde{\cup} \operatorname{int}^{\backprime}(cl^{\backprime}(F))$$

$$\tilde{\subseteq} cl^{\backprime}(\operatorname{int}^{\backprime}(cl^{\backprime}(F))) \tilde{\cup} \operatorname{int}^{\backprime}(cl^{\backprime}(F))$$

$$\tilde{\subseteq} cl^{\backprime}(\operatorname{int}^{\backprime}(cl^{\backprime}(F)))$$

From theorem 5.1.8(1). Therefore, F ∈ supra – βOS(X).

Remark:5.2.2

It is obvious that, supra – POS(X) ∪ supra – SOS(X) ⊆ supra – BOS(X) ⊆ supra-βOS(X).

The following examples shall show that the implications in Theorem cannot be reversed and the converse of these implications are not true in general.

Example:5.2.3

(1) In Example 5.1.3 The soft set G is supra b – open soft set, but it is not supra open soft.

(2) Suppose that there are four alternatives in the universe of dresses X = { h_1, h_2, h_3, h_4} and consider E = {e_1(cotton), e_2(woolen)} be the set of parameters showing the material of the dresses. Let F_1, F_2, F_3 and F_4 be four soft sets over the common universe X which describe the goodness of the dresses, where

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_2\}$,

$F_2(e_1) = \{h_2\}$, $F_2(e_2) = \{h_1\}$,

$F_3(e_1) = \{h_1, h_2\}$, $F_3(e_2) = \{h_1, h_2\}$,

$F_4(e_1) = \{h_1, h_2, h_4\}$, $F_4(e_2) = \{h_1, h_2, h_4\}$.

Then, μ={ $\tilde{X}, \tilde{\Phi}, F_1, F_2, F_3, F_4$ } defines a supra soft topology on X. Hence, the soft set g defined by G(e_1) = {h_1, h_4}, G(e_2) = {h_2, h_4} is supra b-open soft set, but it is not supra pre – open soft.

(3) Suppose that there are three alternatives in the universe of houses X = {h_1, h_2, h_3} and consider E={ e_1, e_2} be two parameter "quality of houses" and "wooden" to be the linguistic variable. Let F_1, F_2 and F_3 be three soft sets over the common universe X which describe the goodness of the houses,

Where

$F_1(e_1) = \{ h_1 \}$, $F_1(e_2) = \{ h_1 \}$,

$F_2(e_1) = \{ h_1, h_2 \}$, $F_2(e_2) = \{ h_1, h_2 \}$,

$F_3(e_1) = \{ h_2, h_3 \}$, $F_3(e_2) = \{ h_2, h_3 \}$.

Then, $\mu=\{ \widetilde{X}, \widetilde{\Phi}, F_1, F_2, F_3, F_4 \}$ defines a supra soft topology on X. Hence, the soft set G which defined as $G(e_1) = \{h_1, h_3\}$, $G(e_2) = \{h_1, h_3\}$ is supra b-open soft set, but it is not supra semi open soft.

(4) Suppose that there are three alternatives in the universe of cars $X = \{h_1, h_2, h_3\}$ and consider $E=\{ e_1, e_2\}$ be two parameter "quality of cars" and "cost" to be the linguistic variable. Let F_1, F_2 and F_3 be three soft sets over the common universe X which describe the goodness of the houses,

Where

$F_1(e_1) = \{ h_4 \}$, $F_1(e_2) = \{ h_1, h_2 \}$,

$F_2(e_1) = \{ h_1, h_2 \}$, $F_2(e_2) = \{ h_4 \}$,

$F_3(e_1) = \{ h_1, h_2, h_3 \}$, $F_3(e_2) = \{ h_1, h_2, h_4 \}$.

Then, $\mu=\{ \widetilde{X}, \widetilde{\Phi}, F_1, F_2, F_3, F_4 \}$ defines a supra soft topology on X. Hence, the soft set G which defined by $G(e_1) = \{h_1\}$, $G(e_2) = \{h_3\}$ is supra β - open soft set, but it is not supra b - open soft.

(5) In (3), the soft set G which defined by: $G(e_1) = \{h_1, h_3\}$, $G(e_2) = \{h_1, h_3\}$ is supra b-open soft set of (X, μ, E), but it is not supra α- open soft.

Corollary:5.2.4

The following implications hold from Theorem 4.1 and remark 5.2, for a supra soft topological space (X, μ, E). These implications are not reversible.

$$\begin{tikzcd}
\text{supra} - OS(X) \arrow[r] \arrow[d] & \text{supra} - \alpha OS(X) \arrow[r] & \text{supra} - SOS(X) \arrow[dl] \\
\text{supra} - POS(X) \arrow[r] & \text{supra} - BOS(X) \arrow[r] & \text{supra} - \beta OS(X)
\end{tikzcd}$$

Theorem:5.2.5

Let (X, τ, E) be a soft topological space and $F \in S_E(X)$. Then, the followings hold.

