

Fuzzy t-continuous mappings

By

T. Gokila

Reg. No. 20PMA006

Supervisor

Dr. D. Jayanthi

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore – 641 043

In Partial Fulfillment of the requirements for the

Degree of Master Science

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N. Balaramani
19.05.2022

Signature of the Head of the Department

[Handwritten Signature]
19/05/2022

Signature of the Supervisor

DECLARATION

I declare that the dissertation, entitled “**Fuzzy t- continuous mappings**” submitted by me for the Degree of Master of Science is a record of work carried out by me during the period from December 2021 to May 2022 under the guidance of **Dr. D. Jayanthi**, Assistant Professor (SS), Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and it has not formed the basis for the award of any Degree, Diploma, Associate ship, Fellowship or other similar title in this University or any other University or Institution of Higher Learning.

A handwritten signature in blue ink, appearing to be 'K. G. X.' with a stylized flourish.

Signature of the Candidate

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INTRODUCTION

TOPOLOGY

Topology was “born” as an identifiable field in the late 19th century. Topology is the branch of mathematics, an extension of geometry. Topology begins with a consideration of the nature of space, investigating both its fine structure and its global structure. The word topology derived from a Greek word TOPOS meaning “place, location” and LOGOS meaning “discourse” or “study”. Topology is the mathematical study of the properties through deformations, twisting and stretching’s of objects. Tearing is not allowed.

Topology, referred to as “rubber sheet geometry”, in which two objects are considered equivalent if they can be continuously deformed into one another through such motions in space as bending, twisting, stretching, and shrinking while disallowing tearing apart or gluing together parts. A set is open if every point in it is an interior point and a set is closed if it contains all of its boundary points.

The main topics of interest in topology are the properties that remain unchanged by such continuous deformations. Topology, while similar to geometry, differs from geometry in that geometrically equivalent objects often share numerically measured quantities, such as lengths or angles, while topologically equivalent objects resemble each other in a more qualitative sense. For example, it is often said that a topologist cannot tell the difference between a doughnut and a coffee cup because one shape can be changed to the other by a sequence of deformations.

In any field, an appropriate perspective, or eyesight, must be defined. That is, what characteristics one is to perceive as the same or different, or what wants to measure. For example, In algebra the notion of sameness is isomorphism (of groups); in geometry it is isometry- a distance preserving transformation. To a topologist, it is homeomorphism: a one – one continuous map with a continuous inverse. Two objects are called the “same” if there is a homeomorphism exists between the two. However, a circle and a line is different. There does not exist a homeomorphism between the two objects.

Another example, Take two strips of paper and glue the ends together, one with no twists at all and the other with a half twists prior to gluing. The first is called annulus; the second is called a Mobius band. So here we conclude these objects are homeomorphic.

Here comes some of the topological problem like the four-colour problem. In colouring a geographic map, we give distinct colours to any two regions that meet along an edge so that we can easily distinguish these regions. What is the minimum number of colours needed to colour any map one might invent. We need at least four colours. Would we ever need more than four? This question was posed by Mobius in 1840 for maps in plane.

In 1890, Five colour problem have introduced. Without any difficulty, we can move to five colour problem. First note that we can assume that the map is trivalent. If there is a vertex with more than three edges, replace it with a small circular face. A new trivalent vertex will be created where each of the edges meet the new face. Now the new map can be coloured with five colours, then by shrinking the newly created circular faces back to points, we have the colouring needed.

Knots, if we take a piece of string, loop it around itself as we please and then splice the ends together, we have a knot. Mathematically, we define a knot to be a subset of \mathbb{R}^3 that is homeomorphic to the circle S^1 . Hence all the knots are homeomorphic to each other. A Knot diagram is simply a projection of the knot into some plane. We project so that it crosses itself at only a finite number of points, with only two strands crossing at each of these points; at those crossings, we indicate over passes and underpasses by leaving a gap in our projection. Although our focus is on knots, there is a closely related notion of links. A link is a set of knots tangled together.

Two knots are equivalent if one can be smoothly deformed to the other in space without cutting or breaking it and without allowing it to pass through itself. For example, if you had a knot made of stretchable rubber, any shape you could twist or bend it into in space is equivalent to the original.

The Latin phrase “analysis situs” may be translated as “analysis of position” and is similar to the phrase geometric sites, meaning “Geometry of position” used in 1735 by the Swiss mathematician Leonhard Euler to describe his solution to Konigsberg bridge problem.

Euler's work on this problem also is cited as the beginning of graph theory, the study of networks of vertices connected by edges, which shares many ideas with topology.

The first practical application of topology was made in the year 1736 by a Swiss mathematician Leonhard Euler, the Konigsberg bridge problem. The solution was purely in the sense of topology.

In Mathematics, a topological space is, roughly speaking, a geometrical space in which closeness is defined but cannot necessarily be measured by a numeric distance. A topological space is a set of points, along with a set of neighborhoods for each point, satisfying a set of axioms relating points and neighborhoods.

In Mathematics, one defines open sets as the members of a given collection of subsets of a given set, a collection that has the property of containing every union of its members, every finite intersection of its members, the empty set and the whole set itself. A set in which such a collection is given is called a topological space and the collection is called a topology.

FUZZY TOPOLOGY

The idea of fuzzy sets was born in July 1964. Lotfi A. Zadeh is a well-respected professor in the department of electrical engineering and computer science at university of California, Berkeley. In the fifties, Professor Zadeh believed that all real – world problems could be solved with efficient, analytical methods or fast electronic computers. In early 1960's, however, he began to feel that traditional system analysis techniques were too precise for many complex real-world problems. In a paper written in 1961, he mentioned that a different kind of mathematics was needed:

“We needed a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not described in terms of probability distribution. Indeed, the need for such mathematics is becoming increasingly apparent, for in most practical cases the a priori data as well as the criteria by which the performance of a man-made system is judged are far from being precisely specified or having accurately known probability distributions.”

Fuzzy set theory is an extension of classical set theory. The word “fuzzy” means “vagueness (ambiguity)”. Fuzziness occurs when the boundary of a piece of information is not clear-cut. Classical set theory allows the membership of the elements in the set in binary terms. Fuzzy set theory permits membership function valued in the interval $[0,1]$. Elements have varying degree of membership. Logic based on two truth values, True and False is sometimes insufficient when describing human reasoning. Fuzzy logic uses the whole interval between 0 (false) and 1 (true) to describe human reasoning. A fuzzy set is any set that allow its members to have different degree of membership, called membership function, having interval $[0,1]$.

A fuzzy set is a set with a smooth boundary. It generalizes the notion of membership from a black and white binary categorization in classical set theory into one that allows partial membership. The notion of membership in fuzzy sets thus becomes a matter of degree, which is a number between 0 and 1. A membership degree of 0 represents complete non membership, while a membership degree of 1 represents the complete membership. A fuzzy set is characterized by a mapping from its universe of discourse into the interval $[0,1]$. This mapping is the membership function of the set.

