Chapter III

Soft Mappings on Soft Topological Spaces

CHAPTER – III

SOFT MAPPINGS ON SOFT TOPOLOGICAL SPACES

Definition : 3.1

Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, f : (X, τ, E) $\rightarrow$ (Y, τ', E) be a mapping. For each soft neighborhood (H, E) of ($f(x)_e$, E), if there exists a soft neighborhood (F, E) of (x_e, E) such that f((F, E)) $\subset$ (H, E), then f is said to be soft continuous mapping at (x_e, E).

If f is soft continuous mapping for all (x_e, E), then f is called **soft continuous mapping**.

Theorem : 3.2

Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, f : (X, τ, E) $\rightarrow$ (Y, τ', E) be a mapping. Then the following conditions are equivalent :

(1) f : (X, τ, E) $\rightarrow$ (Y, τ', E) is a soft continuous mapping,

(2) For each soft open set (G, E) over Y, f^{-1} ((G, E)) is a soft open set over X,

(3) For each soft closed set (H, E) over Y, f^{-1} ((H, E)) is a soft closed set over X,

(4) For each soft set (F, E) over X, $f(\overline{(F,E)}) \subset \overline{(f(F,E))}$,

(5) For each soft set (G, E) over Y, $\overline{(f^{-1}(G,E))} \subset f^{-1}\overline{((G,E))}$,

(6) For each soft set (G, E) over Y, $f^{-1}((G, E)^{o}) \subset (f^{-1}(G, E))^{o}$

Proof

(1) $\Rightarrow$ (2) Let (G, E) be a soft open set over Y and (x_e, E) $\in$ f^{-1}(G, E) be an arbitrary soft point. Then f(x_e, E) = ($f(x)_e$, E) $\in$ (G, E). Since f is soft continuous mapping, there exists $(x_e, E) \in (F, E) \in \tau$ such that $f(F, E) \subset (G, E)$. This implies that $(x_e, E) \in (F, E) \subset f^{-1}((G, E))$ is a soft open set over X.

(2) ⇒ (1) Let (x_e, E) be a soft point and $(f(x)_e, E) \in (G, E)$ be an arbitrary soft neighborhood. Then $(x_e, E) \in f^{-1}(G, E)$ is a soft neighborhood and $f(f^{-1}(G, E)) \subset (G, E)$.

(3) ⇒ (4) Let (F, E) be a soft set over X. Since $(F, E) \subset f^{-1}(f(F, E))$ and $f(F, E) \subset \overline{(f(F,E))}$, we have $(F, E) \subset f^{-1}(f(F, E)) \subset f^{-1}\overline{(f(F,E))}$. By part (3), since $f^{-1}\overline{(f(F,E))}$ is a soft closed set over X, $\overline{(F,E)} \subset f^{-1}\overline{(f(F,E))}$. Thus $f\overline{((F,E))} \subset f(f^{-1}\overline{(f(F,E))}) \subset \overline{f(F,E)}$ is obtained.

(4) ⇒ (5) Let (G, E) be a soft set over Y and $f^{-1}(G, E) = (F, E)$. By part (4), we have $f\overline{((F,E))} = f\overline{(f^{-1}(G,E))} \subset \overline{f(f^{-1}(G,E))} \subset \overline{(G,E)}$. Then $\overline{f^{-1}(G,E)} = \overline{(F,E)} \subset f^{-1}\overline{(f(F,E))} \subset f^{-1}\overline{((G,E))}$.

(5) ⇒ (6) Let (G, E) be a soft set over Y. Substituting $(G,E)'$ for conditon in (5). Then $\overline{f^{-1}((G,E)')} \subset f^{-1}\overline{((G,E)')}$. Since $(G, E)^o = \overline{((G,E)')}'$, then we have $f^{-1}((G, E)^o) = (f^{-1}\overline{((G,E)')})' = f^{-1}\overline{((G,E)')}' \subset (\overline{f^{-1}((G,E)')})' = \overline{((f^{-1}(G,E))')}' = (f^{-1}(G, E))^o$.

