CHAPTER III

CHAPTER-III

FUZZY SET MODEL FOR UNDERWRITING

SECTION – 3.1

SINGLE-OPTION PLANS

In section 3.1, the underwriting a group of employees covered by a single plan of health insurance is examined.

In modeling the group selection process for health insurance via fuzzy sets, the case of an employer that offers a single plan of insurance to its employees is consider.

A. Obtaining insurance is incidental to the purpose of the group. An employer-employee group usually satisfies this criterion.

B. Employment status determines eligibility for health coverage. For example, the employer or insurer may require that an employee be actively at work at least 30 hours per week to receive benefits, and only the spouse and dependent children of such an employee are eligible for dependent coverage. Underwriters use an actively-at work rule because someone who works needs to be healthy.

C. A minimum number of people are in the group. For example, some companies require that the group contain at least five employees.

D. Benefits are determined automatically. For example, this criterion is met in the case of a single-option health plan in which the class of employment determines the benefits.

It is assumed that the above rules are most appropriately described by non fuzzy sets, although C lends itself to fuzzy set representation. As for the remainder of the criteria, the possible fuzzy set characterizations are presented in the following discussion. One difficulty in fuzzy set theory is to create a function that accurately describes the given condition or characteristic. In this chapter 3, the functions essentially derive by working backwards. Four categories of groups: preferred risk; normal, or acceptable, risk; substandard risk; and unacceptable, or declinable, risk. The fuzzy set function has a membership value of 1.0 for a preferred risk. Normal risks have values between 0.5 and 1.0; substandard risks, between 0.25 and 0.5. Finally, unacceptable risks have membership values from 0.0 to 0.25.

The developed functions are illustrative only and are not intended to represent any particular company's underwriting guidelines.

E. Young lives flow constantly into the group and old lives out. Such a requirement helps to ensure stable morbidity. This rule can be verified by determining whether the age/sex factor has been relatively stable and whether the group size has fluctuated greatly during the past few years.

Let α/x equal the annual percentage change in the age/sex factor for the past two years and g/s the annual percentage change in the group size during the same period:

$$e_1(\alpha/x) = \begin{cases} 1, & \alpha/x \leq 0.05 \\ -5\alpha/x + 1.25, & 0.05 \leq \alpha/x \leq 0.25 \\ 0, & else. \end{cases}$$

$$e_2(g/s) = \begin{cases} 1, & -0.05 \le g/s \\ 5g/s + 1.25, & -0.25 \le \frac{g}{s} \le -0.05 \\ 0, & else. \end{cases}$$

The information contained in the above functions are examined: An increase of up to 5 percent or any decrease in the age/sex factor describes a preferred risk. A group is normal if the percentage increase lies between 5 and 15 percent ($0.5 \le e_1 \le 1.0$), while an unacceptable group has an age/sex factor increase of 20 percent or more ($0.0 \le e_1 \le 0.25$). For a change in group size, opposite viewpoint is taken; that is, a decrease of down to 5 percent or any increase is preferred, while a 5 to 15 percent decrease is acceptable. These two functions are linear, but for a particular insurance company's underwriting guidelines, the functions are not necessarily expected to be of that form.

One way to combine these two functions, which give equal weight to e_1 and e_2, is

$$e(\alpha/x, g/s) = \sqrt{e_1(\alpha/x) \times e_2(g/s)}$$

F. There is a minimum participation in the plan. An insurer usually require that all eligible employees enroll in a noncontributory, or employer- pay-all, plan. Since there is no reason for an employee not to join such a plan, we focus more on contributory ones. A typical rule in this case requires 100 percent enrollment for groups of five or fewer grading to 75 or 85 percent for groups of 10 or more.