(1) $b\operatorname{int}^s(F^c) = (bcl^s(F))^c$

(2) $bcl^s(F^c) = (b\operatorname{int}^s(F))^c$

Proof:

(1) From Definition 3.2(1),

$$bcl^s(F^c) = (\bigcap\{G : F \tilde{\subseteq} G, G \in \text{supra}-BCS(X)\})^c$$
$$= \bigcup\{G^c : G^c \subseteq F^c, G^c \in \text{supra}-BOS(X)\}$$
$$= b\,\text{int}^s(F^c).$$

(2) From Definition 3.2(2),

$$(b\,\text{int}^s(F))^c = (\bigcup\{G : G \tilde{\subseteq} F, G \in \text{supra}-BOS(X)\})^c$$
$$= (\bigcap\{G^c : F^c \tilde{\subseteq} G^c, G^c \in \text{supra}-BCS(X)\})$$
$$= bcl^s(F^c).$$

Theorem:5.2.6

Let (X, τ, E) be a soft topological space and F∈ supra-BOS(X).

(1) If $\text{int}^s(F) = \tilde{\Phi}$, then F is a supra pre open soft set.

(2) If $\text{cl}^s(F) = \tilde{\Phi}$, then F is a supra semi open soft set.

Theorem:5.2.7

Let (X, μ, E) be a soft topological space and F∈ $S_E(X)$. Then, F∈ supra-BCS(X) if and only if $cl^s(\text{int}^s(F)) \tilde{\bigcap} \text{int}^s(cl^s(F)) \tilde{\subseteq} F$.

Section 5.3

Supra b-Continuous Soft Functions

Definition: 5.3.1

Let (X, τ_1, E) and (X, τ_2, K) be soft topological spaces, μ_1 be an associated supra soft topology with τ_1 and $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function. Then, the soft function f_{pu} is called a **supra b-continuous soft (supra b-cts soft)** if $f_{pu}^{-1}(G) \in$ supra-BOS(X) for all G∈ τ_2.

Theorem 5.3.2

Let (X, τ_1, E) and (X, τ_2, K) be soft topological spaces, μ_1 be an associated supra soft topology with τ_1 and $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function. Then, the followings are equivalent:

(1) f_{pu} is a supra b-continuous soft function

(2) $f_{pu}^{-1}(H) \in$ supra-BCS(X) for every closed soft set H over Y.

(3) $f_{pu}\big(bcl^s(G)\big) \tilde{\subseteq} cl_{\tau_2}(f_{pu}(G))$ for all G$\in S_E(X)$.

(4) $bcl^s(f_{pu}^{-1}(H)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H))$ for all H$\in S_K(Y)$.

(5) $f_{pu}^{-1}(int_{\tau_2}(H)) \tilde{\subseteq} bint^s(f_{pu}^{-1}(H))$ for all H$\in S_K(Y)$.

Proof:

(1⇒2) Let H be a closed soft set over Y. Then, $H^{\tilde{c}} \in \tau_2$ and $f_{pu}^{-1}(H^{\tilde{c}})$ ∈supra-BOS(X) from Definition 5.3.1. Since $f_{pu}^{-1}\big(H^{\tilde{c}}\big) = (f_{pu}^{-1}(H))^{\tilde{c}}$. Thus, $f_{pu}^{-1}(H)$ ∈supra-BCS(X).

(2⇒3) Let G$\in S_E(X)$. Since $G \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(G)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G)))\in$ supra-BCS(X) from (2). Then, $G \tilde{\subseteq} bcl^s(G) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G)))$.

Hence, $f_{pu}\big(bcl^s(G)\big) \tilde{\subseteq} f_{pu}(f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G)))) \tilde{\subseteq} cl_{\tau_2}(f_{pu}(G))$.

Thus, $f_{pu}\big(bcl^s(G)\big) \tilde{\subseteq} cl_{\tau_2}(f_{pu}(G))$.

(3⇒4) Let H$\in S_K(Y)$ and G= $f_{pu}^{-1}(H)$.

Then, $f_{pu}(bcl^s(f_{pu}^{-1}(H))) \tilde{\subseteq} cl_{\tau_2}(f_{pu}(f_{pu}^{-1}(H)))$ from (3).

Hence, $bcl^s(f_{pu}^{-1}(H)) \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(bcl^s(f_{pu}^{-1}(H)))\tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(f_{pu}\ (f_{pu}^{-1}(H))))$

$\tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H))$. .

Thus, $bcl^s(f_{pu}^{-1}(H)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H))$.

(4⇒2) Let H be a closed soft set over Y.

Then $bcl^s((f_{pu}^{-1}(H)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H))$ from(4).