(6) ⇒ (2) Let (G, E) be a soft open set over Y. Then since $(f^{-1}(G, E))^o \subset f^{-1}(G, E) = f^{-1}((G, E)^o) \subset (f^{-1}(G, E))^o$, $(f^{-1}(G, E))^o = f^{-1}(G, E)$ is obtained. This implies that $f^{-1}(G, E)$ is a soft open set over X.

Example : 3.3

Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E)\}$, $\tau' = \{\Phi, \tilde{X}, (G_1, E), (G_2, E)\}$ be two soft topologies defined on X, where (F_1, E), (F_2, E), (G_1, E) and (G_2, E) are soft sets over X, defined as follows :

$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = \{h_3\}$, $F_2(e_1) = X$, $F_2(e_2) = \{h_3\}$,

and

$G_1(e_1) = \{h_1\}$, $G_1(e_2) = \{h_3\}$, $G_2(e_1) = \{h_1, h_3\}$, $G_2(e_2) = \{h_2, h_3\}$,

If we get the mapping $f : X \to X$ defined as

$f(h_1) = f(h_2) = h_1$, $f(h_3) = h_3$

then since $f^{-1}(G_1, E) = (F_1, E)$ and $f^{-1}(G_2, E) = (F_2, E)$, f is a soft continuous mapping.

Example : 3.4

Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, $\tau' = \{\Phi, \tilde{X}, (G_1, E), (G_2, E), (G_3, E), (G_4, E)\}$ be two soft topologies defined on X where (F_1, E), (F_2, E), (F_3, E), (F_4, E), (G_1, E), (G_2, E), (G_3, E) and (G_4, E) are soft sets over X, defined as follows :

$F_1(e_1) = \{h_2\}$, $F_1(e_2) = \{h_1\}$, $F_2(e_1) = \{h_2, h_3\}$, $F_2(e_2) = \{h_1, h_2\}$,

$F_3(e_1) = \{h_3\}$, $F_3(e_2) = \{h_1, h_2\}$, $F_4(e_1) = \Phi$, $F_4(e_2) = \{h_1\}$,

$F_5(e_1) = X$, $F_2(e_2) = \{h_1, h_2\}$

and

$G_1(e_1) = \{h_2\}$, $G_1(e_2) = \{h_1\}$, $G_2(e_1) = \{h_2, h_3\}$, $G_2(e_2) = \{h_1, h_2\}$,

$G_3(e_1) = \{h_1, h_2\}$, $G_3(e_2) = X$, $G_4(e_1) = \{h_2\}$, $G_4(e_2) = \{h_1, h_2\}$,

Then (X, τ, E), (X, τ', E) are two soft topological spaces and $f = 1_X : X \to X$ is not soft continuous mapping.

Definition : 3.5

Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : X \to Y$ be a mapping.

(a) If the image $f((F, E))$ of each soft open set (F, E) over X is a soft open set in Y, then f is said to be a **soft open mapping**.

(b) If the image $f((H, E))$ of each soft closed set (H, E) over X is a soft close set in Y, then f is said to be a **soft closed mapping**.

Theorem : 3.6

If $f : (X, \tau, E) \to (Y, \tau', E)$ is soft open (closed), then for each $\alpha \in E$, $f_\alpha: (X, \tau_\alpha) \to (Y, \tau'_\alpha)$ is an open (closed) mapping.

Note : 3.7

The concepts of soft continuous, soft open and soft closed mappings are all independent of each other.

Example : 3.8

Let (X, τ, E) be soft discrete topological space and (X, τ', E) be soft indiscrete topological space. Then $1_X : (X, \tau, E) \to (X, \tau', E)$ is a soft open and soft closed mapping. But it is not soft continuous mapping.

Example : 3.9

Let $X=\{h_1, h_2, h_3\}$, $E=\{e_1, e_2\}$ and $\tau=\{\Phi, \tilde{X}, (F_1, E), (F_2, E),\ldots,(F_7, E)\}$, $\tau' = \{\Phi, \tilde{X}, (G_1, E), (G_2, E), (G_3, E), (G_4, E)\}$ be two soft topologies defined on X where $(F_1, E), (F_2, E), (F_3, E), \ldots, (F_7, E), (G_1, E), (G_2, E), (G_3, E)$ and (G_4, E) are soft sets over X, defined as follows :