In addition, insurers often want a minimum percentage of employees who have dependents to cover them. For example, suppose there is a group of 20 eligible employees; then 15 employees must elect coverage if the insurer mandates

75 percent participation. If 16 of the 20 are enrolled and 12 of them are eligible to cover their dependents, then at least 9 must do so. If at least 90 percent of the employees participate, then the group is preferred. A normal group has 80 to 90 percent participation; a substandard group, 75 to 80 percent. This distribution corresponds to the requirement that a group has 85 percent participation with possible bending of the rule down to 75 percent. Let p be the proportion of employees that select group coverage:

$$f(p) = \begin{cases} 1, & 0.90 \leq p \\ 5p - 3.5, & 0.70 \leq p \leq 0.90 \\ 0, & else. \end{cases}$$

For small groups, higher percentages of participation than those given in the above equation may be desired. Also, the same or a similar function may account for participation of employees who are eligible for dependent coverage; then some intersection of the two could integrate the criteria.

G. The employer pays some or all of the cost of insurance; contributing at least one-fourth of the premium is often a requirement. A preferred risk is one for which the employer pays all of the employee's cost and 75 percent or more of the dependent's cost. For an acceptable risk, the percentages go down to 75 and 50 percent, respectively; for a substandard risk, down to 50 and 25 percent, respectively. Let r_1 equal the proportion of the employee's premium paid by the employer and r_2 the proportion of the dependent's premium paid by the employer:

$$g_1(r_1) = \begin{cases} 0, & r_1 \leq 0.25 \\ r_1 - 0.25, & 0.25 \leq r_1 \leq 0.75 \\ 2r_1 - 1, & 0.75 \leq r_1 \leq 1.0 \end{cases}$$

$$g_2(r_2) = \begin{cases} r_2, & r_2 \leq 0.50 \\ 2r_2 - 0.50, & 0.50 \leq r_2 \leq 0.75 \\ 1, & 0.75 \leq r_2 \end{cases}$$

As in criterion E, The two functions are linked by

$$g(r_1, r_2) = \sqrt{g_1(r_1) \times g_2(r_2)}$$

The function g allows trade-offs between r_1 and r_2. For example, if an employer pays all of the employee's cost but only 40 percent of the dependent's cost, then

$$g(1.0, 0.4) = \sqrt{1 \times 0.4} \cong 0.6325 \in [0.5, 1.0],$$

from which it's infer that the group is an acceptable risk. If the employee's contribution decreases, then the participation is likely to increase.

H. A strong, central department of the employer helps the insurer administer billing, enrollment, and certification of eligibility. Fuzziness arises by the use of the term "strong"; such a word cannot be quantified easily. An underwriter will use judgment to assign a value between 0 and 1 to represent the administrative ability of the group. Some factors to consider include the size and qualifications of the personnel staff, the function it normally performs, and any knowledge about previous experience of the insurer with the employer's staff. Call the developed fuzzy set function h.

I. The industry of the group is acceptable. A company may either use a list of industries it considers uninsurable or apply a load to the medical manual rates. Note that the size of the load may be restricted for small groups due to state regulation or law.

If the underwriting department has a list of unacceptable industries, then any group in one of those industries has a membership value of 0 in the fuzzy set of groups in industries that can be underwritten. Similarly, a group in a questionable industry has a membership value between 0 and 1. Call the corresponding fuzzy set function i.

J. The policyholder has a good credit rating that helps to ensure that the premium will be paid in a timely manner. The Dun & Bradstreet credit reports or financial statements of the prospect can be examined to determine whether the prospect is a good credit risk. Based upon the willingness of the company to accept this risk, develop a fuzzy set function for this criterion and call it j.

K. Ongoing claims are not large as a proportion of total expected claims. For the initial underwriting of a small group, no employees are health risks as determined by a health questionnaire or by an attending physician's statement or physical examination. In renewal underwriting, information is available from the claim file. The number and size of ongoing claims that an insurer can tolerate depend upon the group size and the nature of the claims. An acute condition may be resolved in a relatively short time, whereas a chronic condition may continue much longer. The function we define presupposes the following: The expected payments for ongoing claims are less than 1/2 percent of the total expected paid claims for a preferred group. For a normal group, the percentage lies between $1/2$ and $1\frac{1}{2}$ percent. If the percentage of ongoing claims is above 2 percent, then the group is unacceptable. Let c equal the percentage of ongoing claims as a proportion of the total expected claims:

$$k(c) = \begin{cases} 1, & c \leq \frac{1}{2} \\ -\frac{1}{2}c + \frac{5}{4}, & \frac{1}{2} \leq c \leq 2\frac{1}{2} \\ 0, & 2\frac{1}{2} \leq c. \end{cases}$$

(Note that 100 times the decimal representation of the proportion of ongoing claims is used to evaluate k; for example, if ongoing claims =3/4 percent, then use c=0.75 to determine k.)