Fuzzy sets were introduced by Lotfi zadeh in 1965 to provide means to describe situations mathematically which give rise to ill-defined classes, i.e., collections of objects for which there is no precise criteria for membership. Collections of this type have vague or “fuzzy” boundaries; there are objects for which it is impossible to determine whether or not they belong to the collection. The classical mathematical theories, by which certain types of certainty can be expressed, are the classical set theory and the probability theory.

A family τ of fuzzy sets A of X is called a fuzzy topology on X if $0, 1$ belong to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complement are fuzzy closed sets.

Continuity is one of the most important and fundamental properties that have been widely used in mathematical analysis. In the recent years, a considerable amount of research has been done on many types of continuity in general topology.

In this thesis the concept of fuzzy t -continuous mapping are introduced and studied. Some of the interesting remarks are proved with examples and their inter-relations with already existing fuzzy continuous mapping are investigated and analyzed.

Chapter 1, begins with preliminary definitions, remarks followed by examples required for fuzzy t -continuous mapping.

Chapter 2, deals with the introduction of fuzzy t -continuous mapping. Comparison between fuzzy t -continuous mapping with already existing fuzzy continuous mappings, such as fuzzy semi-continuous mapping, fuzzy pre-continuous mapping, fuzzy α -continuous mapping, fuzzy β -continuous mapping, fuzzy γ -continuous mapping are examined and their inter-relation is given by a neat diagram.

Throughout this paper (X, τ) represent non-empty fuzzy topological space on which no separation axiom are assumed, unless otherwise explicitly mentioned. If A is a subset of a space (X, τ) , $\text{cl}(A)$ denotes the closure of A and $\text{int}(A)$ denotes the interior of A respectively. The extension of the notions of union, intersection, contained in and contains in fuzzy topological spaces are denoted as \vee, \wedge, \leq, \geq respectively.

REVIEW OF THE LITERATURE

In 1965, L. A. Zadeh introduced the concept of fuzzy sets as a new approach for modelling uncertainties. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. The concept of fuzzy topological space was introduced by C. L. Chang in 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In 1969, N. Biwas introduced the concept of simply continuity by means of the notion of simply open sets and some properties. In 1987, Supriti Saha introduced the idea of fuzzy δ - continuous mapping of a function in the fuzzy sets.

The concept of regular open sets and regular closed sets in topological spaces are introduced by Stone in 1937 and the concept of semi- open set in topological space are introduced by Levine in 1963. Semi closed set was introduced by Biwas in 1969 in topological space. Njastad (1965) has introduced the concept of α -open set which are weaker than open sets in topological space. Characterization of Semi- continuous functions are introduced by Biwas in 1970. Andrijevic (1986) has introduced the semi- pre open sets (β – open set and β -open set). Abd El-Monsef et. al. (1983) has introduced β –open set, β –closure and β –continuous mapping in topological spaces. Hatir and Noiri (1996) introduced t-sets in topological spaces.

K. K. Azad (1981) introduced fuzzy semi-continuity, fuzzy almost and weekly continuity, Abd. El. Monsef. et. al. (1982) introduced fuzzy pre-continuous and weak pre-continuous mappings, Mashhour. A .S and et. al., (1982) introduced semi-continuity and pre- continuity. Abdulla. S. Bin Shahna (1991) introduced the fuzzy strong semi-continuity and pre-continuity. Singal M.K. and Parkash. N (1991) introduced the fuzzy pre-open sets and fuzzy sets and systems. M. K. Singal and Niti Rajvanshi (1991) introduced the fuzzy alpha sets and fuzzy alpha continuous mappings. Jin Han Park, Jin Kuen Park and Bu- young lee (1998) introduced the regular generalized fuzzy closed sets and generalization of fuzzy continuous mapping. S. S. Thakur and Surendar Singh (1998) introduced the concept of fuzzy semi-pre open sets and fuzzy semi-pre continuous mapping in topological space. I. M. Hanafy (1999) introduced fuzzy

γ –open sets and fuzzy γ –continuity. B. Ahmed and Athar Khan (2001) introduced fuzzy S-open and S-closed mappings. Yalcav. T .H (1998) introduced the semi- interior and semi-closure fuzzy set. Erdal Ekici (2003) introduced the fuzzy θ –continuous mapping, fuzzy almost super continuous mapping and properties of fuzzy weekly continuous mapping. Yuskel. S. et. al., (2005) introduced δ -I continuous mapping. Yuskel. S and Keskin, A. (2006) introduced irregular sets and decomposition of continuity. Yuskel, et. al., (2004) introduced decomposition of I-continuity. Shukla. M (2012) introduced the fuzzy strongly continuous mapping. Thakur. S .S and Manoj Mishra (2012) introduced fuzzy w- continuous mapping. Athar Kharal (2013) introduced fuzzy α –continuous mapping. G. Thangaraj and K. Dinakaran (2016) introduced fuzzy simply* continuous mapping and G. Thangaraj and S. Senthil (2021) introduced somewhat fuzzy continuous mappings. Warren, R. H. (1978) discussed and analysed the properties of continuous mappings in fuzzy topological space.

1. ON SOME MAPPINGS IN TOPOLOGICAL SPACE

[N. Biwas, 1969]

In this paper, the author introduced some mappings in topological spaces, and discontinuity of functions are investigated and continuity of functions are also discussed.

2. ON SOME TYPES OF CONTINUOUS FUZZY FUNCTIONS

[Erdal Ekici, 2003]

In this paper, the author have introduced and proved some characterizations and some properties of fuzzy weekly continuous, fuzzy θ -continuous, fuzzy almost super continuous, fuzzy δ – continuous mappings.

3. ON FUZZY SIMPLY* CONTINUOUS FUNCTIONS

[G. Thangaraj and K. Dinakaran, 2016]

In this paper, the concepts of fuzzy simply* open sets, fuzzy simply* continuous and fuzzy simply* open functions are introduced and some of their basic properties are studied, the inter-relations between fuzzy simply* continuous functions and fuzzy simply continuous functions are obtained.

4. ON SEMI - PRE - OPEN SETS

[D. Andrijevic, 1986]

In this paper, the author has introduced a new class of sets called semi- pre open sets and established relationships between various topological sets and discussed their fundamental properties.

5. ON FUZZY δ –CONTINUOUS MAPPING

[Supriti Saha, 1987]

In this paper, the author has introduced the fuzzy continuity, fuzzy δ – continuity and proved that fuzzy almost continuity are equivalent when the domain of definition and range space of the function are in both fuzzy semi-regular spaces.

6. ON REGULAR GENERALIZED FUZZY CLOSED SETS AND GENERALIZATION OF FUZZY CONTINUOUS MAPPING

[Jin Han Park, Jin Kuen Park and Bu-Young, 1998]

In this paper, the author defined and studied another various generalization of fuzzy

continuous functions by using the concept of regular generalized fuzzy closed sets and also mutual inter-relations among these functions along with the functions.

