CHAPTER - I

CHAPTER I

FUNDAMENTAL DEFINITIONS AND NOTATIONS

Definition: 1.1 [76]

A ***fuzzy set A*** in a universe of discourse X is defined by

$$A = \{(x, \mu_A(x)) : x \in X, \mu_A(x) \in [0,1]\}$$

where the function $\mu_A(x) : X \rightarrow [0,1]$.

For each $x \in X$, the value $\mu_A(x)$ is called the grade of membership/ membership function of x in X.

Definition: 1.2 [60]

A fuzzy set A of the universe of discourse X is ***convex*** if and only if for all $x_1$and x_2 in X we have $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$ where $\lambda \in [0,1]$.

Definition: 1.3 [60]

A fuzzy set A of the universe of discourse X is called a ***normal*** fuzzy set implying that $\exists x \in X : \mu_A(x) = 1$

Definition: 1.4 [9]

A ***fuzzy number*** $\tilde{A}$ is a convex, normal fuzzy set $A \subseteq \mathbb{R}$ whose membership function is at least segmentally continuous and has functional value $\mu_A(x) = 1$ at precisely one element.

Definition: 1.5 [60]

The ***α-cut of a fuzzy number*** $\tilde{A}$ is defined as $\tilde{A}_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X\}$ where $\alpha \in [0,1]$.

$\tilde{A}_\alpha$ is a non-empty bounded closed interval contained in X and it can be denoted by $\tilde{A}_\alpha = [A_l, A_u]$ where A_l and A_u are the lower and upper bounds of the closed interval, respectively.

Definition: 1.6 [12]

D is called a ***fuzzy matrix***, if all its entries are fuzzy numbers.

Definition: 1.7 [6]

Let a crisp set X *be* fixed. An ***Intuitionistic Fuzzy Set (IFS)*** A in X is defined as an object of the following form:

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$$

where the functions:

$$\mu_A : X \to [0,1]$$

and

$$\nu_A : X \to [0,1]$$

denotes the degree of membership and degree of non-membership of each element $x \in X$ to the set A, respectively, and for every $x \in X$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

Definition: 1.8[6]

For each IFS A on X, define the function $\pi_A : X \to [0,1]$ as:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is called the ***Degree of non-determinacy/ hesitancy/ uncertainty*** of the element $x \in X$ to the set A.

In the case of ordinary fuzzy set $\pi_A(x) = 0$ for every $x \in X$. Also $\pi_A(x)$ cannot be omitted when calculating the distance between two Intuitionistic Fuzzy Sets. Obviously $\pi_A(x) \in [0,1]$ $\forall\, x \in X$. For convenience we call $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ an ***Intuitionistic Fuzzy Value (IFV),*** where $\mu_\alpha \in [0,1]$, $\nu_\alpha \in [0,1]$, $\pi_\alpha \in [0,1]$ and $\mu_\alpha + \nu_\alpha \le 1$.

In order to rank the IFVs, Szmidt and Kacprzyt [61] proposed a function, which is in the mathematical form:

$$\rho(\alpha) = 0.5(1 + \pi_\alpha)(1 - \mu_\alpha) \qquad (1)$$

The smaller the value of $\rho(\alpha)$, the greater the IFV α in the sense of the amount of positive information included and reliability of information.

Definition: 1.9 [35, 40]

A ***Triangular Fuzzy Number*** is a special case of fuzzy number. It is defined by a triplet $\tilde{A} = (a, b, c)$ shown in Fig. 1. This representation is interpreted as membership function $\mu_{\tilde{A}}$ and holds the following conditions:

1. a to b is an increasing function
2. b to c is an decreasing function
3. $a \le b \le c$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 \quad if \quad x \le a \\ \dfrac{x-a}{b-a} \quad if\ a \le x \le b \\ \dfrac{c-x}{c-b} \quad if\ b \le x \le c \\ 0 \quad if \quad x > c \end{cases}$$

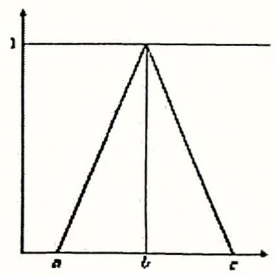

Fig 1

Alternatively, by defining the interval of confidence α, the triangular fuzzy number $\tilde{A}$ can be characterized as:

$$\tilde{A}_\alpha = [a^\alpha, c^\alpha] = [(b-a)\alpha + a, -(c-b)\alpha + c] \qquad \forall\, \alpha \in [0,1]$$

Definition: 1.10 [40]

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. If $\tilde{A} = \widetilde{B}$, then $a_1 = b_1$, $a_2 = b_2$ and $a_3 = b_3$.