$F_1(e_1) = \{h_2\}$, $F_1(e_2) = \{h_1\}$, $F_2(e_1) = \{h_1, h_3\}$, $F_2(e_2) = \{h_2, h_3\}$,

$F_3(e_1) = \{h_2\}$, $F_3(e_2) = X$, $F_4(e_1) = \Phi$, $F_4(e_2) = \{h_1\}$,

$F_5(e_1) = \{h_1, h_3\}$, $F_5(e_2) = X$, $F_6(e_1) = \Phi$, $F_6(e_2) = \{h_2, h_3\}$,

$F_7(e_1) = \Phi$, $F_7(e_2) = X$

and

$G_1(e_1) = \{h_2\}$, $G_1(e_2) = \{h_1\}$, $G_2(e_1) = \{h_2, h_3\}$, $G_2(e_2) = \{h_1, h_2\}$,

$G_3(e_1) = \{h_1, h_2\}$, $G_3(e_2) = X$, $G_4(e_1) = \{h_2\}$, $G_4(e_2) = \{h_1, h_2\}$,

If we get the mapping $f : X \to X$ defined as $f(h_i) = h_i$, for $1 \leq i \leq 3$. It is clear that $f^{-1}(G_1)\ (e_1) = f^{-1}(G_4)\ (e_1) = \Phi$, $f^{-1}(G_1)\ (e_2) = f^{-1}(G_4)\ (e_2) = X$, $f^{-1}(G_3)\ (e_1) = X$, $f^{-1}(G_3)\ (e_2) = X$.

Then f is a soft continuous mapping, but

$f(F_1)\ (e_1) = \{h_1\}$, $f(F_1)\ (e_2) = \{h_1\}$, $f(F_1')(e_1) = \{h_1\}$, $f(F_1')(e_2) = \{h_1\}$.

Hence it is not both soft open and soft closed mapping.

Example : 3.10

Let $X = \{h_1, h_2, h_3\}$, $Y = \{a, b\}$ and $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E)\}$, $\tau' = \{\Phi, \tilde{Y}, (G_1, E), (G_2, E)\}$ be two soft topologies defined on X and Y, respectively. Here (F_1, E), (F_2, E), (G_1, E), (G_2, E) are soft sets over X and Y, respectively. The soft sets are defined as follows :

$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = \{h_3\}$, $F_2(e_1) = X$, $F_2(e_2) = \{h_3\}$,

and

$G_1(e_1) = Y$, $G_1(e_2) = \{b\}$, $G_2(e_1) = \{a\}$, $G_2(e_2) = \{b\}$,

If we get the mapping $f : X \to Y$ defined as

$f(h_1) = \{a\}$, $f(h_2) = f(h_3) = \{b\}$.

It is clear that

$f(F_1)\ (e_1) = Y$, $f(F_1)\ (e_2) = \{b\}$, $f(F_2)\ (e_1) = Y$, $f(F_2)\ (e_2) = \{b\}$.

Then the mapping $f : X \to Y$ is a soft open mapping. Also since $f(F_1')(e_1) = \{b\}$, $f(F_1')(e_2) = Y$, it is not soft closed mapping and $f^{-1}(G_1)(e_1) = X$, $f^{-1}(G_1)(e_2) = \{h_2, h_3\}$. Hence it is not soft continuous mapping.

Example : 3.11

Let $X = \{h_1, h_2, h_3\}$, $Y = \{a, b\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, $\tau' = \{\Phi, \tilde{Y}, (G_1, E), (G_2, E)\}$ be two soft topologies defined on X and Y, respectively. Here (F_1, E), (F_2, E), (F_3, E), (G_1, E), (G_2, E) are soft sets over X and Y, respectively. The soft sets are defined as follows :

$F_1(e_1) = \{h_1, h_3\}$, $F_1(e_2) = \{h_2\}$, $F_2(e_1) = X$, $F_2(e_2) = \{h_2, h_3\}$, $F_3(e_1) = \{h_3\}$, $F_3(e_2) = \{h_2\}$

and

$G_1(e_1) = \Phi$, $G_1(e_2) = \{a\}$, $G_2(e_1) = \{a\}$, $G_2(e_2) = Y$.