7. ON A SOME WHAT FUZZY CONTINUOUS FUNCTIONS

[G. Thangaraj and S. Senthil, 2021]

In this paper, a new form of somewhat fuzzy continuity between the topological spaces is introduced. Several characteristics of these functions are obtained. Somewhat fuzzy continuous functions for the fuzzy hyper connected and fuzzy strongly irresolvable spaces to become fuzzy, baire spaces are also obtained.

8. FUZZY α - CONTINUOUS MAPPING

[Athar Kharal, 2013]

In this paper, the author has studied the notion of fuzzy α -continuous mapping and several fundamental properties of fuzzy α -open sets. Using different results many characterizations of fuzzy α -continuous, fuzzy almost α -continuous and fuzzy semi-weekly continuous mapping are obtained.

9. β - OPEN SETS AND β - CONTINUOUS MAPPINGS

[Abd El-Monsef et.al, 1983]

In this paper, the author has introduced β -open sets in topological spaces and also introduced the β -continuous mapping in topological spaces.

10. DECOMPOSITION OF CONTINUITY AND COMPLETE CONTINUITY

[E. Hatir and T. Noiri, 1996]

In this paper, the author first introduced the notation of α^* set, B set and t-set and obtained decomposition of continuity and complete continuity.

11. SEMI OPEN SETS AND SEMI CONTUNITY IN TOPOLOGICAL SPACES

[N. Levine, 1963]

In this paper, the author has introduced and studied a new class of semi- open sets and semi-continuity in topological spaces. Also, he discussed several characterizations of semi-open and semi-continuity in topological spaces.

12. ON PRE- CONTINUOUS AND WEEK PRE- CONTINOUS MAPPINGS

[A. S. Mashhour. et. al, 1982]

In this paper, the author have introduced the notions of pre-open sets, pre-continuous, pre- open and week pre-open mappings in topological spaces. Also, they investigated the connections of these notions with other existing topological concepts.

13. FUZZY SETS

[Lofti A. Zadeh, 1965]

In this paper, the author has introduced a new class of sets namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership

ranging between zero and one. Further the author has provided the notions of inclusion, union intersections, complement etc., with respect to the fuzzy sets.

14. FUZZY TOPOLOGICAL SPACES

[C. L. Chang, 1968]

In this paper, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. The basic concepts such as fuzzy open sets, fuzzy closed set, fuzzy neighborhood, fuzzy continuity etc., are also given.

15. FUZZY SETS, FUZZY S-OPEN AND S- CLOSED MAPPINGS

[B. Ahmed and Athar khan, 2001]

In this paper, the author have defined fuzzy semi-open and fuzzy semi- closed sets. Also, they worked on the characterizations of fuzzy semi- continuous and fuzzy semi-open mappings and their relationships to properties of fuzzy semi-open sets.

16. ON FUZZY SEMI CONTINUITY, FUZZY ALMOST CONTINUITY AND FUZZY WEAKLY CONTINUITY

[K. K. Azad, 1981]

In this paper, the author has first introduced fuzzy semi-open, fuzzy semi-closed, fuzzy regular open and fuzzy regular closed sets and then considered generalizations of fuzzy semi-continuous mapping, fuzzy semi-open mapping, fuzzy semi-closed mapping and fuzzy pre-continuous mappings.

17. ON FUZZY γ - OPEN SETS AND FUZZY γ - CONTINUITY

[I. M. Hanafy, 1999]

In this paper, the author has introduced the notion of fuzzy γ -open set and established relationships between fuzzy semi-open (fuzzy semi closed) set and fuzzy pre-open (fuzzy pre-closed) set in fuzzy topological spaces and fuzzy γ -open and fuzzy γ -closed functions are discussed with their basic properties.

18. ON FUZZY SEMI- PRE- OPEN SETS AND FUZZY SEMI- PRE- CONTINUITY

[S.S. Thakur and Surendar Singh, 1998]

In this paper, the author have introduced and studied the concept of fuzzy semi-pre-open sets and fuzzy semi-pre-continuous mappings in fuzzy topological spaces.

19. FUZZY ALPHA SETS AND FUZZY ALPHA CONTINUOUS MAPS

[M. K. Singal and Niti Rajvanshi, 1991]

In this paper, the authors generalized the notion of α -sets to fuzzy spaces. They also introduced the concepts of fuzzy α -continuous and fuzzy α -open mappings and discussed their relations with fuzzy continuous and other weaker forms of fuzzy continuous mappings.

20. ON FUZZY STRONG SEMI- CONTINUITY AND PRE- CONTINUITY

[Abdulla. S. Bin Shahna, 1991]

In this paper, the author has introduced fuzzy α -open sets, fuzzy α -closed sets, fuzzy pre-open sets, and fuzzy pre closed sets. Further, he introduced fuzzy separation axioms and investigated with the help of fuzzy pre-open sets.

CHAPTER 1

PRELIMINARIES

Definition 1.1: [L. A. Zadeh, 1965]

Let X be a non-empty set. The **fuzzy set** A in X can be described in the form

$$A = \{ \langle x, \mu_A(x) \rangle / x \in X \}$$

where the function $\mu_A: X \rightarrow [0,1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and $0 \leq \mu_A(x) \leq 1$ for each $x \in X$.

A membership function assigned the values to the elements of the universal set fall within a specified range and indicates the membership grade of these elements in the set. Larger values denote higher degrees of the set membership. A set defined by membership function is a fuzzy set. The commonly used range of values of membership functions is the unit interval $[0,1]$.

Definition 1.2: [L.A. Zadeh, 1965]

Let A and B be two fuzzy sets of the form

$$A = \{ \langle x, \mu_A(x) \rangle : x \in X \}$$

and

$$B = \{ \langle x, \mu_B(x) \rangle : x \in X \}.$$

Then inclusion, equal, complement, union and intersection and etc., with respect to the fuzzy

sets are

- (i) $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$.
- (ii) $A = B$ if and only if $A \leq B$ and $A \geq B$.
- (iii) $A^c = \{ \langle x, (1 - \mu_A(x)) \rangle : x \in X \}$.
- (iv) $A \vee B = \{ \langle x, \mu_A(x) \vee \mu_B(x) \rangle : x \in X \}$.
- (v) $A \wedge B = \{ \langle x, \mu_A(x) \wedge \mu_B(x) \rangle : x \in X \}$.

The fuzzy sets $0_{\sim} = \langle x, 0 \rangle$ and $1_{\sim} = \langle x, 1 \rangle$ are respectively the empty set and the whole set of X .

Definition 1.3: [C. L. Chang, 1968]

The **fuzzy topology** τ on X is a family of subsets in X satisfying the following axioms:

- (i) $0_{\sim}, 1_{\sim} \in \tau$,
- (ii) $G_1 \wedge G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- (iii) $\bigvee G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the **fuzzy topological space** and any fuzzy set in τ is known as the **fuzzy open** set in X . The complement A^c of a fuzzy open set A in fuzzy topological space (X, τ) is called the **fuzzy closed set** in X .