L. The claim experience of the group has been good. One measure of such a characteristic is the loss ratio. Experience for small groups fluctuates more than that for large groups; therefore, a high loss ratio for a large group is more indicative of poor experience in the future than a high loss ratio for a small group.

Let LR equal the previous year's loss ratio and s the group size. Here, the loss ratio refers to incurred claims divided by expected claims for the given time, that is, the actual to expected ratio. The inclusion of retention in the denominator would distort the loss ratio because retention, as a percentage of gross premiums, varies with the size of the premium:

$$L_1(LR) = \begin{cases} 1, & LR \leq 0.95 \\ -5LR + 5.75, & 0.95 \leq LR \leq 1.15 \\ 0, & 1.15 \leq LR. \end{cases}$$

In this function, a group is preferred if its loss ratio is 95 percent or less. An acceptable group has a loss ratio between 95 and 105 percent. If a given company's guidelines are looser than the above indicates, then the underwriter will broaden the intervals.

Because the loss ratio is not as indicative of future claims for small groups as it is for large ones, we account for size through

$$L_2(s) = \begin{cases} \sqrt{\frac{s}{500}}, & 0 \leq S \leq 500 \\ 1, & 500 \leq S. \end{cases}$$

Another function for L_2 could be derived from a credibility table. Combining the two functions:

$L(LR, s) = L_1(LR) \times L_2(s) + [1.0 - L_2(s)]$.

Note that L(LR, s) is a weighted average of $L_1(LR)$ and 1.0 with $L_2(s)$ as the weight.

M. The group does not change insurers often. A high turnover rate may mean that group has poor experience and frequently shops around for better rates.

Let n equal the number of insurers the group has had in the past five years.

$$m(n) = \begin{cases} 1, & n = 1 \\ 0.5, & n = 2 \\ 0, & n \geq 3. \end{cases}$$

The remainder of this section deals with the problem of combining the above rules so that we can decide whether to underwrite the group. In other words, we create a single fuzzy set to describe the set of good risks. We assume that criteria A, B, C, and D are satisfied from the outset and do not consider them further.

First, contemplate whether a 0 in any one of the categories leads to outright rejection of the group. If not, then include the operation of convex combination together with intersection; otherwise, use only intersection. Suppose that a grade of 0 in any of the following rules does not automatically disqualify a group from being insured:

H. The employer has a strong, central administrative department.

I. The industry of the group is acceptable.

K. The ongoing claims are not large.

The manual rates can be loaded to account for the effects of criteria I and K, and an extra margin can be added for expenses to compensate for any deficiency concerning criterion H. Create the linear combination:

$$P = \frac{1}{6}h + \frac{1}{3}i + \frac{1}{2}k.$$

By the choice of weights, we view the amount of ongoing claims to be somewhat more important than the industry of the group but judge strong administration less important than either. We can intersect P with the remaining criteria in many ways as in Section 2.2. The fuzzy set used in deciding whether to accept the group can have the form:

$$Q = P * e * f * g * j * L * m,$$

in which * is the selected intersection. We present two options below:

3.1.1. Option One

$$Q_1 = P \cap e \cap f \cap g \cap j \cap L \cap m,$$

in which $\cap$ denotes the minimum operator. Note that a group preferred in each category is preferred in total. Since the minimum operator does not allow the variables to interact, the relations among them are missed.