Now we define the mapping $f : X \to Y$ as $f(h_1) = f(h_2) = \{a\}$, $f(h_3) = \{b\}$. It is clear that $f(F_1'(e_1)) = f(h_2) = \{a\}$, $f(F_1'(e_2)) = f(\{h_1, h_3\}) = Y$, $f(F_2'(e_1)) = \Phi$, $f(F_2'(e_2)) = f(\{h_1\}) = \{a\}$, $f(F_3'(e_1)) = \{a\}$, $f(F_3'(e_2)) = Y$. This implies that f is soft closed mapping. Also $f(F_1(e_1)) = Y$, $f(F_1(e_2)) = \{a\}$, $f^{-1}(G_1(e_1)) = \Phi$, $f^{-1}(G_1(e_2)) = \{h_1, h_2\}$. Then it is not soft open and soft continuous mapping, respectively.

Theorem : 3.12

Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : X \to Y$ be a mapping.

(a) f is a soft open mapping if and only if for each soft set (F, E) over X, $f((F, E)^o) \subset (f(F, E))^o$ is satisfied.

(b) f is a soft closed mapping if and only if for each soft set (F, E) over X, $\overline{(f(F, E))} \subset f\overline{((F, E))}$ is satisfied.

Proof

(a) Let f be a soft open mapping and (F, E) be a soft set over X. $(F, E)^{o}$ is a soft open set and $(F, E)^{o} \subset (F, E)$. Since f is a soft open mapping, $f((F, E)^{o})$ is a soft open set in Y and $f((F, E)^{o}) \subset f((F, E))$. Thus $f((F, E)^{o}) \subset f((F, E))^{o}$ is obtained.

Conversely, let (F, E) be any soft open set over X.

Then $(F, E) = (F, E)^{o}$. From the condition of theorem, we have $f((F, E)^{o}) \subset (f(F, E))^{o}$. Then $f((F, E)) = f((F, E)^{o}) \subset (f(F, E))^{o} \subset f((F, E))$. This implies that $f((F, E)) = (f(F, E))^{o}$.

This completes the proof.

(b) Let f be a soft closed mapping and (F, E) be any soft set over X. Since f is a soft closed mapping, $f(\overline{(F,E)})$ is a soft closed set over Y and $f((F, E)) \subset f(\overline{(F,E)})$.

Thus $\overline{f(F,E)} \subset f(\overline{(F,E)})$ is obtained.

Conversely, let (F, E) be any soft closed set over X. From the condition of theorem, $\overline{(f(F,E))} \subset f(\overline{(F,E)}) = f((F, E)) \subset \overline{(f(F,E))}$. This means that $\overline{(f(F,E))} = f((F, E))$. This completes the proof.

Definition : 3.13

Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : X \rightarrow Y$ be a mapping. If f is a bijection, soft continuous and f^{-1} is a soft continuous mapping, then f is said to be **soft homeomorphism** from X to Y. When a homeomorphism f exists between X and Y, we say that X is soft homeomorphic to Y.

Theorem : 3.14

Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : X \to Y$ be a bijective mapping. Then the following conditions are equivalent :

1) f is a soft homeomorphism.
2) f is a soft continuous and soft closed mapping.
3) f is a soft continuous and soft open mapping.

SOFT pu-CONTINUOUS FUNCTIONS

Definition : 3.15

Let (U, τ, A) and (V, τ^*, B) be soft topological spaces. Let $u : U \to V$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(U)_A \to SS(V)_B$ be a function and $e_F \tilde{\in} U_A$.

(a) f_{pu} is soft pu-continuous at $e_F \tilde{\in} U_A$ if for each (G, B) $\in N_{\tau^*}(f_{pu}(e_F))$, there exists a (H, A) $\in N_\tau(e_F)$ such that $f_{pu}(H, A) \tilde{\subseteq} (G, B)$.

(b) f_{pu} is soft pu-continuous on U_A if f_{pu} is soft continuous at each soft point in U_A.

Theorem : 3.16

Let (U, τ, A) and (V, τ^*, B) be soft topological spaces. Let $f_{pu} : SS(U)_A \to SS(V)_B$ be a function and $e_F \tilde{\in} U_A$. Then the following statements are equivalent.