Definition 1.4: [C. L. Chang, 1968]

Let (X, τ) be the fuzzy topological space and $A = \langle x, \mu_A \rangle$ be the fuzzy set in X . Then the **fuzzy interior** and the **fuzzy closure** are defined by

$$\text{int}(A) = \vee \{G / G \text{ is a fuzzy open set in } X \text{ and } G \leq A\},$$

$$\text{cl}(A) = \wedge \{K / K \text{ is a fuzzy closed set in } X \text{ and } A \leq K\},$$

It is to be noted that for any fuzzy set A in (X, τ) , we have

$$\text{cl}(A^c) = (\text{int}(A))^c \text{ and } \text{int}(A^c) = (\text{cl}(A))^c.$$

Definition 1.5: [K. K. Azad, 1981]

The fuzzy set $A = \langle x, \mu_A \rangle$ in a fuzzy topological space (X, τ) is said to be a

- (i) **fuzzy regular open set** if $A = \text{int}(\text{cl}(A))$
- (ii) **fuzzy regular closed set** if $\text{cl}(\text{int}(A)) = A$

Definition 1.6:

The fuzzy set $A = \langle x, \mu_A \rangle$ in a fuzzy topological space (X, τ) is said to be a

- (i) **fuzzy semi – open set** if $A \leq \text{cl}(\text{int}(A))$ [Azad. K. K, 1981]
- (ii) **fuzzy pre – open set** if $A \leq \text{int}(\text{cl}(A))$ [Abdulla S. Bin shahna, 1991]
- (iii) **fuzzy α – open set** if $A \leq \text{int}(\text{cl}(\text{int}(A)))$ [Abdulla S. Bin shahna , 1991]

- (iv) **fuzzy β – open set** if $A \leq \text{cl}(\text{int}(\text{cl}(A)))$ [Thakur. S. S and Singh. S, 1998]
- (v) **fuzzy γ – open set** if $A \leq \text{cl}(\text{int}(A)) \vee \text{int}(\text{cl}(A))$. [Hanafy. I. M, 1999]

Definition 1.7:

The fuzzy set $A = \langle x, \mu_A \rangle$ in the fuzzy topological space (X, τ) is said to be a

- (i) **fuzzy semi – closed set** if $\text{int}(\text{cl}(A)) \leq A$ [Azad. K. K, 1981]
- (ii) **fuzzy pre – closed set** if $\text{cl}(\text{int}(A)) \leq A$ [Abdulla S. Bin shahna , 1991]
- (iii) **fuzzy α – closed set** if $\text{cl}(\text{int}(\text{cl}(A))) \leq A$ [Abdulla S. Bin shahna , 1991]
- (iv) **fuzzy β – closed set** if $\text{int}(\text{cl}(\text{int}(A))) \leq A$ [Thakur. S. S and Singh. S, 1998]
- (vi) (v) **fuzzy γ – closed set** if $\text{int}(\text{cl}(A)) \wedge \text{cl}(\text{int}(A)) \leq A$ [Hanafy. I. M, 1999]

Definition 1.8: [C. L. Chang, 1968]

Let (X, τ) be a fuzzy topological space and A, B be fuzzy sets in X .

Then the following properties hold:

- (i) $\text{int}(A) \leq A$
- (ii) $A \leq \text{cl}(A)$
- (iii) $A \leq B \Rightarrow \text{int}(A) \leq \text{int}(B)$
- (iv) $A \leq B \Rightarrow \text{cl}(A) \leq \text{cl}(B)$

- (v) $\text{int}(\text{int}(A)) = \text{int}(A)$
- (vi) $\text{cl}(\text{cl}(A)) = \text{cl}(A)$
- (vii) $\text{int}(A \wedge B) = \text{int}(A) \wedge \text{int}(B)$
- (viii) $\text{cl}(A \vee B) = \text{cl}(A) \vee \text{cl}(B)$
- (ix) $\text{int}(1_{\sim}) = 1_{\sim}$
- (x) $\text{cl}(0_{\sim}) = 0_{\sim}$

Definition 1.9: [K. Annapoorani and D. Jayanthi, 2021]

- (i) A fuzzy set A of a fuzzy topological space (X, τ) is said to be a **fuzzy t- open set** if

$$\text{int}(\text{cl}(A)) = \text{int}(A).$$

- (ii) A fuzzy set A of a fuzzy topological space (X, τ) is said to be a **fuzzy t- closed set** if

$$\text{cl}(\text{int}(A)) = \text{cl}(A).$$

Definition 1.10: [Palaniappan, 2002]

Let X and Y be two non empty sets and $f: X \rightarrow Y$ be a mapping. If $A = \{ \langle x, \mu_A(x) \rangle : x \in X \}$ is a fuzzy set in X , then the **image** of A under f , denoted by $f(A)$, is the fuzzy set in Y defined by,

$$f(A) = \{ \langle y, f(\mu_A)(y) / y \in Y \rangle \}$$

Definition 1.11: [Palaniappan, 2002]

Let X and Y be two non empty sets and $f : X \rightarrow Y$ be a mapping. If $B = \{ \langle y, \mu_B(y) \rangle : y \in Y \}$ is a fuzzy set in Y , then the **preimage** of B under f , denoted by $f^{-1}(B)$, is the fuzzy set in X defined by

$$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x) / x \in X \rangle \}$$

where

$$f^{-1}(\mu_B)(x) = \mu_B(f(x)) \text{ for every } x \in X.$$

Definition 1.12: [C. L. Chang, 1968]

Let f be a mapping from a fuzzy topological space (X, τ) into a fuzzy topological space (Y, σ) . Then f is said to be a **fuzzy continuous mapping** if $f^{-1}(V)$ is a fuzzy open set in (X, τ) for every fuzzy open set V of (Y, σ) .

Definition 1.13:

Let f be a mapping from a fuzzy topological space (X, τ) into a fuzzy topological space (Y, σ) . Then f is said to be a

- (i) **fuzzy semi continuous mapping** if $f^{-1}(V)$ is a fuzzy semi open set in (X, τ) for every fuzzy open set V of (Y, σ) . [Azad. K .K, 1981]
- (ii) **fuzzy α – continuous mapping** if $f^{-1}(V)$ is a fuzzy α - open set in (X, τ) for every fuzzy open set V of (Y, σ) . [Abdulla S. Bin Shahna, 1991]

- (iii) **fuzzy pre continuous mapping** if $f^{-1}(V)$ is a fuzzy pre open set in (X, τ) for every fuzzy open set V of (Y, σ) . [Abdulla S. Bin Shahna, 1991]
- (iv) **fuzzy β continuous mapping** if $f^{-1}(V)$ is a fuzzy β - open set in (X, τ) for every fuzzy open set V of (Y, σ) . [Abd El Monsef, Et. al, 1983.]
- (v) **fuzzy γ continuous mapping** if $f^{-1}(V)$ is a fuzzy γ - open set in (X, τ) for every fuzzy open set V of (Y, σ) . [I. M. Hanafy, 1999]

