Chapter IV

CHAPTER IV

APPLICATION OF BIMATRICES TO SOME FUZZY MODELS

This chapter illustrates how bimatrices are utilized in fuzzy models. Here, a model which is analyzing the problem faced by an industry is considered. All problems faced while running an industry or a factory cannot be put as a statistical data. Several of them are feelings involving a great deal of uncertainty and impreciseness. In order to run the industry smoothly and with atleast some profit, one should try to analyze the problem. To get some sort of frictionless feelings among workers, among the financers and above all the relation between the workers and boss i.e., what we mean the relation between the employee and the employers. Thus to have good profit, the sales should be good which indirectly means the impact of their products in the public has a good standing and rapport. So the problem involved is multidimensional. Illustration of this model is given in this chapter.

Suppose the industry on one side analyzes the factors promoting business and considers the following five attributes.

C_1 – Good business

C_2 – Good investment

C_3 – Customer Satisfaction

C_4 – Establishment

C_5 – Marketing strategies

and also at the same time wishes to study the employee problems .

Employer relationship with employee so that the factory runs smoothly. The attributes given by an expert in the analysis of the employee-employer model is as follows:

E_1 – Maximum profit to the employer

E_2 – Just profit to the employer

E_3 – Neither profit nor loss to the employer

E_4 – Loss to the employer

E_5 – Best performance by the employee

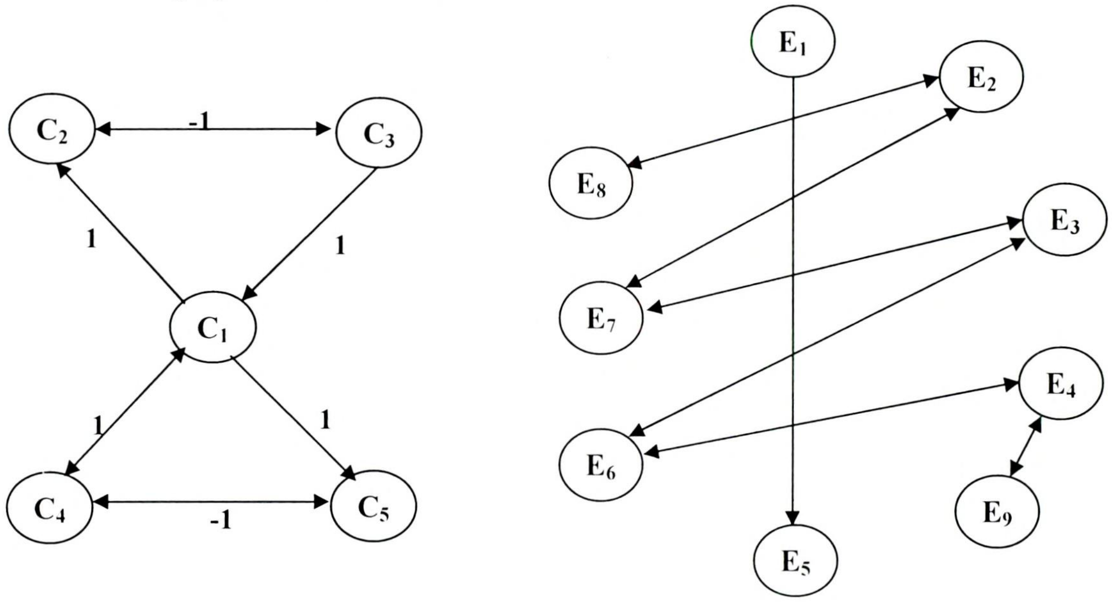
E_6 – Only pay to employee

E_7 – Employee workers more number of hours.

E_8 – Average performance by few employee

E_9 – Poor performance by some employee

The directed bigraphs related with the model is as follows.



Here, the attributes $\{C_1, C_2, C_3, C_4, C_5\}$ are disjoint with the attributes $\{E_1, E_2, \ldots, E9\}$.

This bigraph is a disjoint bigraph as we have no common attributes with the given set of concepts.