The following matrix represents the pair-wise interactions between the criteria:

Interaction between criteria	P	e	f	g	j	L	m
P					Some		Little
Flow of lives (e)			Some				
Participation (f)		Some	Important	Max			
Employer Contribution (g)			Max				
Good credit (j)	Some						Little
Loss ratio (L)							
Turnover (m)	Little				Little		

We symbolize the label of "some" by the Hamacher operator (p = 0.5); "little" by the Hamacher operator (p = 0); "max" by the algebraic product; and no interaction by the minimum operator. Finally, "important," means that the criterion warrants special emphasis.

One possible fuzzy set function representing the above table is as follows:

3.1.2. Option Two

$Q_2 = \{H\,[H(P, j; 0.5), m; 0]\}^{1/3} \cap [f \times g]^{1/2} \cap [H(f, e; 0.5)]^{1/2} \cap [L^{\alpha}]$,

in which a changes the importance of the fuzzy set representing the loss ratio. The less important it is, the less α will be.

To develop this function, the variables are first grouped according to their interaction. In this case, the variables are partitioned as follows: {*P, j, m*}, {*e, f*}, {*f, g*}, and {*L*}. It is noted that it is not a true partition because f appears in two distinct subsets.

Since *P* and *j* interact somewhat and each interacts little with *m*, first combine *P* and *j* via the Hamacher operator (p=0.5), then join the result with *m* by using the Hamacher operator (p=0). Take the cube root of the outcome to negate the effect of the product $P \times j \times m$, making the result comparable to other terms that involve fewer than three factors. If, instead, the interaction between *P* and *m* had been "some" and not "little," *m* and *j* could first be combined via the Hamacher operator (*p*=0) and then that result could be joined with *P* by using the Hamacher operator (*p*=0.5).

As an aside, note that the Hamacher operator is not associative if the parameter changes; that is, H[H(A, B; *p*), C; *q*] is not necessarily equal to H[A, H(B, C; *q*); *p*]. This inequality can be seen by taking $f_B \equiv 1$; in this case, the first term is H(A, C; *q*), while the second term is H(A, C; *p*). For a general fuzzy set B, if *p*=0.5 and *q*=0, then an interpretation of the first term is that A and B interact somewhat and each interacts little with C. Similarly, the second term may mean that B and C interact little and each interacts somewhat with A. From this perspective, the two terms are not expected to be equal.

Because f and g interact maximally, intersect them through the algebraic product. Form the square root of this product to make the term commensurate with the others.

The functions f and e interact to some extent; therefore, combine them by using the Hamacher operator (p=0.5); again, take the square root of this result. The importance of f is reflected in its appearance in the above two terms. Alternatively, the concentration CON(f ; 2) could be used in either or both of the two terms in place of f, or an extra term of the form CON(f ; 2) could be included, in which the importance off is made explicit. Ambiguity arises in this example because e and g are each related to f but not to each other.

Since L does not interact with any other variable, it is a term in the final intersection by itself. However, L can be concentrated or dilated to reflect the significance of the loss ratio. The degree of dilation or concentration is influenced by any considerations of credibility not taken into account by the variable of group size, for example, a high turnover rate within the plan.

Finally, intersect the four terms, $H[H(P, j; O.5), m; O]^{1/3}$, $(fg)^{(1/2)}$, $H(f, e; 0.5)^{(1/2)}$, and L^{α}, through the minimum operator. Apply this operator because the four terms do not interact, except by means of f.

In general, the cutoff point for choosing or not choosing a group depends upon the function selected for Q. An alternative to using a particular number between 0 and 1 is to implement a fuzzy decision scheme. For example, if Q lies between 0.75 and 1.0, then the group is definitely acceptable. If Q is in the range from 0.50 to 0.75, then the group is most likely acceptable, and if Q is less than 0.25, then the group is definitely unacceptable. Otherwise, if Q lies between 0.25 and 0.50, rely upon the discretion of the underwriter. (This outline roughly follows the concepts of preferred, normal, substandard, and unacceptable risks introduced at the beginning of this section.)

Within this fuzzy decision strategy, an insurer can use the value of Q to indicate how to load the manual rates, change the plan of benefits, or require an alternative funding scheme. This process of making decisions could be modeled by using the method.