But clearly, $f_{pu}^{-1}(H) \tilde{\subseteq} bcl^s((f_{pu}^{-1}(H))$.This means that, $f_{pu}^{-1}(H) = bcl^s((f_{pu}^{-1}(H))$, and so $f_{pu}^{-1}(H)$ ∈supra-BCS(X).

(1⇒5) Let H$\in S_K(Y)$. Then, $f_{pu}^{-1}(int_{\tau_2}(H))$ ∈supra-POS(X) from (1).

Hence. $f_{pu}^{-1}(int_{\tau_2}(H)) = bint^s(f_{pu}^{-1}(int_{\tau_2}(H))) \widetilde{\subseteq}\ bint^s(f_{pu}^{-1}(H))$.

Thus. $f_{pu}^{-1}(int_{\tau_2}(H))\ \widetilde{\subseteq}\ bint^s(f_{pu}^{-1}(H))$.

(5⇒1) Let H be an open soft set over Y. Then,

$int_{\tau_2}(H) = H$ and $f_{pu}^{-1}(int_{\tau_2}(H)) = f_{pu}^{-1}(H)\ \widetilde{\subseteq}\ bint^s(f_{pu}^{-1}(H))$ from (5). This means that, $bint^s(f_{pu}^{-1}(H)) = f_{pu}^{-1}(H)$ ∈supra- POS(X).

Thus, f_{pu} is a supra b-continuous soft function.

Theorem: 5.3.3

Let (X, τ_1, A) and (X, τ_2, B) be soft topological spaces and $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function. Then,

(1) Every supra cts soft function is supra b-cts soft function .

(2) Every supra pre-cts soft function is supra b-cts soft function .

(3) Every supra semi-cts soft function is supra b-cts soft function .

(4) Every supra b-cts soft function is supra β-cts soft function .

(5) Every supra α-cts -cts soft function is supra b-cts soft function .

Remark : 5.3.4

The converse of the Theorem 5.3.3 is not true and is shown in the following examples.

Example:5.3.5

(1) Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$, $E = \{e_1, e_2\}$ and $K = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as follows:

$u(a) = x$, $u(b) = z$, $u(c) = y$,

$p(e_1) = k_2$, $p(e_2) = k_1$.

Let (X, τ_1, E) be a soft topological space over X where, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, F\}$, where F is a soft set over X defined as $F(e_1) = \{a, b\}$ and $F(e_2) = \{a, b\}$. The supra soft topology μ_1 is defined as $\mu_1 = \{\tilde{X}, \tilde{\emptyset}, F_1, F_2, F_3\}$, where F_1, F_2 and F_3 are soft sets over X defined as follows:

$F_1(e_1) = \{a\}$, $F_1(e_2) = \{a\}$

$F_2(e_1) = \{a,b\}$ $F_2(e_2) = \{a,b\}$

$F_3(e_1) = \{b,c\}$ $F_3(e_2) = \{b, c\}$.

Let (Y, τ_2, K) be a soft topological space over Y where $\tau_2 = \{\tilde{Y}, \tilde{\emptyset}, G\}$, where G is a soft set over Y defined as $G(k_1) = \{x, y\}$ and $G(k_2) = \{x, y\}$. Let $f_{pu}: (X, \tau_1, E) \rightarrow (Y, \tau_2, K)$ be a soft function.

Then, $f_{pu}^{-1}(G) = \{(e_1, \{a, c\}), (e_2, \{a, c\})\}$ is a supra b-open soft set, but it is not supra open soft. Hence , f_{pu} is a supra b-continuous soft function, but it is not supra continuous soft.

(2) Let $X = \{a, b, c, d\}$, $Y = \{x, y, z, w\}$, $E = \{e_1, e_2\}$ and $K = \{k_1, k_2\}$. Define $u: X \rightarrow Y$ and $p: E \rightarrow K$ as follows:

$u(a) = x,$ $u(b) = y,$ $u(c) = y,$ $u(d) = x$

$p(e_1) = k_1,$ $p(e_2) = k_2.$

Let (X, τ_1, E) be a soft topological space over X where, $\tau_1 = \{\tilde{X}, \tilde{\emptyset}, F\}$, where F is a soft set over X defined as $F(e_1) = \{a, b\}$ and $F(e_2) = \{a, b\}$. The supra soft topology μ_1 is defined as $\mu_1 = \{\tilde{X}, \tilde{\emptyset}, F_1, F_2, F_3, F_4\}$, where F_1, F_2, F_3 and F_4 are soft sets over X defined as follows:

$F_1(e_1) = \{a\}$, $F_1(e_2) = \{a\}$

$F_2(e_1) = \{a,b\}$ $F_2(e_2) = \{a,b\}$

$F_3(e_1) = \{b,c\}$ $F_3(e_2) = \{b, c\}$

$F_4(e_1) = \{a,b,c\}$ $F_4(e_2) = \{a, b, c\}$.