The related bimatrix or the connection bimatrix is a mixed square bimatrix B.

$$B = B_1 \cup B_2.$$

$$
B = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix} \end{array} \cup \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \\ E_7 \\ E_8 \\ E_9 \end{array}\!\!\begin{array}{c} \begin{array}{ccccccccc} E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_8 & E_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}
$$

Now this bimatrix can be used to find the effect of any state vector. Here it is important to note that a state vector for any bimatrix which will only be a row bivector. If we wish to make only one system to work then in the initial row bivector we use one of the row vectors to be a zero vector. Now we explain how this bimodel functions.

Any initial state vector would be a pair of row bivector in this case any state bivector

$$X = \{(a_1^1, a_2^1, a_3^1, a_4^1, a_5^1) \cup (a_1^2, a_2^2, a_3^2, a_4^2, a_5^2, \ldots, a_9^2,)\}$$

$$a_j^1 = \begin{cases} 0 \text{ is the } j^{th} \text{ state is off} \\ 1 \text{ is the } j^{th} \text{ state is on} \end{cases}$$

$$a_j^2 = \begin{cases} 0 \text{ is the } j^{th} \text{ state is off} \\ 1 \text{ is the } j^{th} \text{ state is on} \end{cases}$$

This bivector X will be known as the instantaneous state bivector.

Now we study the effect of any state bivector on the dynamical bisystem

$$B = B_1 \cup B_2.$$

Now we find the effect of the initial state bivector

$$X = (1\ 0\ 0\ 0\ 0) \cup (0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$$

$$= X_1 \cup X_2 \text{ on } B = B_1 \cup B_2.$$

$$XB = (X_1 \cup X_2)\, B$$

$$= (X_1 \cup X_2)\,(B_1 \cup B_2)$$

$$= X_1 B_1 \cup X_2 B_2$$

$$= [(0\ 1\ 0\ 1\ 1) \cup (0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0)].$$

Now updating and thresholding the bivector we get

$$XB = [(1\ 1\ 0\ 1\ 1\ ) \cup (0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0)]$$
$$= Y = Y_1 \cup Y_2.$$
$$YB = (Y_1 \cup Y_2\ ) (B)$$
$$= (Y_1 \cup Y_2\ ) (B_1 \cup B_2)$$
$$= Y_1 B_1 \cup Y_2 B_2$$
$$= [(1\ 1\ -1\ 0\ 0)] \cup [(0\ 2\ 1\ 0\ 0\ 0\ 1\ 1\ 0)];$$

after updating and thresholding we get the resultant as

$$= Z$$
$$= [(1\ 1\ 0\ 0\ 0) \cup (0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0)]$$
$$= Z_1 \cup Z_2.$$

Consider

$$ZB = (Z_1 \cup Z_2) B$$
$$= (Z_1 \cup Z_2) (B_1 \cup B_2)$$
$$= Z_1B_1 \cup Z_2B_2$$
$$= [(0\ 1\ -1\ 1\ 1)] \cup [(0\ 2\ 1\ 0\ 0\ 1\ 1\ 1\ 0)]$$

after updating and thresholding we get the resultant bivector as S where

$$S = [(1\ 1\ 0\ 1\ 1) \cup (0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0)]$$
$$= S_1 \cup S_2.$$

Now the effect of S on the dynamical system B is given by

$$BS = (1\ 1\ -1\ 0\ 0) \cup (0\ 2\ 2\ 1\ 0\ 2\ 2\ 1\ 1)$$

after updating and thresholding the state bivector we get the resultant bivector as

$$R = (1\ 1\ 0\ 0\ 0) \cup (0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1)$$
$$= R_1 \cup R_2\ .$$

Now we study the effect of R on the dynamical system B,

$$RB = (0\ 1\ -1\ 1\ 1) \cup (0\ 2\ 2\ 1\ 0\ 2\ 2\ 1\ 1)$$

after updating and thresholding resultant state bivector we get the resultant bivector; we get the resultant vector T as

$$T = (1\ 1\ 0\ 1\ 1) \cup (0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1)$$
$$= T_1 \cup T_2.$$

