
Introduction

Topology is a branch of mathematics which is good at extracting global qualitative features from complicated geometric structures. General topology known as point set topology is the branch of topology dealing with the basic set theoretic definitions and constructions used in topology. It is the foundation of most other branches of topology, including differential topology, geometric topology, algebraic topology and network topology. The fundamental concepts in point set topology are continuity, compactness and connectedness. The topological structures are modelled suitably in the fields of computer graphics, pattern recognition, artificial intelligence, data mining, information systems, rough set theory, quantum physics etc.

The notion of fuzzy set theory has caused great interest among both pure and applied mathematics. After the introduction of fuzzy sets by Zadeh (1965), several researches were conducted on the generalizations of the notion of fuzzy set. In fuzzy set theory the membership of an element to a fuzzy set is a single value between zero and one. However in reality it may not be always true that the degree of non membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree.

A fuzzy set is created to describe linguistic variables in more detail. The linguistic variable 'temperature' for instance, may have categories (members) of cold, very cold, moderate, warm and very hot. Chang (1968) gave the topological structure to these fuzzy sets called fuzzy topological spaces and studied the topological properties. Most basic concepts like open set, closed set, neighbourhood, interior of a set, continuity and compactness etc. Fuzzy sets are designed to handle a particular kind of uncertainty, namely degree-vagueness, which results a property that can be possessed by objects to varying degrees.

Intuitionistic fuzzy set, an extension of fuzzy set, has been introduced by Atanassov (1986). Intuitionistic fuzzy set has been found to be more efficient in dealing with vagueness and ambiguity. It is characterized by a membership function ($\mu_A(x)$) and a non membership function ($\nu_A(x)$) with their sum being less than or equal to one

$(0 \leq \mu_A(x) + \nu_A(x) \leq 1)$. This relaxes the enforced duality $\nu_A(x) = 1 - \mu_A(x)$ from fuzzy set theory. Intuitionistic fuzzy set allows one to address the positive and negative side of an imprecise concept separately. Intuitionistic fuzzy sets can be more precisely expressed. For example, the fact that the temperature of a patient changes, and other symptoms are not quite clear. There is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object.

Intuitionistic fuzzy set is useful in decision making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. Recently various applications of intuitionistic fuzzy set like artificial intelligence, intuitionistic fuzzy expert systems, intuitionistic fuzzy neural networks, intuitionistic fuzzy decision making, intuitionistic fuzzy machine learning, intuitionistic fuzzy semantic representations etc., have appeared.

Using the notion of intuitionistic fuzzy sets, Coker (1997) has constructed the basic concepts of intuitionistic fuzzy topological spaces. After giving the fundamental definitions and the necessary examples he introduced the definitions of intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, intuitionistic fuzzy connectedness and obtained several preservation properties and some characterizations concerning intuitionistic fuzzy connectedness.

In this research work, a new generalization of closed set called intuitionistic fuzzy γ generalized closed set in intuitionistic fuzzy topological spaces is introduced. Further the corresponding open sets, theoretical applications, continuous mappings, almost continuous mappings, contra continuous mappings, almost contra continuous mappings, irresolute mappings, M-irresolute mappings, completely continuous mappings, closed mappings, open mappings, almost closed mappings, contra open mappings, almost contra open mappings, homeomorphism and connectedness are being introduced and their respective properties, preservation theorems, interrelations are discussed with necessary counter examples.

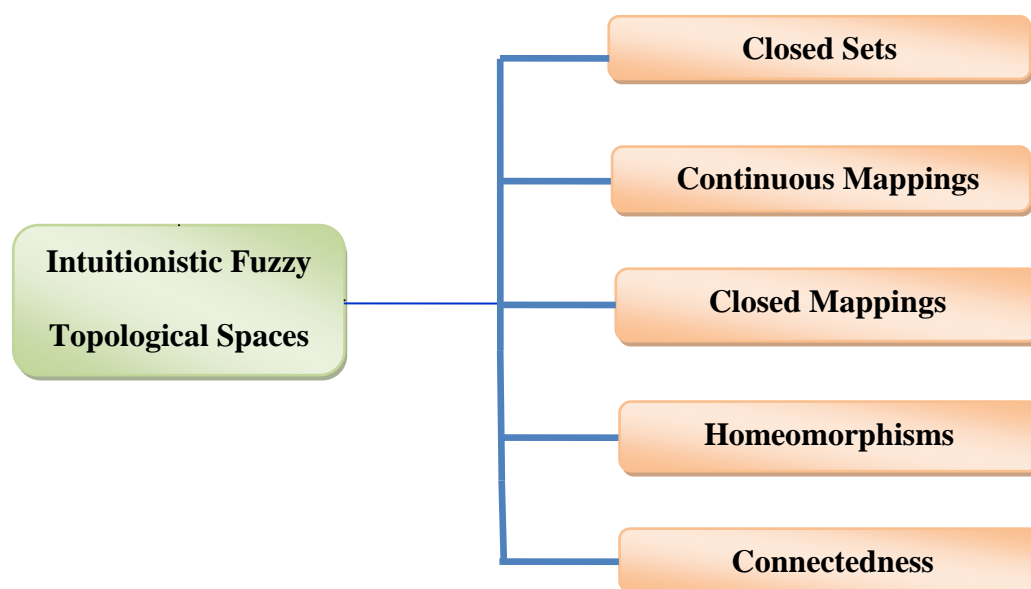
Throughout the thesis, the following notations are used.