Example: 3.1.3

Consider a group with the characteristics:

Group size (s)	= 250
Age/sex factor change (α/x)	= 10%
Size change (g/s)	= -15%
Participation (p)	= 85%
Employer contribution (r_1/r_2)	= 100%/40%
Strong administration (h)	= 0.9
Industry (i)	= 1.0
Credit rating (j)	= 0.95
Ongoing claims (c)	= 0.75%
Loss ratio (LR/α)	= 1.05/0.75
Number of carriers (n)	= 1.

After applying the fuzzy functions presented above, we obtain:

$Q_2 = \min [0.8765^{1/3}, 0.4743^{1/2}, 0.4827^{1/2}, 0.6464^{3/4}]$

$= 0.6887.$

Since $Q_2 = 0.6887$ lies between 0.50 and 0.75, the group is most likely acceptable. The employer may be interested in knowing how to improve the group's acceptability. Determine this information by examining the function $\sqrt{(fg)}$ because its value yields the minimum Q_2. The contribution made by the employer to the single rate is 100 percent, so g_1 cannot be improved. To measure the effect of an increase in the participation rate or in the employer's contribution to the dependent's cost, consider the following first partial derivatives:

$$\frac{\partial}{\partial p}\sqrt{fg} = 5/2\sqrt{\frac{g}{f}} = 2.2958.$$

$$\frac{\partial}{\partial r_2}\sqrt{fg} = 1/4\sqrt{\frac{f}{g}} \times \sqrt{\frac{g_1}{g_2}} = 0.4305.$$

The greater improvement in $\sqrt{(fg)}$ comes from an increase in participation rate because 2.2958>0.4305. On the other hand, this change may be effected most easily if the employer contributes more. Suppose the variable r_2=0.50 and, as a result, p=0.90, then

$Q_2 = \min [0.8765^{1/3}, 0.7071^{1/2}, 0.6124^{1/2}, 0.6464^{3/4}]$

$= 0.7209.$

The above process may be continued by considering how the importance of *L* can be decreased; for example, a strong preexisting-condition exclusion could be implemented or the plan design could be changed to lower utilization.

Another application of the first partial derivatives inherent in the above discussion is sensitivity analysis. In addition to calculating Q for various scenarios to determine its appropriateness, the partial derivatives can be evaluated to determine whether Q's sensitivity to the variables follows the underwriting guidelines of the company.

Such an evaluation is important if several factors interact intricately.

For instance, suppose the fuzzy sets A, B, C, and D interact as follows:

variable	A	B	C	D
A		Some	Little	Some
B	Some		Some	Little
C	Little	Some		Some
D	Some	Little	Some	

Ambiguity exists because B and D interact to some degree with each of A and C, but B and D interact little with each other, as do A and C. These relationships can be represented by H[H(A, C; O), H(B, D; O); 0.5].

The vagueness of this example emphasizes the importance of verifying that the chosen functions characterize the qualities correctly and that the combination of those functions accurately reflects the given underwriting process.

SECTION – 3.2

MULTIPLE-OPTION PLANS

In section 3.2, the work with single plan is extended to the group selection in a multichoice environment. Since more employees offer multiple plans to their employees, an underwriting scheme for such situations becomes important.

The factors that receive the most attention in the underwriting of multiple-option plans are those that affect participation, such as the level of access to care, the employee contribution, the plan of benefits, and the age, sex, and dependent coverage of the employee. These factors are also at work in single-option selection because an employee can choose whether to accept coverage. The existence of a working spouse's plan and that plan's benefits, employee contribution, and access to care influence an employee's decision. Underwriting against shadow plans is nearly impossible; therefore, external factors that affect participation in single-option plans are not usually considered. They are more visible, however, in multiple-option cases because the underwriter is normally aware of the competition.