Now the effect of T on the bimatrix B is given by

$$TB = (T_1 \cup T_2)\ (B_1 \cup B_2)$$
$$= T_1\ B_1 \cup T_2\ B_2$$
$$= (1\ 1\ -1\ 0\ 0) \cup (0\ 2\ 2\ 2\ 0\ 2\ 2\ 1\ 1)$$

after thresholding and updating the resultant bivector we get the bivector as

$$A = [(1\ 1\ 0\ 0\ 0\ ) \cup (0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1)]$$
$$= A_1 \cup A_2$$

which is a hidden pattern. The hidden pattern is a limit cycle combined with the fixed point given by

$$\{(1\ 1\ 0\ 1\ 1) \cup (0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1)\} \text{ or}$$
$$\{(1\ 1\ 0\ 0\ 0) \cup (0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1)\}.$$

So in the system B when we consider a initial state bivector $(1\ 0\ 0\ 0\ 0\ ) \cup (0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ )$ i.e. Good business coupled with just profit to the employer gives good investment with other states like neither profit nor loss to the employer is on, when he contemplates on good business.

One can expect to give only pay to employee and for good business with just profit, the employee works for more number of hours, it still gives only an average performance by few employee and poor performance by some employee.

Thus this state bivector $(1\ 0\ 0\ 0\ 0) \cup (0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ )$ gives a unique form of the hidden pattern i.e. one row vector in the birow vector is a limit cycle and the other happens to be a fixed point.

It is important to note here that at times, the hidden pattern is such that both the bivectors are fixed point or both of them are limit cycles. In case when both are limit cycle it is still interesting to note that the interpretation of the state bivectors vary from stage to stage, which is pertinent for in practical situation such types of solutions are possible in the real world problems that too when we use an unsupervised data. Thus we can get four possibilities as the hidden pattern which we may from now onwards call as bihidden pattern. It may be a fixed bipoint or limit bicycle or fixed point and limit cycle.

Now we consider the state bivector

$$Y = (0\ 0\ 1\ 0\ 0) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)$$
$$= Y_1 \cup Y_2$$

i.e. only the state C3 i.e. the customer satisfaction and E5 – Best performance by the employees are in the on state all other state vectors in the bivector is in the off state. Now we

analyze the effect of the state bivector Y on the dynamical

system B.

$$YB = (Y_1 \cup Y_2)(B_1 \cup B_2)$$
$$= Y_1 B_1 \cup Y_2 B_2$$
$$= (1\ -1\ 0\ 0\ 0) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)$$

(after thresholding and updating the resultant bivector) we get Z

$$Z = (1\ 0\ 1\ 0\ 0) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)$$
$$= Z_1 \cup Z_2 .$$
$$ZB = Z_1 B_1 \cup Z_2 B_2$$
$$= (1\ 0\ 0\ 1\ 1) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0) .$$

After updating we get the resultant state bivector

$$X = (1\ 0\ 1\ 1\ 1) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)$$

The effect of the bivector X on the dynamical system B gives

$$XB = (2\ 0\ 0\ 0\ 0) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)$$

After updating and thresholding we get the resultant bivector as

$$T = T_1 \cup T_2$$
$$= (1\ 0\ 1\ 0\ 0) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)$$

The effect the birow vector T on B gives

$$TB = T_1 B_1 \cup T_2 B_2$$
$$= (1\ 0\ 0\ 1\ 1) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)$$

After thresholding and updating the resultant bivector is

$$U = (1\ 0\ 1\ 1\ 1) \cup (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)$$
$$= U_1 \cup U_2 .$$

Thus the bihidden pattern is a fixed point and a limit cycle, we see when the concept / attribute, Best Performance by the employee is in the on state the system remains static on other attribute ever becomes on. But on the other hand when the customer satisfaction is in the on state we see the bihidden pattern is a limit cycle which at one point makes all the states to be or C_2 alone is in the off state. Thus it fluctuates from (1 0 1 0 0) and (1 0 1 1 1).

It is still important to note that when we analyze a problem with a FCBMs both the sets of attributes need not always be disjoint. It can also be a over lapping set. For even to analyze the same problem one can use FCBMs.

