Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test II – April 2025
VI Semester

Class: III UG **Time: 2 Hours**
Branch: Mathematics/ Special Education & Mathematics **Max. Marks: 60**

21BMAC23/21BSMC19 -Complex Analysis - II

Course Outcomes:

CO1: Classify the singularities of a function.
CO2: Analyze the behavior of a function at its singularities.
CO3: Expand complex functions as Laurent's and Taylor's series.
CO4: Determine the residues of a function.
CO5: Apply residue theorem to evaluate integrals.

Part A **6 x 1 = 6**
Choose the Correct Answer

1. If $z=a$ is a pole of a function $f(z)$, then $\lim_{z\to a} f(z) =$ ____________ CO3K1
   a. 0 b. ∞ c. 1 d. $-\infty$
2. If $f(z)=\frac{e^{z-a}}{(z-a)^2}$, then the residue of $f(z)$ at $z=a$ is CO4K2
   a. 1 b.3 c. 3! d. 2!
3. Find the residues at $z=0$ of the function $f(z)=\frac{sinz}{z^4}$ CO4K2
   a. 1 b. $-\frac{1}{2}$ c. $-\frac{1}{6}$ d. 4
4. The residue of the function cot z at the simple pole $z=0$ is ______ CO4K2
   a. -1 b. 1 c. π d. $-\pi$
5. On the unit circle $C:|Z|=1$, the value of $\cos\theta$ is __________ CO5K1
   a. $\frac{1}{2}(z+\frac{1}{z})$ b. $\frac{1}{2}(z-\frac{1}{z})$ c. $\frac{1}{2i}(z+\frac{1}{z})$ d. $\frac{1}{2i}(z-\frac{1}{z})$
6. $\int_0^{2\pi}\frac{1}{1-2a\cos\theta+a^2}\,d\theta =$ CO5K2
   a. $\frac{2\pi}{3-a^2}$ b. $\frac{2\pi}{1+a^2}$ c. $\frac{2\pi}{1-a^2}$ d. $\frac{4\pi}{1-a^2}$

Part B **3 x 6 = 18**
Answer ALL questions

7. a. If a function f(z) is analytic in a deleted neighbourhood of z = a and if $\lim_{z\to\infty} f(z) = \infty$, then prove that z = a is a pole of f(z). CO3K4

(or)

7. b. Prove that, if z = a is a pole of order m of a function f (z), then z = a is a zero of same order m of the function $\frac{1}{f(z)}$. CO3K4

8. a. (i) Find the poles and residues of f (z) = cot z.
   (ii) Find the residue of f (z) = $\frac{1+e^z}{\sin z+z\cos z}$ CO4K3

(or)

8. b. (i) Find the residue of f (z) = $\frac{1}{(1+z^2)^4}$ at z = i
   (ii) Show that the residue of $\frac{1}{z(e^z-1)}$ at z = 0 is $-\frac{1}{2}$ CO4K3

9. a. State and prove Residue Theorem. CO5K3

(or)

9. b. Evaluate I= $\int_0^{2\pi}\frac{1}{5-4\sin\theta}\,d\theta$ CO5K4

Part C **3 x 12 = 36**

Answer ALL questions

10. a. State and prove Weierstrass Theorem. CO3K4

(or)

10. b. Find the poles of the following functions and find the residues of the functions at them: (i) $\frac{1}{z^2 e^z}$ (ii) $\frac{z+1}{z^2-2z}$ (iii) $\frac{z^2-1}{(z^2+1)^2}$ (iv) $\frac{1}{z^3(z+4)}$ CO4K4

11. a. Find the residues of $f(z)=\frac{z^4}{(z-1)^4(z-2)(z-3)}$ at its singularities. CO4K4

11. b. By contour integration show that $\int_0^{\pi}\frac{1}{a^2+\cos^2\theta}\,d\theta, = \frac{\pi}{a\sqrt{1+a^2}}$ CO5K3

(or)

12. a. Using contour integration, evaluate $I=\int_0^{2\pi}\frac{\cos 3\theta}{5-4\cos\theta}\,d\theta$ CO5K3

(or)

12. b. Evaluate the improper integral $I = \int_0^{\infty}\frac{x^2}{x^4+1}\,dx$ CO5K3

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