- ❖ (X, τ) , (Y, σ) and (Z, η) denote non empty intuitionistic fuzzy topological spaces on which no separation axioms are mentioned unless it is stated specifically.
- ❖ The closure and interior of a subset A of an intuitionistic fuzzy topological space is denoted by $cl(A)$ and $int(A)$ respectively.
- ❖ In all the diagrams $A \rightarrow B$ represents A implies B but not conversely and $A \leftrightarrow B$ represents A and B are independent with each other.

PROFILE OF THE PRESENT WORK

In the present work, a new class of generalized closed sets called intuitionistic fuzzy γ generalized closed sets is defined and their properties are analyzed in intuitionistic fuzzy topological spaces.

Various concepts discussed in this thesis are given in the following diagram.



For this, the thesis is organized in to six chapters, as given below:

Chapter 1 is divided into five sections which deals with the study of preliminary definitions and results on various already existing closed sets, continuous mappings, closed mappings, homeomorphisms and connectedness in intuitionistic fuzzy topological spaces which are used to accomplish the work.

In Chapter 2, the first section gives a short introduction to intuitionistic fuzzy γ closed sets. In the second section we have introduced and studied a new class of sets called intuitionistic fuzzy γ generalized closed sets in intuitionistic fuzzy topological spaces. This section deals with the interrelation of our newly introduced sets with other type of intuitionistic fuzzy closed sets such as intuitionistic fuzzy semi closed sets, intuitionistic fuzzy pre closed sets, intuitionistic fuzzy regular closed sets, intuitionistic fuzzy α closed sets, intuitionistic fuzzy semi-pre closed sets. Also it is shown that the converses are not true in general and they are proved with necessary counter examples. In section three, intuitionistic fuzzy γ generalized open sets are introduced and their interrelations with other intuitionistic fuzzy open sets are established. Also some of their properties are studied. In section four, we have introduced some theoretical application of intuitionistic fuzzy γ generalized closed sets and we have proved some interesting propositions.

In Chapter 3, a new type of intuitionistic fuzzy continuous mapping called intuitionistic fuzzy γ generalized continuous mapping has been introduced. This chapter is divided into six sections. In the first section, we give a short introduction to intuitionistic fuzzy continuous mappings and irresolute mappings. In the second section, we have introduced intuitionistic fuzzy γ generalized continuous mappings and analyzed the interrelations between intuitionistic fuzzy γ generalized continuous mapping with other existing continuous mappings. Also the composition of two intuitionistic fuzzy γ generalized continuous mappings and their respective properties, preservation theorems are discussed with necessary counter examples. Some fascinating theorems concerning intuitionistic fuzzy γ generalized continuous mappings are discussed and we have provided some characterization of intuitionistic fuzzy γ generalized irresolute mappings. In section three, intuitionistic fuzzy contra γ generalized continuous mappings have been

introduced and some of their properties are studied. In section four, intuitionistic fuzzy almost γ generalized continuous mappings and intuitionistic fuzzy almost contra γ generalized continuous mappings have been introduced and some of their properties are studied. We have analyzed the interrelations of the newly defined continuous mappings with other existing continuous mappings and proved the converse relations are not true by giving counter examples. In section five, intuitionistic fuzzy γ generalized irresolute mappings and intuitionistic fuzzy γ generalized M-irresolute mappings have been introduced and some of their properties are proved. In section six, completely γ generalized continuous mappings in intuitionistic fuzzy topological spaces are established and their properties are discussed.

In Chapter 4, the first section presents a short introduction to intuitionistic fuzzy closed mappings and intuitionistic fuzzy open mappings. In the second section, intuitionistic fuzzy γ generalized closed mappings and intuitionistic fuzzy γ generalized open mappings have been introduced and some of their properties are discussed. In this section some characterizations of intuitionistic fuzzy γ generalized closed mappings and intuitionistic fuzzy γ generalized open mappings are proved. Third section deals with the study of intuitionistic fuzzy contra γ generalized open mappings and intuitionistic fuzzy contra M- γ generalized open mappings. Some of their properties and characterizations are studied. Interrelations between them with other open mappings are established with counter examples. In the fourth section, intuitionistic fuzzy almost γ generalized closed mappings and intuitionistic fuzzy almost contra γ generalized closed mappings are introduced and some of their properties are analyzed. Also some interesting characterizations are acquired.

In Chapter 5, the first section begins with the introduction of homeomorphisms in intuitionistic fuzzy topological spaces. In the second section we have introduced intuitionistic fuzzy γ generalized homeomorphism. The interrelations between intuitionistic fuzzy γ generalized homeomorphism with other existing homeomorphism have been obtained. Third section deals with the study of intuitionistic fuzzy M- γ generalized homeomorphism. Furthermore we have discussed some of their properties and preservation theorems with necessary counter examples.

In Chapter 6, the first section gives an introduction to connectedness in intuitionistic fuzzy topological spaces. In the second section, we have introduced the notion of intuitionistic fuzzy γ generalized connected space and intuitionistic fuzzy γ generalized super connected space. Also we have investigated some of their properties and characterized the intuitionistic fuzzy γ generalized super connected space.