The application of FCBMs to analyze the factors promoting business. The opinion of two experts is taken simultaneously. The attributes given by the first expert is

E_1 – Good business

E_2 – Good investment

E_3 – Customer satisfaction

E_4 – Establishment

E_5 – Good Marketing Strategies.

The attributes given by the second expert is as follows

E'_1 – Good business

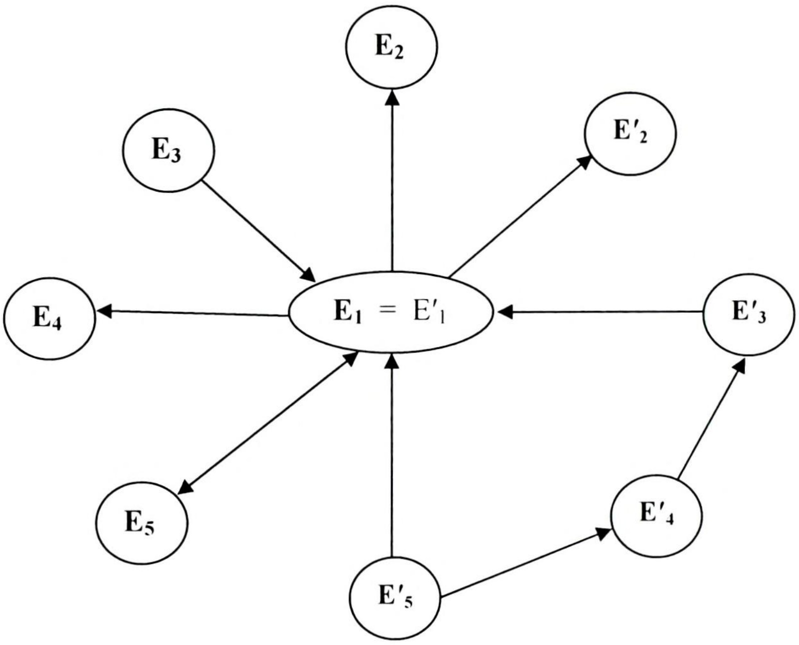
E'_2 – Appropriate locality

E'_3 – Selling quality products

E'_4 – Updation of techniques

E'_5 – Knowledge about the policies of the government.

Now the related directed bigraph is as follows.



Clearly this is a connected directed bigraph.

Now the associated connection bimatrix is given by $B = B_1 \cup B_2$

$$\begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{array} \begin{array}{c} \begin{array}{ccccc} E_1 & E_2 & E_3 & E_4 & E_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \cup \begin{array}{c} \\ E'_1 \\ E'_2 \\ E'_3 \\ E'_4 \\ E'_5 \end{array} \begin{array}{c} \begin{array}{ccccc} E'_1 & E'_2 & E'_3 & E'_4 & E'_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Suppose one is interested in studying the effect of the row bivector

$$X = X_1 \cup X_2$$
$$= (1\ 0\ 0\ 0\ 0) \cup (1\ 0\ 0\ 0\ 0).$$

The effect of X on B is given by

$$XB = X (B_1 \cup B_2)$$
$$= X_1 B_1 \cup X_2 B_2$$
$$= (0\ 1\ 0\ 1\ 1\ ) \cup (0\ 1\ 0\ 0\ 0\ )$$

After updating and thresholding we get

$$Y = (1\ 1\ 0\ 1\ 1) \cup (1\ 1\ 0\ 0\ 0)$$
$$= Y_1 \cup Y_2$$

The effect of Y on B is given by

$$YB = Y_1 B_1 \cup Y_2 B_2.$$

$$= (1\ 1\ 0\ 1\ 1) \cup (0\ 1\ 0\ 0\ 0).$$

After updating and thresholding we get the resultant vector as

$$Z = (1\ 1\ 0\ 1\ 1) \cup (1\ 1\ 0\ 0\ 0);$$

which is a fixed bipoint. Good business according to the first expert has no impact on Customer satisfaction but it has influence on good investment, establishment and good Marketing strategies. But at the same time we see according to the second expert good business has to do only with the appropriate locality and nothing to do with selling quality products or updation or knowledge about the polices of the government. As they are the opinion given by the expert we have no right to change or modify these effects.