The level of access to care encompasses the size of the provider panel relative to the provider population in the area, the geographic distribution of the panel relative to the location of the employees, the operating hours of the panel, the particular gatekeeper mechanism used, and the ease of obtaining referrals. In general, a staff model or small, closed-panel health maintenance organization (HMO) with tight controls has restricted access. Moderate control is found in a large, open-panel HMO, for example, and lose control in an HMO that is based upon an independent practice association. Little or no control describes a preferred provider organization or an indemnity plan.

We measure the richness of the plan of benefits relative to the competing plans and consider the scope of coverage and any copayments, deductibles, and coinsurance. The contribution to the premium required from the employee may or may not be tied to the benefits and access to care. For example, the employer may contribute a flat dollar amount for each employee-say, the cost of single coverage for the lowest-priced option. In this case, the excess paid by the employee varies with the benefit design and access to care, assuming that the tighter the control, the lower the premium. On the other hand, the employer may seek to direct employees to a particular option by requiring that employees contribute less for that option.

In what follows, we use the above variables to develop an age factor that denotes possible participation in the plan. To simplify the presentation of the age factor table, we combine the categories of restricted access and moderate control into one of limited access; the categories of loose and no control, into one of free access. Also, we confine benefit design to either rich or poor; employee contribution to the premium, to high or low. We consider only three employee age groups: younger (under 40), middle-aged (40 to 55), and older (over 55).

We associate a number between 0 and 1 with each age group that varies with the level of access to care, the plan design, the employee contribution, and whether the employee has single or family coverage. The number represents the possibility of the employee's participating in the described plan.

We obtain single and family age factors by forming the weighted averages of these individual factors, where the weights are the proportions of employees in the corresponding single and family age brackets. Sex can be taken into account if the composition of the block of business warrants doing so.

For example, in some areas, a married female employee is more likely to be covered through her husband's plan than vice versa.

			Single			Family		
Access	Benefits	Cost	Young	Middle	Older	Young	Middle	Older
Free	Rich	High	0.2	0.5	0.8	0.3	0.6	0.9
		Low	0.9	0.9	0.9	1.0	1.0	1.0
	Poor	High	0.1	0.4	0.7	0.1	0.5	0.8
		Low	0.5	0.6	0.7	0.6	0.7	0.8
Limited	Rich	High	0.2	0.2	0.2	0.1	0.1	0.1
		Low	0.8	0.5	0.2	0.7	0.4	0.2
	Poor	High	0.1	0.1	0.1	0.1	0.1	0.1
		Low	0.7	0.4	0.1	0.6	0.3	0.1

We emphasize that these possibility distributions are theoretical only and are not based upon empirical data. The following is a list of assumptions implicit in the table:

(1) Older people prefer less control to more control regardless of the cost, because they have established physician relations and want no restrictions of the providers they use. Within a given level of access, benefits are more important than the amount of employee contribution.

(2) Younger people are more interested in low-cost options. Within a particular cost bracket, less control and richer benefits are preferred, with the latter taking precedent.

(3) An employee with family coverage is more interested in easy access to care than one with single coverage.

(4) Middle-aged people balance the preferences of younger and older employees.

The health status of an individual also plays a role in selection: Less healthy employees and dependents favor greater access to care, and more healthy ones look for lower required contributions. In fact, the tacit assumption in the age factors is that health status is related to age. For small groups, an underwriter may know the health status of individuals, but for large groups, gaining such information would be administratively costly.

The relative differences among the factors in the various categories can be adjusted based on the actual variations among the plans. For example, if the average difference in the employee contributions between two given plans is $50 and between two other plans is $25, then the relativities between the factors in the low and high employee contribution categories for the first case should be greater than those for the second. Indeed, the level of access to care, the plan of benefits, and the employee contribution can be represented by fuzzy set functions and the participation factors varied according to the membership values of those fuzzy sets

Example: 3.2.1

Employees may choose from two plans: plan 1 with free access, rich benefits, and high employee contribution; and plan 2 with limited access, rich benefits, and low employee contribution. The census for the group is

	Single	Family
Younger	10	20
Middle-aged	20	35
Older	15	15

	Single			Family		
	Employees	Factor	Product	Employees	Factor	Product
Younger	10	0.2	2.0	20	0.3	6.0
Middle-aged	20	0.5	10.0	35	0.6	21.0
Older	15	0.8	12.0	15	0.9	13.5
Total	45		24.0	70		40.5

Calculate the single/family age factors for plan 1:

Single factor = 24/45 = 0.533; family factor = 40.5/70 = 0.579.