Definition 1.14: [C. L. Chang, 1968]

Let A, A_i ($i \in J$) be fuzzy set in X and B, B_j ($j \in K$) be fuzzy sets in Y and $f : X \rightarrow Y$ be a function . Then,

- (i) $A_1 \subset A_2 \Rightarrow f(A_1) \subset f(A_2)$
- (ii) $B_1 \subset B_2 \Rightarrow f^{-1}(B_1) \subset f^{-1}(B_2)$
- (iii) $A \subset f^{-1}(f(A))$ [if f is injective, then $A = f^{-1}(f(A))$]
- (iv) $f(f^{-1}(B)) \subset B$ [if f is surjective, then $B = f(f^{-1}(B))$]
- (v) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
- (vi) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
- (vii) $f^{-1}(0_{\sim}) = 0_{\sim}$
- (viii) $f^{-1}(1_{\sim}) = 1_{\sim}$
- (ix) $f^{-1}(B^c) = (f^{-1}(B))^c$.

Definition 1.15: [K. Annapoorani and D. Jayanthi, 2021]

Let A be a fuzzy set in a fuzzy topological space (X, τ) then the **t - interior** and **t - closure** of A are defined as

$$t - \text{int}(A) = \vee \{G / G \text{ is a fuzzy } t - \text{open set in } X \text{ and } G \leq A\},$$

$$t - \text{cl}(A) = \wedge \{K / K \text{ is a fuzzy } t - \text{closed set in } X \text{ and } A \leq K\}.$$

CHAPTER 2

Fuzzy t- continuous mapping

2.1 Introduction

In 1965, L. A. Zadeh introduced the concept of fuzzy sets. Fuzzy Topology is used in all branches of mathematics. The concept of fuzzy topological space was introduced by C. L. Chang in 1968. Hatir and Noiri (1996) introduced t-sets in topological spaces.

In 1969, N. Biwas introduced the concept of simply continuity by means of the notion of simply open sets and some properties. In 1986, Natkaniec. T introduced I-continuity and I-semi-continuity points. In 1987, L. A. Zadeh, has introduced fuzzy δ -continuous mapping. In 1987, Supriti Saha introduced the idea of fuzzy δ - continuous mapping of a function in the fuzzy sets. In 2010, Min. W. K introduced the α M-open sets and α M- continuous mapping. In 2021, Annapoorani. K and Jayanthi. D introduced t- open set and t- closed set in fuzzy topological space.

2.2 t-continuous mapping in fuzzy topological spaces

In this section, we have introduced a new type of fuzzy continuous mapping called fuzzy t-continuous mapping. We have established the inter-relation between the newly introduced fuzzy t- continuous mapping and already existing fuzzy pre continuous mapping, fuzzy semi continuous mapping, fuzzy α - continuous mapping, fuzzy β - continuous mapping and fuzzy γ - continuous mapping. Some interesting remarks based on their properties are established and examples are given.

Definition 2.2.1:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from a fuzzy topological space (X, τ) into a fuzzy topological space (Y, σ) , then f is said to be a **fuzzy t - continuous mapping** if $f^{-1}(V)$

is a fuzzy t - open set in (X, τ) for every fuzzy open set V of (Y, σ) .

Example 2.2.2:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b) \rangle$ and $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$.

Consider the fuzzy open set $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$ in Y . Then $f^{-1}(G_2) = \langle x, (0.2_a, 0.4_b) \rangle$ is a fuzzy t - open set in (X, τ) as,

$$\begin{aligned} \text{int}(\text{cl}(f^{-1}(G_2))) &= \text{int}(G_1^c) \\ &= 0_{\sim} \end{aligned}$$

and $\text{int}(f^{-1}(G_2)) = 0_{\sim}$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(f^{-1}(G_2))$.

Hence, f is a fuzzy t -continuous mapping.

Remark 2.2.3:

Every fuzzy continuous mapping and every fuzzy t -continuous mapping are independent to each other.

Example 2.2.4:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy

topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b) \rangle$ and $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$.

Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy t -open continuous mapping but not a fuzzy continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.2_a, 0.4_b) \rangle$ is a fuzzy t -open set in (X, τ) as,

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(G_1^c)$$

$$= 0_{\sim}$$

$$\text{int}(f^{-1}(G_2)) = 0_{\sim}$$

and

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(f^{-1}(G_2)).$$

But f is not a fuzzy continuous mapping as, G_2 is fuzzy open in Y , but $f^{-1}(G_2)$ is not a fuzzy open set in X

$$\text{For, } \text{int}(f^{-1}(G_2)) = 0_{\sim}$$

$$\neq (f^{-1}(G_2))$$

$$\text{and } \text{int}(f^{-1}(G_2)) \neq (f^{-1}(G_2)).$$

Hence, f is a fuzzy t -continuous mapping but not a fuzzy continuous mapping.

Example 2.2.5:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.8_a, 0.9_b) \rangle$ and $G_2 = \langle y, (0.8_u, 0.9_v) \rangle$.

Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy continuous mapping but not a fuzzy t -continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.8_u, 0.9_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.8_a, 0.9_b) \rangle$ is a fuzzy open set in (X, τ) as

$$\text{int}(f^{-1}(G_2)) = G_1 = f^{-1}(G_2)$$

But f is not a fuzzy t -continuous mapping as,

$$\begin{aligned} \text{int}(\text{cl}(f^{-1}(G_2))) &= \text{int}(1_{\sim}) \\ &= 1_{\sim} \end{aligned}$$

and $\text{int}(f^{-1}(G_2)) = f^{-1}(G_2)$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) \neq \text{int}(f^{-1}(G_2))$

Hence, f is a fuzzy continuous mapping but not a fuzzy t -continuous mapping.

Remark 2.2.6:

Every fuzzy semi continuous mapping and every fuzzy t -continuous mapping are independent to each other.

Example 2.2.7:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b) \rangle$ and $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy t -continuous mapping but not a fuzzy semi continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.2_a, 0.4_b) \rangle$ is a fuzzy t - open set in (X, τ) as,

$$\begin{aligned} \text{int}(\text{cl}(f^{-1}(G_2))) &= \text{int}(G_1^e) \\ &= 0_{\sim} \end{aligned}$$

and $\text{int}(f^{-1}(G_2)) = 0_{\sim}$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(f^{-1}(G_2))$

But f is not fuzzy semi continuous mapping as,

$$\begin{aligned} \text{cl}(\text{int}(f^{-1}(G_2))) &= \text{cl}(0_{\sim}) \\ &= 0_{\sim} \end{aligned}$$

But $f^{-1}(G_2) \not\subseteq 0_{\sim}$

Therefore, $f^{-1}(G_2) \not\subseteq \text{cl}(\text{int}(\text{cl}(G_2)))$

Hence, f is a fuzzy t -continuous mapping but not a fuzzy semi continuous mapping.