Next we go for the opinion of the third expert. He gives the attributes as

E''_1 – Good business

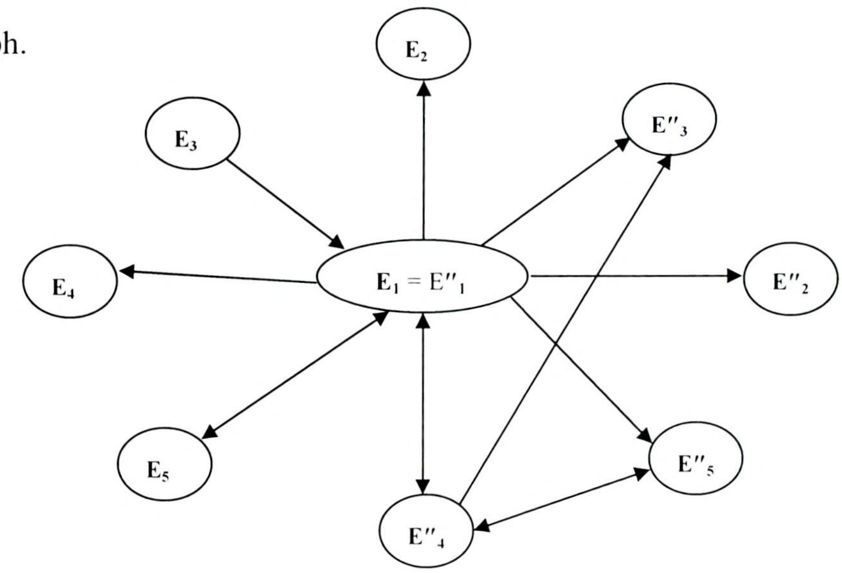
E''_2 – Geographical situation

E''_3 – Rendering good service

E''_4 – Previous experience of the owner

E''_5 – Demand and supply.

Now using the first and third experts opinions we have the following directed bigraph.



The related connection bimatrix. $B = B_1 \cup B_2$.

$$\begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{array} \begin{array}{c} \begin{array}{ccccc} E_1 & E_2 & E_3 & E_4 & E_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \cup \begin{array}{c} \\ E''_1 \\ E''_2 \\ E''_3 \\ E''_4 \\ E''_5 \end{array} \begin{array}{c} \begin{array}{ccccc} E''_1 & E''_2 & E''_3 & E''_4 & E''_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Now we study the effect of the state bivector on the dynamical bisystem.

Let $X = (0\ 1\ 0\ 0\ 1) \cup (1\ 0\ 1\ 0\ 0)$

i.e., the first expert considers good investment and good marketing strategies in the on state and good business and rendering good service to be in the on states i.e., the bivector given by them has E_2, E_5, E''_1 and E''_5 to be in the on state and all other nodes are in the off state. We study the effect of

$$X = X_1 \cup X_2 \text{ on } B.$$

$$XB = (X_1 \cup X_2)(B_1 \cup B_2)$$

$$= X_1 B_1 \cup X_2 B_2.$$

$$= (1\ 0\ 0\ 0\ 0) \cup (1\ 1\ 1\ 1\ 1).$$

after updating we get the resultant

$$Y = (1\ 1\ 0\ 0\ 1) \cup (1\ 1\ 1\ 1\ 1)$$

$$= Y_1 \cup Y_2.$$

the effect of Y on B gives

$$YB = Y_1 B_1 \cup Y_2 B$$

$$= (1\ 1\ 0\ 1\ 1) \cup (2\ 1\ 1\ 2\ 2)$$

after thresholding and updating we get

$$Z = (1\ 1\ 0\ 1\ 1) \cup (1\ 1\ 1\ 1\ 1)$$

$$= Y.$$

Thus the bihidden pattern is a fixed bipoint. Thus good investment and good marketing strategies has no impact on other nodes; where as the on state of good business and rendering good service makes on all other states.