Similarly, the factors for plan 2 are:

Single factor = 21/45 = 0.467; family factor = 31/70 = 0.443.

Plan 1 is the favorite; if neither were clearly preferred, then we would combine the single and family factors to obtain one age factor for each plan. One possible combination is the weighted average, with the weight equal to the relative premium size, say, 2.7 = (family rate)/(single rate).

If we follow this scheme, the age factor for plan 1 is

$$(0.533 + 2.7 * 0.579)/(1 + 2.7) = 0.567,$$

and that for plan 2 is

$$(0.467 + 2.7 * 0.443)/(1 + 2.7) = 0.450.$$

Since there is a 26 percent difference between the factors, roughly 5/9 of the participants can be expected to choose plan 1 and 4/9, plan 2; however, the desirability of a certain level of participation can differ from plan to plan. A given participation may be good for plan 2 because it expects to attract the younger and supposedly healthier lives, whereas the same participation may be less than adequate for plan 1 because older and presumably sicker lives will be drawn to it. (Percentage participation in the different options is with respect to those employees selecting some type of coverage, and the corresponding age factors are also based upon that subset.)

To account for variable levels of participation for different types of acceptable plans, we define a fuzzy set for each plan to reflect the desired participation. For instance, create the following fuzzy set for plan 1 in the above example:

$$f_{11}(p_1) = \begin{cases} 1, & 0.6 \leq p_1 \\ 5p_1 - 2, & 0.25 \leq p_1 \leq 0.6 \\ 0, & p_1 \leq 0.4 \end{cases}$$

in which p~ is the expected percentage participation in plan 1. Similarly, for plan 2, define

$$f_{12}(p_2) = \begin{cases} 1, & 0.5 \le p_2 \\ 4p_2 - 1, & 0.25 \le p_2 \le 0.5 \\ 0, & p_2 \le 0.25, \end{cases}$$

in which P_2 is the expected percentage participation in plan 2. Calculate the variables Pl and P2 based upon the participation age factors of the plan. For example, assume that p~ =5/9 and p2=4/9 for the given group.

In addition to or in modification of the underwriting rules presented in Section 3 for single-option plans, we discuss the following guidelines in the case of multiple-option plans [41, pp. 4-9], [44, pp. 48-52]. Again, we contemplate only an employer-employee group.

D1. Benefits within each option are determined automatically. Since this rule is nonfuzzy in nature, we assume that D1 is satisfied from the beginning.

E1. The group has an average age less than a stated number of years and female participation less than a given percentage. A large proportion of females and older participants leads to high claim costs and may induce selection against the plan that has greater access to care or richer benefits. To combat higher expected claims, the manual rates can be adjusted by an appropriate age/sex factor.

In addition, the age/sex factor may have to be close to the average of one's block of business or to the nationwide average for the industry of the group.

Such a rule is more important for a plan against which selection is more likely. For example, a plan with freer access to care may attract those with higher expected claim costs, as mentioned above. Different plans therefore will require distinct fuzzy sets to represent the above criterion.

Let *asf* be the age/sex factor of the group. Define a fuzzy set representation for plan 1 by:

$$e_{11}(asf) = \begin{cases} 1, & asf \leq 1.1 \\ -5asf + 6.5, & 1.1 \leq asf \leq 1.3 \\ 0, & 1.3 \leq asf. \end{cases}$$

Similarly, define one for plan 2 by:

$$e_{12}(asf) = \begin{cases} 1, & asf \leq 1.3 \\ -2.5asf + 4.25, & 1.3 \leq asf \leq 1.7 \\ 0, & 1.7 \leq asf. \end{cases}$$

F1. There is a minimum participation when all benefit options are being considered, as well as a minimum enrollment in the given option. We discuss this topic at the beginning of this section and mention the rule here for the sake of completeness.