(a) F_{pu} is soft pu-continuous at e_F ;

(b) For each (G, B) $\in N_{\tau^*}(f_{pu}(e_F))$, there exists a (H, A) $\in N_\tau(e_F)$ such that (H, A) $\tilde{\subseteq} f_{pu}^{-1}(G, B)$;

(c) For each (G, B) $\in N_{\tau^*}(f_{pu}(e_F))$, $f_{pu}^{-1}(G, B) \in N_\tau(e_F)$.

Theorem : 3.17

Let (U, τ, A) and (V, τ^*, B) be soft topological spaces. Let f_{pu} : $SS(U)_A \to SS(V)_B$ be a function. Then the following statements are equivalent.

(a) f_{pu} is soft pu-continuous ;

(b) For each (H, B) $\in \tau^*$, f^{-1}((H, B)) $\in \tau$;

(c) For each soft closed set (F, B) over V, f_{pu}^{-1}(F, B) is soft closed over U.

Proof : (a) $\Rightarrow$ (b)

Let (H, B) $\in$ τ^* and e_F $\tilde{\in}$ f_{pu}^{-1}(H, B). We will show that f_{pu}^{-1}(H, B) $\in$ $N_\tau(e_F)$. Since $f_{pu}(e_F)$ $\tilde{\in}$ (H, B) and (H, B) $\in$ τ^*, (H, B) $\in$ N_{τ^*} ($f_{pu}(e_F)$). Since f_{pu} is soft pu-continuous at e_F, there exists (M, A) $\in$ $N_\tau(e_F)$ such that f_{pu}(M, A) $\tilde{\subseteq}$ (H, B). Therefore, we have e_F $\tilde{\in}$ (M, A) $\tilde{\subseteq}$ f_{pu}^{-1}(H, B) and so f_{pu}^{-1}(H, B) $\in$ $N_\tau(e_F)$.

(b) $\Rightarrow$ (c). Let (F, B) be soft closed over V. Then $(f, B)^c \in \tau^*$ and by (b), $f_{pu}^{-1}((F, B)^c) \in \tau$. Since $f_{pu}^{-1}((F, B)^c)$ = $(f_{pu}^{-1}((F, B))^c$, we have that f_{pu}^{-1}(F, B) is soft closed over U.

(c) $\Rightarrow$ (b). It is similar to that of (b) $\Rightarrow$ (c).

(b) $\Rightarrow$ (a). Let e_F $\tilde{\in}$ U_A and (G, B) $\in$ N_{τ^*} ($f_{pu}(e_F)$). Then there is a soft open set (H, B) $\in \tau^*$ such that $f_{pu}(e_F)$ $\tilde{\in}$ (H, B) $\tilde{\subseteq}$ (G, B). By (b), f_{pu}^{-1}(H, B) $\in \tau$ and e_F $\tilde{\in}$ f_{pu}^{-1}(H, B) $\tilde{\subseteq}$ f_{pu}^{-1}(G, B). This shows that f_{pu}^{-1}(G, B) $\in$ $N_\tau(e_F)$. Therefore, we have f_{pu} is soft pu-continuous at every point e_F $\tilde{\in}$ U_A.

Theorem : 3.18

Let (U, τ, A) and (V, τ^*, B) be soft topological spaces. For a function f_{pu} : $SS(U)_A \rightarrow SS(V)_B$, consider the following statements :

(a) f_{pu} is soft pu-continuous ;

(b) for each soft set (F, A) over U, the inverse image of every neighborhood of f_{pu}(F, A) is a neighborhood of (F, A) ;

(c) for each soft set (F, A) over U and each neighborhood (H, B) of f_{pu}(F, A), there is a neighborhood (G, A) of (F, A) such that f_{pu}(G, A) $\tilde{\subseteq}$ (H, B) ;

(d) For each sequence {(F_n, A) : n = 1, 2, ...} of soft sets over U which converges to a soft set (F, A) over U, the sequence {f((F_n, A)) : n = 1, 2, ...} converges to f_{pu}(F, A).

Then we have (a) $\Leftrightarrow$ (b) $\Leftrightarrow$ (c) $\Rightarrow$ (d). Moreover, if the neighborhood system of each soft set over U is countable, then (d) implies (a) and hence all of the above statements are equivalent.