Example 2.2.8:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.7_b) \rangle$ and $G_2 = \langle y, (0.7_u, 0.8_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy semi continuous mapping but not a fuzzy t -continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.7_u, 0.8_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.7_a, 0.8_b) \rangle$ is a fuzzy semi open set in (X, τ) as,

$$\text{cl}(\text{int}(f^{-1}(G_2))) = \text{cl}(G_1)$$

$$= 1_{\sim}$$

$$\text{and} \quad f^{-1}(G_2) \leq 1_{\sim}$$

$$\text{Therefore, } f^{-1}(G_2) \leq \text{cl}(\text{int}(f^{-1}(G_2)))$$

Hence, f is a fuzzy semi continuous mapping.

But f is not a fuzzy t -continuous mapping as,

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(1_{\sim})$$

$$= 1_{\sim}$$

$$\text{and} \quad \text{int}(f^{-1}(G_2)) = G_1$$

$$\text{Therefore, } \text{int}(\text{cl}(f^{-1}(G_2))) \neq \text{int}(f^{-1}(G_2))$$

Hence, f is a fuzzy semi continuous mapping but not a fuzzy t -continuous mapping.

Remark 2.2.9:

Every fuzzy pre continuous mapping and every fuzzy t -continuous mapping are independent to each other.

Example 2.2.10:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.2_a, 0.4_b) \rangle$ and $G_2 = \langle y, (0.5_u, 0.6_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy t -continuous mapping but not a fuzzy pre continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.5_u, 0.6_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.5_a, 0.6_b) \rangle$ is a fuzzy t -open set in (X, τ) as,

$$\begin{aligned} \text{int}(\text{cl}(f^{-1}(G_2))) &= \text{int}(G_1^c) \\ &= G_1 \end{aligned}$$

and $\text{int}(f^{-1}(G_2)) = G_1$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(f^{-1}(G_2))$

Hence f is a fuzzy t - continuous mapping.

But f is not a fuzzy pre continuous mapping as, and G_2 is a fuzzy open set in Y but $f^{-1}(G_2)$ is not a fuzzy pre open set in X since,

$$\begin{aligned} \text{int}(\text{cl}(f^{-1}(G_2))) &= \text{int}(G_1^c) \\ &= G_1 \end{aligned}$$

and $f^{-1}(G_2) \not\geq G_1$

Therefore, $f^{-1}(G_2) \not\subseteq \text{int}(\text{cl}(f^{-1}(G_2)))$

Hence, f is a fuzzy t -continuous mapping but not a fuzzy pre continuous mapping.

Example 2.2.11:

Let $X = \{a, b\}$ $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b) \rangle$ and $G_2 = \langle y, (0.3_u, 0.7_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy pre open continuous mapping but not a fuzzy t - continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.3_u, 0.7_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.3_a, 0.7_b) \rangle$ is a fuzzy pre open set in (X, τ) as,

$$\begin{aligned} \text{int}(\text{cl}(f^{-1}(G_2))) &= \text{int}(1_{\sim}) \\ &= 1_{\sim} \\ &\geq f^{-1}(G_2) \end{aligned}$$

Therefore, $f^{-1}(G_2) \leq \text{int}(\text{cl}(f^{-1}(G_2)))$

Hence f is a fuzzy pre continuous mapping.

But f is not a t -continuous mapping as,

$$\begin{aligned} \text{int}(\text{cl}(f^{-1}(G_2))) &= \text{int}(1_{\sim}) \\ &= 1_{\sim} \end{aligned}$$

and $\text{int}(f^{-1}(G_2)) = 0_{\sim}$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) \neq \text{int}(f^{-1}(G_2))$

Hence, f is a fuzzy pre continuous mapping but not a fuzzy t -continuous mapping.

Remark 2.2.12:

Every fuzzy α continuous mapping and every fuzzy t-continuous mapping are independent to each other.

Example 2.2.13:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.7_b) \rangle$ and $G_2 = \langle y, (0.7_u, 0.8_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy α - continuous mapping but not a fuzzy t- continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.7_u, 0.8_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.7_a, 0.8_b) \rangle$ is a fuzzy α - open set in (X, τ) , as

$$\begin{aligned} \text{int}(\text{cl}(\text{int}(f^{-1}(G_2)))) &= \text{int}(\text{cl}(G_1)) \\ &= \text{int}(1_{\sim}) \\ &= 1_{\sim} \end{aligned}$$

$$f^{-1}(G_2) \leq 1_{\sim}$$

Therefore, $f^{-1}(G_2) \leq \text{int}(\text{cl}(\text{int}(f^{-1}(G_2))))$.

Hence, f is a α - continuous mapping.

But, f is not fuzzy t-continuous mapping as,

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(1_{\sim})$$

$$= 1$$

$$\text{int}(f^{-1}(G_2)) = G_1$$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) \neq \text{int}(f^{-1}(G_2))$ and f is not a fuzzy t -continuous mapping.

Hence, f is a fuzzy α -continuous mapping but not a fuzzy t -continuous mapping.

Example 2.2.14:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.1_a, 0.2_b) \rangle$ and $G_2 = \langle y, (0.3_u, 0.4_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy t -continuous mapping but not a fuzzy α -continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.3_u, 0.4_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.3_a, 0.4_b) \rangle$ is a fuzzy t -open set in (X, τ) , as

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(G_1^c)$$

$$= G_1$$

$$\text{int}(f^{-1}(G_2)) = G_1$$

$$\text{Therefore, } \text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(f^{-1}(G_2))$$

Hence f is a fuzzy t -continuous mapping.

But, f is not fuzzy α -continuous mapping as,

$$\text{int}(\text{cl}(\text{int}(f^{-1}(G_2)))) = \text{int}(\text{cl}(G_1))$$

$$= \text{int}(G_1^c)$$

$$= G_1$$

$$f^{-1}(G_2) \geq G_1$$

Therefore, $f^{-1}(G_2) \not\leq \text{int}(\text{cl}(f^{-1}(G_2)))$

Therefore, f is a fuzzy t -continuous mapping but not a fuzzy α -continuous mapping.

Remark 2.2.15:

Every fuzzy β -continuous mapping and every fuzzy t -continuous mapping are independent to each other.

Example 2.2.16:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.4_a, 0.6_b) \rangle$ and $G_2 = \langle y, (0.6_u, 0.8_v) \rangle$.

Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy β -continuous mapping but not a fuzzy t -continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.6_u, 0.8_v) \rangle$ is an fuzzy open set in Y , then $f^{-1}(G_2) = \langle x, (0.6_a, 0.8_b) \rangle$ is a fuzzy β - open set in (X, τ) , as

$$\text{cl}(\text{int}(\text{cl}(f^{-1}(G_2)))) = \text{cl}(\text{int}(1_-))$$

$$= \text{cl}(1_-)$$

$$= 1_{\sim}$$

$$f^{-1}(G_2) \leq 1_{\sim}$$

Therefore, $f^{-1}(G_2) \leq \text{cl}(\text{int}(\text{cl}(f^{-1}(G_2))))$, and f is a fuzzy β - continuous mapping.