N. The employee contributions do not differ greatly among the plans. A large difference may lead to selection against a higher-cost plan. Let *ec* be the difference between the employee contributions to plan 1 and plan 2.

For the higher-cost plan, plan 1, define the fuzzy set:

$$n_1(ec) = \begin{cases} 1, & ec \leq 25 \\ -0.04ec + 2, & 25 \leq ec \leq 50 \\ 0, & 50 \leq ec. \end{cases}$$

For the lower-cost plan, plan 2, define the fuzzy set identically equal to 1: $n_2 \equiv 1$.

Note that we may use the same function for both plans by allowing *ec* to assume negative values.

O. The benefits of one plan are not overly rich in relation to the other(s), particularly with respect to selected benefits, such as prescription drugs and organ transplants. This requirement helps to reduce selection against the plan and premium differentials.

One measure of the relative richness is the ratio of the manual claims of one plan divided by the other, where provider discounts are not considered. Let *rr* be this ratio, and define the fuzzy set:

$$o(\mathrm{rr}) = \begin{cases} 1, & \mathrm{rr} \le 1.2 \\ -2.5\mathrm{rr} + 4, & 1.2 \le \mathrm{rr} \le 1.6 \\ 0, & 1.6 \le \mathrm{rr}. \end{cases}$$

As in Section 3, we use a matrix to relate the above rules:

Interaction between criteria	e_{1i}	f_{1i}	n_i	o	Q_2
Age/Sex (e_{1i})		Some			Some
Participation (f_{1i})	Some	Important	Max	Max	Some
Contribution (n_i)		Max			Some
Benefits (o)		Max			Some
Q_2	Some	Some	Some	Some	

To define a fuzzy set function representing the above table, first group the variables associated with an option's characteristics according to whether they interact: $\{e_{1i}, f_{1i}\}$ and $\{f_{1i}, n_i, o\}$. Since e_{1i} and f_{1i} interact somewhat, merge them by means of the Hamacher operator (p=0.5). Take the square root to make the result comparable to the next term.

Because f_{1i}, n_i, and o interact maximally, intersect them through the algebraic product. Form the cube root of the product, so that the two terms are commensurate. The importance of f_{1i} is implicit in its appearance in both of the above terms. As in Section 3, this importance becomes explicit by using the concentration of f_{1i}, either as a substitute for f_{1i} in the two terms or as an extra term.

Combine the two terms through the minimum operator; the resulting function represents an option's qualities. Join this function with Q_2 by the Yager operator (p=2); the Hamacher operator (p=0.5) also may be used. The resulting combination is

$$R_i = Y(Q_2, [f_{1i} \times n_i \times o]^{1/3} \cap [H(f_{1i}, e_{1i}; 0.5)]^{1/2}; 2)$$

in which i=1,2. Note that R_i generalizes the results of Section 3 by embedding Q_2, the fuzzy set related to group characteristics, within it. The other four functions, e_{1i}, f_{1i}, n_i, and o, depend upon the plan's characteristics as well as those of the group.

Example: 3.2.2

Assume the group is as given in Example 3.1.3, and use the plans and single/family age distributions from Example 3.2.1. Let αsf = 1.2, *ec* = 40,

and *rr*= 1.

$$R_1 = Y(0.6887, \min[0.6776, 0.6417]; 2)$$

$$= 1 - \min\{1, [(1 - 0.6887)^2 + (1 - 0.6417)^2]^{1/2}\}$$

$= 1 - \min\{1, 0.4746\}$

$= 0.5254.$

$R_2 = Y(0.6887, \min[0.9196, 0.8819]; 2)$

$= 1 - \min\{1, [(1 - 0.6887)^2 + (1 - 0.8819)^2]^{1/2}\}$

$= 1 - \min\{1, 0.3329\}$

$= 0.6671.$

According to the above results, the sponsor of plan 2 is happier about offering that plan against plan 1 than vice versa.