Definition: 1.11 [3]

A Fuzzy Number $\tilde{A} = (a, b, c, d)$ is said to be ***Trapezoidal Fuzzy Number*** if its membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for \quad x < a \\ \dfrac{x-a}{b-a} & for \quad a \le x < b \\ 1 & for \quad b < x < c \\ \dfrac{d-x}{d-c} & for \quad c \le x < d \\ 0 & for \quad x > d \end{cases}$$

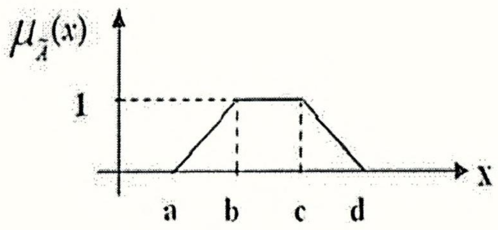

Fig 2

where a ≤ b ≤ c ≤ d.

Alternatively, by defining the interval of confidence α, the trapezoidal fuzzy number $\tilde{A}$ can be characterized as:

$$\tilde{A}_\alpha = [(b-a)\alpha + a, -(d-c)\alpha + d] \qquad \forall\, \alpha \in [0,1]$$

Definition: 1.12[24, 1]

***Algebraic Operations*:**

Let $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ be two triangular fuzzy numbers.

i) Addition of Triangular Fuzzy Numbers, $\oplus$:

$$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

ii) Subtraction of Triangular Fuzzy Numbers, $\ominus$:

$$\tilde{A} \ominus \tilde{B} = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$$

iii) Multiplication of Triangular Fuzzy Numbers, $\otimes$:

$$\tilde{A} \otimes \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2) \qquad a_1 \geq 0, a_2 \geq 0$$

$$k \otimes \tilde{A} = (ka_1, kb_1, kc_1) \qquad k \in \mathbb{R}, k \geq 0$$

iv) Division of Triangular Fuzzy Numbers, $\oslash$:

$$\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right) \qquad a_1 > 0, a_2 > 0$$

$$\tilde{A}^{-1} = (a_1, b_1, c_1)^{-1} = \left(\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1}\right) \qquad a_1 > 0$$

Let $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers

i) Addition of Trapezoidal Fuzzy Numbers, $\oplus$:

$$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

ii) Subtraction of Trapezoidal Fuzzy Numbers, $\ominus$:

$$\tilde{A} \ominus \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$

iii) Multiplication of Trapezoidal Fuzzy Numbers, $\otimes$:

$$\tilde{A} \otimes \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2) \qquad a_1 \geq 0, a_2 \geq 0$$

$$k \otimes \tilde{A} = (ka_1, kb_1, kc_1, kd_1) \qquad k \in \mathbb{R}, k \geq 0$$

iv) Division of Trapezoidal Fuzzy Numbers, $\oslash$:

$$\tilde{A} \oslash k = \left(\frac{a_1}{k}, \frac{b_1}{k}, \frac{c_1}{k}, \frac{d_1}{k}\right)$$

Definition: 1.13 [5]

A ***Triangular Fuzzy Number Matrix*** of order $n \times m$ is defined as $A = \left(\widetilde{a_{ij}}\right)_{n \times m}$ where $\widetilde{a_{ij}}$ is a triangular fuzzy number.

For two Triangular Fuzzy Number Matrices $A = (\widetilde{a_{ij}})_{n\times m}$ and $B = (\widetilde{b_{ij}})_{n\times m}$ the *Addition* $A + B$ is defined as

$$A + B = (\widetilde{a_{ij}} \oplus \widetilde{b_{ij}})_{n\times m}$$

Definition: 1.14 [12]

A ***linguistic variable*** is a variable whose values are linguistic terms. The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions. For example, "weight" is a linguistic variable and its values are very low, low, medium, high, very high, etc. These linguistic values can also be represented by fuzzy numbers. It can also be represented as triangular fuzzy numbers as

Very low (VL)	(0, 0.1, 0.3)
Low (L)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)