But, f is not a fuzzy t -continuous mapping as,

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(1_{\sim})$$

$$= 1_{\sim}$$

$$\text{int}(f^{-1}(G_2)) = G_1$$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) \neq \text{int}(f^{-1}(G_2))$

Hence, f is a fuzzy β -continuous mapping but not a fuzzy t -continuous mapping.

Example 2.2.17:

Let $X = \{a, b\}$. $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.3_a, 0.6_b) \rangle$ and $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy t - continuous mapping but not a fuzzy β - continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.2_a, 0.4_b) \rangle$ is a fuzzy t - open set in (X, τ) , as

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(G_1^c)$$

$$= 0_{\sim}$$

$$\text{and } \text{int}(f^{-1}(G_2)) = 0_{\sim}$$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(f^{-1}(G_2))$ and f is a fuzzy t -continuous mapping.

But, f is not a fuzzy β -continuous mapping as,

$$\begin{aligned} \text{cl}(\text{int}(\text{cl}(f^{-1}(G_2)))) &= \text{cl}(\text{int}(G_1^c)) \\ &= \text{cl}(0_{\sim}) \\ &= 0_{\sim} \end{aligned}$$

$$\text{and } f^{-1}(G_2) \geq 0_{\sim}$$

$$\text{Therefore, } f^{-1}(G_2) \not\subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(G_2))))$$

Hence, f is a fuzzy t -continuous mapping but not a fuzzy β -continuous mapping.

Remark 2.2.18:

Every fuzzy γ -continuous mapping and every fuzzy t -continuous mapping are independent to each other.

Example 2.2.19:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.8_a, 0.9_b) \rangle$ and $G_2 = \langle y, (0.3_u, 0.4_v) \rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy γ - continuous mapping but not a fuzzy t - continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.3_u, 0.4_v) \rangle$ in Y , Then $f^{-1}(G_2) = \langle x, (0.3_a, 0.4_b) \rangle$ is a fuzzy t -open set in (X, τ) .

$$\text{cl}(\text{int}(f^{-1}(G_2))) \vee (\text{int}(\text{cl}(f^{-1}(G_2)))) = \text{cl}(0_{\sim}) \vee \text{int}(1_{\sim})$$

$$= 0_{\sim} \vee 1_{\sim}$$

$$= 1_{\sim}$$

$$f^{-1}(G_2) \leq 1_{\sim}$$

$$\text{Therefore, } f^{-1}(G_2) \leq \text{cl}(\text{int}(f^{-1}(G_2))) \vee \text{int}(\text{cl}(f^{-1}(G_2)))$$

Hence, f is a fuzzy γ - continuous mapping.

But f is not fuzzy t -continuous mapping as,

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(1_{\sim})$$

$$= 1_{\sim}$$

$$\text{and } \text{int}(f^{-1}(G_2)) = 0_{\sim}$$

$$\text{Therefore, } \text{int}(\text{cl}(f^{-1}(G_2))) \neq \text{int}(f^{-1}(G_2))$$

Hence, f is a fuzzy γ -continuous mapping but not a fuzzy t -continuous mapping.

Example 2.2.20:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ be fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b) \rangle$ and $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$.

Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is a fuzzy t - continuous mapping but not a fuzzy γ - continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.2_u, 0.4_v) \rangle$ in Y , then $f^{-1}(G_2) = \langle x, (0.2_a, 0.4_b) \rangle$ is a fuzzy t - open set in (X, τ) as,

$$\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(G_1^e)$$

$$= 0_{\sim}$$

$$\text{and } \text{int}(f^{-1}(G_2)) = 0_{\sim}$$

Therefore, $\text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(f^{-1}(G_2))$ and f is a fuzzy t - continuous mapping.

But, f is not a fuzzy γ -continuous mapping as, G_2 is a fuzzy open set in Y but $f^{-1}(G_2)$ is not a fuzzy γ - open set in X , since

$$\text{cl}(\text{int}(f^{-1}(G_2))) \vee \text{int}(\text{cl}(f^{-1}(G_2))) = \text{cl}(0_{\sim}) \vee \text{int}(G_1^e)$$

$$= 0_{\sim} \vee 0_{\sim}$$

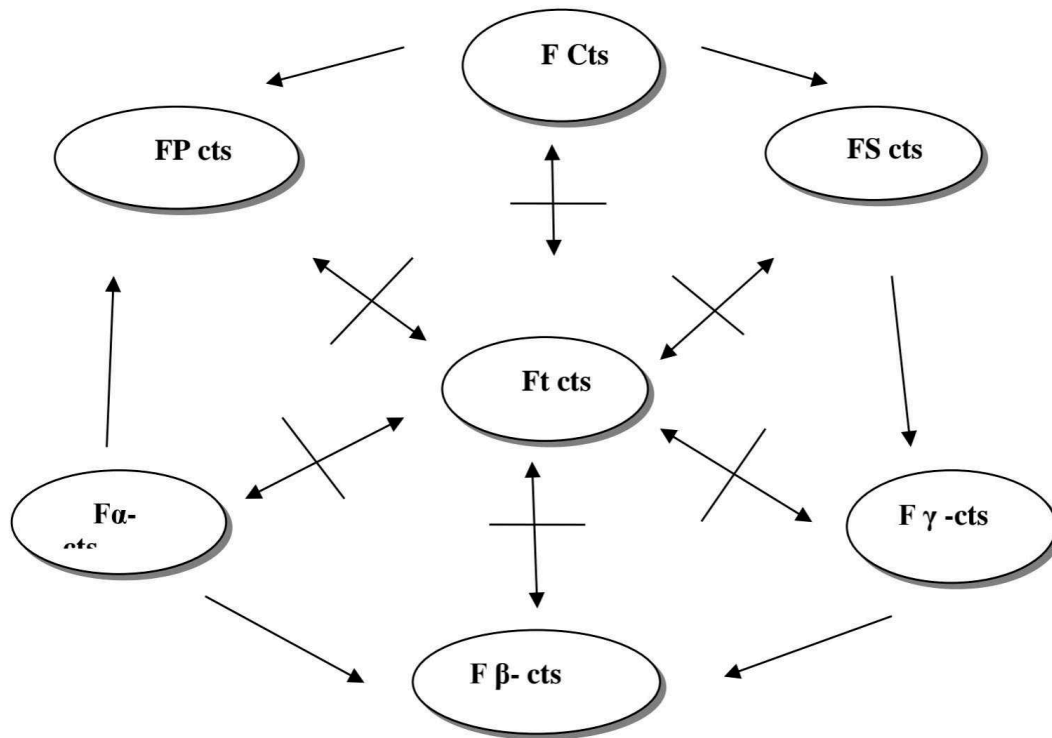
$$= 0_{\sim}$$

$$\text{and } f^{-1}(G_2) \not\geq 0_{\sim}$$

Therefore, $f^{-1}(G_2) \not\subseteq \text{cl}(\text{int}(f^{-1}(G_2))) \vee \text{int}(\text{cl}(f^{-1}(G_2)))$.