Let (Y,τ_2,K) be a soft topological space over Y where $\tau_2 = \{\tilde{Y},\widetilde{\emptyset},G\}$, where G is a soft set over Y defined as $G(k_1) = \{x,z\}$ and $G(k_2) = \{x,w\}$. Let $f_{pu}:(X,\tau_1,E) \rightarrow (Y,\tau_2,K)$ be a soft function.

Then, $f_{pu}^{-1}(G) = \{(e_1,\{a,d\}),(e_2,\{a,d\})\}$ is a supra b-open soft set, but it is not supra pre open soft. Hence , f_{pu} is a supra b-continuous soft function, but it is not supra pre- continuous soft function.

In (2), let $u: X \rightarrow Y$ defined as follows: $u(a) = y$, $u(b) = x$,$u(c) = y$ and $u(d) = y$. Let (Y,τ_2,K) be a soft topological space over Y where $\tau_2 = \{\tilde{Y},\widetilde{\emptyset},G\}$, where G is a soft set over Y defined as $G(k_1) = \{x,y\}$ and $G(k_2) = \{x,w\}$. Let $f_{pu}:(X,\tau_1,E) \rightarrow (Y,\tau_2,K)$ be a soft function.

Then, $f_{pu}^{-1}(G) = \{(e_1,\{b\}),(e_2,\{b\})\}$ is a supra b-open soft set, but it is not supra semi open soft. Hence , f_{pu} is a supra b-continuous soft function, but it is not supra semi- continuous soft.

(3) Let $X = \{a,b,c,d\}$, $Y = \{x,y,z,w\}$, $E = \{e_1,e_2\}$ and $K = \{k_1,k_2\}$. Define $u: X \rightarrow Y$ and $p: E \rightarrow K$ as follows:

$$u(a) = x, \quad u(b) = y, \quad u(c) = w, \quad u(d) = z,$$

$$p(e_1) = k_1, \quad p(e_2) = k_2.$$

Let (X,τ_1,E) be a soft topological space over X where, $\tau_1 = \{\tilde{X},\widetilde{\emptyset},F\}$, where F is a soft set over X defined as $F(e_1) = \{a,b\}$ and $F(e_2) = \{a,b\}$. The supra soft topology μ_1 is defined as $\mu_1 = \{\tilde{X},\widetilde{\emptyset},F_1,F_2,F_3,F_4\}$, where F_1,F_2,F_3 and F_4 are soft sets over X defined as follows:

$F_1(e_1) = \{d\}$, $F_1(e_2) = \{a,b\}$

$F_2(e_1) = \{a,b\}$ $F_2(e_2) = \{d\}$

$F_3(e_1) = \{a,b,d\}$ $F_3(e_2) = \{a,b,d\}$

$F_4(e_1) = \{a,b,d\}$ $F_4(e_2) = \emptyset$.

Let (Y, τ_2, K) be a soft topological space over Y where $\tau_2 = \{\tilde{Y}, \widetilde{\emptyset}, G\}$. where G is a soft set over Y defined as $G(k_1) = \{x\}$ and $G(k_2) = \{w\}$.

Let $f_{pu}: (X, \tau_1, E) \rightarrow (Y, \tau_2, K)$ be a soft function.

Then. $f_{pu}^{-1}(G) = \{(e_1, \{a\}), (e_2, \{c\})\}$ is a supra b-open soft set. but it is not supra pre open soft. Hence , f_{pu} is a supra β-continuous soft function. but it is not supra b- continuous soft. Then, $\mu = \{\tilde{X}, \widetilde{\emptyset}, F_1, F_2, F_3, F_4\}$ defines a supra soft topology on X. Hence the soft set G which is defined by $G(e_1) = \{h_1\}$and $G(e_2) = \{h_3\}$ is supra β-open soft set, but it is not supra b-open soft.

(5) In (2), let $u: X \rightarrow Y$ defined as follows: $u(a) = y$, $u(b) = x$,$u(c) = x$ and $u(d) = x$. Let (Y, τ_2, K) be a soft topological space over Y where $\tau_2 = \{\tilde{Y}, \widetilde{\emptyset}, G\}$, where G is a soft set over Y defined as $G(k_1) = \{x, y\}$ and $G(k_2) = \{x, w\}$. Let $f_{pu}: (X, \tau_1, E) \rightarrow (Y, \tau_2, K)$ be a soft function.

Then, $f_{pu}^{-1}(G) = \{(e_1, \{b, c, d\}), (e_2, \{b, c, d\})\}$ is a supra b-open soft set, but it is not supra α- open soft. Hence , f_{pu} is a supra b-continuous soft function, but it is not supra α-continuous soft.