Proof : (a) $\Rightarrow$ (b)

Let f_{pu} be soft pu-continuous. If (H, B) is a neighborhood of f_{pu}(F, A), then (H, B) contains a soft open neighborhood (G, B) of f_{pu}(F, A). Since f_{pu}(F, A) $\tilde{\subseteq}$ (G, B) $\tilde{\subseteq}$ (H, B), f_{pu}^{-1}(f_{pu}(F, A)) $\tilde{\subseteq}$ f_{pu}^{-1}(G, B) $\tilde{\subseteq}$ f_{pu}^{-1}(H, B). But (F, A) $\tilde{\subseteq}$ f_{pu}^{-1} (f_{pu}(F, A)) and f_{pu}^{-1}(G, B) is soft open. Consequently, f_{pu}^{-1}(H, B) is a neighborhood of (F, A).

(b) $\Rightarrow$ (a). We will use theorem 3.17. Let (G, B) be soft open over V. Then f_{pu}^{-1}(G, B) is a soft subset of U_A. Let (F, A) be any soft subset of f_{pu}^{-1}(G, B). Then (G, B) is a soft open neighborhood of f_{pu}(F, A), and by (b), f_{pu}^{-1}(G, B) is a soft neighborhood of (F, A). This shows that f_{pu}^{-1}(G, B) is a soft open set by Theorem 2.24.

(b) ⇒ (c). Let (F, A) be any soft set over U and let (H, B) be any neighborhood of f_{pu}(F, A). By (b), f_{pu}^{-1}(H, B) is a neighborhood of (F, A). Then there exists a soft open set (G, A) in U_A such that (F, A) $\tilde{\subseteq}$ (G, A) $\tilde{\subseteq}$ f_{pu}^{-1}(H, B). Thus, we have a soft open neighborhood (G, A) of (F, A) such that f_{pu}(F, A)$\tilde{\subseteq}$ f_{pu}(G, A) $\tilde{\subseteq}$ (H, B).

(c) ⇒ (b). Let (H, B) be a neighborhood of f_{pu}(F, A). Then there is a neighborhood (G, A) of (F, A) such that f_{pu}(G, A) $\tilde{\subseteq}$ (H, B). Hence f_{pu}^{-1}(f_{pu}(G, A)) $\tilde{\subseteq}$ f_{pu}^{-1}(H, B). Furthermore, since (G, A) $\tilde{\subseteq}$ f_{pu}^{-1} (f_{pu} (G, A)), f_{pu}^{-1}(H, B) is a neighborhood of (F, A).

(c) ⇒ (d). If (H, B) is a neighborhood of f_{pu}(F, A), there is a neighborhood (G, A) of (F, A) such that f_{pu}(G, A) $\tilde{\subseteq}$(H, B). Since {(F_n, A) : n = 1, 2, …} is eventually in (G, A), we have f_{pu}(F_n, A) $\tilde{\subseteq}$ f_{pu}(G, A) $\tilde{\subseteq}$ (H, B) for n ≥ m ; i.e., there is an m such that for n ≥ m, (F_n, A) $\tilde{\subseteq}$ (G, A). Therefore, {f_{pu}(F_n, A) : n = 1, 2, …} converges to f_{pu}(F, A).

(d) ⇒ (a). Suppose that the neighborhood system of each soft set over U is countable. Let (G, B) be any soft open set over V. Then f_{pu}^{-1}(G, B) is a soft subset of U_A. Let (F, A) be any soft subset of f_{pu}^{-1}(G, B), and let (F_1, A), (F_2, A), …, (F_n, A), … be the neighborhood system (F, A). Let (H_n, A) = $\tilde{\cap}_{i=1}^{n}$ (F_i, A). Then (H_1, A), (H_2, A), …, (H_n, A), … is a sequence which is eventually contained in each neighborhood of (F, A), i.e., (H_1, A), (H_2, A), …, (H_n, A), … converges to (F, A). Hence, there is an m such that for n ≥ m, (H_n, A) $\tilde{\subseteq}$ f_{pu}^{-1}(G, B). Since for each n, (H_n, A) is a neighborhood of (F, A), f_{pu}^{-1}(G, B) is a neighborhood of (F, A). This shows that f_{pu}^{-1}(G, B) is soft open.