Hence, f is a fuzzy t -continuous mapping but not a fuzzy γ -continuous mapping.

In the following diagram, the inter-relation between various types of fuzzy continuous mapping with fuzzy t- continuous mapping is established.



In this diagram the reverse implications are not true in general.

Proposition 2.2.21:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a fuzzy t -continuous mapping if and only if the inverse image $f^{-1}(A)$ of each fuzzy closed set A in Y is a fuzzy t -closed set in X .

Proof:**Necessity:**

Let A be a fuzzy closed set in Y . This implies that A^c is a fuzzy open set in Y . Then $f^{-1}(A^c)$ is fuzzy t -open set in X , by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is a fuzzy t -open set in X and hence $f^{-1}(A)$ is a fuzzy t -closed set in X .

Sufficiency:

Let A be a fuzzy open set in Y . Then A^c is a fuzzy closed set in Y . By hypothesis, $f^{-1}(A^c)$ is fuzzy t -closed set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is a fuzzy t -closed set in X and hence, $f^{-1}(A)$ is a fuzzy t -open set in X . This implies that f is a fuzzy t -continuous mapping.

Proposition 2.2.22:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a fuzzy t -continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a fuzzy continuous mapping. Then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a fuzzy t -continuous mapping

Proof:

Let V be a fuzzy open set in Z . Then $g^{-1}(V)$ is a fuzzy open set in Y , by hypothesis. Since f is a fuzzy t -continuous mapping, $f^{-1}(g^{-1}(V))$ is a fuzzy t -open set in X . Hence $g \circ f$ is a fuzzy t -continuous mapping.

Remark 2.2.23:

The composition of the two fuzzy t-continuous mapping need not to be a fuzzy t-continuous mapping in general.

Example 2.2.24:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and $Z = \{p, q\}$ and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$, $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ and $\delta = \{0_{\sim}, G_3, 1_{\sim}\}$ be fuzzy topologies on X , Y and Z respectively, where $G_1 = \langle x, (0.2_a, 0.7_b) \rangle$ and $G_2 = \langle y, (0.2_u, 0.3_v) \rangle$, $G_3 = \langle z, (0.6_p, 0.7_q) \rangle$. Then (X, τ) , (Y, σ) and (Z, δ) are fuzzy topological spaces.

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$.

Now, consider the fuzzy set $G_2 = \langle y, (0.2_u, 0.3_v) \rangle$ which is a fuzzy open set in Y . Then $f^{-1}(G_2) = \langle x, (0.2_a, 0.3_b) \rangle$ is a fuzzy t- open set in (X, τ) as,

$$\begin{aligned} \text{int}(\text{cl}(f^{-1}(G_2))) &= \text{int}(G_1^c) \\ &= 0_{\sim} \end{aligned}$$

$$\text{and } \text{int}(f^{-1}(G_2)) = 0_{\sim}$$

$$\text{Therefore, } \text{int}(\text{cl}(f^{-1}(G_2))) = \text{int}(f^{-1}(G_2))$$

Hence, f is a fuzzy t- continuous mapping.

Define a mapping $g : (Y, \sigma) \rightarrow (Z, \delta)$ by $g(u) = p$ and $g(v) = q$.

Now, consider the fuzzy set $G_3 = \langle z, (0.6_p, 0.7_q) \rangle$ which is a fuzzy open set in Z . Then $g^{-1}(G_3) = \langle y, (0.6_u, 0.7_v) \rangle$ is a fuzzy t- open set in (Y, σ) as,

$$\begin{aligned} \text{int}(\text{cl}(g^{-1}(G_3))) &= \text{int}(G_2^c) \\ &= G_2 \end{aligned}$$

$$\text{and } \text{int}(g^{-1}(G_3)) = G_2$$

$$\text{Therefore, } \text{int}(\text{cl}(g^{-1}(G_3))) = \text{int}(g^{-1}(G_3))$$

Hence, g is a fuzzy t - continuous mapping.

Define a mapping $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ by $(g \circ f)(a) = p$ and $(g \circ f)(b) = q$.

Now, Consider the fuzzy set $G_3 = \langle z, (0.6_p, 0.7_q) \rangle$ which is a fuzzy open set in Z . Then

$(g \circ f)^{-1}(G_3) = \langle x, (0.6_a, 0.7_b) \rangle$ is not a fuzzy t - open set in (X, τ) as,

$$\begin{aligned} \text{int}(\text{cl}((g \circ f)^{-1}(G_3))) &= \text{int}(1_-) \\ &= 1_{\sim} \end{aligned}$$

$$\text{and } \text{int}((g \circ f)^{-1}(G_3)) = G_1$$

$$\text{Therefore, } \text{int}(\text{cl}((g \circ f)^{-1}(G_3))) \neq \text{int}((g \circ f)^{-1}(G_3))$$

Hence, $(g \circ f)$ is not a fuzzy t - continuous mapping.

Proposition 2.2.25:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from a fuzzy topological space X into a fuzzy topological space Y that satisfies $f^{-1}(B) = \text{int}(\text{cl}(f^{-1}(B)))$ for every B in Y , then f is a fuzzy t - continuous mapping.

Proof:

Let B be a fuzzy open set in Y , then $\text{int}(B) = B$.

By hypothesis,

$$\begin{aligned} f^{-1}(\text{int}(B)) &= f^{-1}(B) \\ &= \text{int}(\text{cl}(f^{-1}(B))), \end{aligned}$$

Then

$$\begin{aligned} \text{int}(f^{-1}(B)) &= \text{int}(\text{int}(\text{cl}(f^{-1}(B)))) \\ &= \text{int}(\text{cl}(f^{-1}(B))). \end{aligned}$$

This implies that $f^{-1}(B)$ is a fuzzy t - open set in X .

Hence, f is a fuzzy t - continuous mapping.

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

In **chapter 1**, the preliminary definitions are discussed.

In **chapter 2**, we have introduced fuzzy t-continuous mapping and made an attempt to compare fuzzy t-continuous mapping in fuzzy topological spaces with some of the existing fuzzy continuous mappings such as fuzzy continuous mapping, fuzzy pre- continuous mapping, fuzzy semi-continuous mapping, fuzzy α - continuous mapping, fuzzy β - continuous mapping, fuzzy γ - continuous mapping are examined.

For future study, one can apply our results for the comparison of various fuzzy mappings in different number intervals especially through numerical examples. It can extended to a next level of connectedness, separation axioms, homeomorphisms, compactness and etc., The research of fuzzy mappings between the fuzzy topological spaces are purely mathematical perspective so that all the definitions and theorems described in this thesis are accessible to Applied Mathematicians and Engineers for developing its practical applications.

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