



Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD
Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore-641 043, Tamil Nadu, India

Continuous Internal Assessment Test II – October 2024
Semester V

Class : III UG

Branch : Mathematics & Spl Edn.& Mathematics

Time : 2 Hours

Max. Marks : 60

21BMAC14/21BSMC14 – Real Analysis-I

Course Outcomes:

CO1: Recognize the basic properties of Real Number System.

CO2: Identify closed and open sets in Euclidean Space.

CO3: Prove standard theorems in Real Analysis.

CO4: Understand the concept of compactness and metric spaces.

CO5: Acquire abstract and logical thinking that pervades modern analysis

Part A

6 x 1 = 6

Choose the Correct Answer

1. A set S in \mathbb{R}^n is said to be -----if and only if every open covering as a finite sub cover. **CO3K1**

- a. open covering b. compact c. closed d. bounded

2. If S is contained in M , a point a in M is called -----of S if some ball $B_M(a;r)$ is contained in S . **CO3K1**

- a. interior point b. accumulation point c. base point d. None of these

3. The boundary of S , $\partial S =$ ----- . **CO4K2**

- a. M b. $S \cap \bar{S}$ c. $\bar{S} \cap \overline{M - S}$ d. None of these

4. Every convergent sequence is a ----- **CO3K1**

- a. Cauchy s sequence b. increasing sequence c. divergent sequence d. None

5. In \mathbb{R}^1 , $\{1/n\}$ ----- to 0 as $n \rightarrow \infty$ in $(0,1]$. **CO4K2**

- a. converges b. does not converge c. bounded d. None

6. Every Euclidean subspace $(0,1]$ is ----- **CO3K1**

- a. closed b. complete c. not complete d. None

Part B

3 x 6 = 18

Answer ALL questions

7. a. State and prove Lindleof covering Theorem . **CO3K3**

(or)

7. b. Assume that $G=\{A_1,A_2,\dots\}$ denote the countable collection of n -balls having rational radii and centers at points with rational co-ordinates. Assume x belongs to \mathbb{R}^n and let S be and open set in \mathbb{R}^n which contains X . Then prove that atleast one of the n -balls in G contains X and is contained in S . **CO3K4**

8. a. Assume that (S,d) as a metric subspace of (M,d) and let Y be a subset of S . Then prove that

Y is closed in S iff $Y = B \cap S$ for some set B which is closed in M. **CO4K4**

(or)

8. b. Let X be a closed subset of a compact metric space M then prove that X is compact. **CO4K3**

9. a. Prove that in any metric space (S, d) every compact subset T is complete. **CO4K3**

(or)

9. b. Let p is an accumulation point of A and assume b belongs to T then $\lim_{x \rightarrow p} f(x) = b$

iff $\lim_{x \rightarrow \infty} f(x_n) = b$, $b \in T$ for every sequence $\{x_n\}$ of points $A - \{p\}$ which converges to p.

Prove the statement. **CO3K4**

Part C

3 x 12 = 36

Answer ALL questions

10. a. Let S be a subset of \mathbb{R}^n the following statements are equivalent:

a) S is compact

b) S is closed and bounded

c) Every infinite subset of S has an accumulation point in S. **CO3K3**

(or)

10. b. State and prove Heine - Borel covering theorem. **CO3K3**

11. a. Prove the following statement. Let (S,d) be a metric subspace of (M,d), and let X be a subset of S. Then X is open in S iff $X = A \cap S$ for some set A which is open in M. **CO4K4**

(or)

11. b. i) S.T a sequence $\{x_n\}$ in a Metric Space (S,d) can converge to atmost one point in S. **CO4K4**

ii) Prove the following statement. Let $f: S \rightarrow T$ be a function from (S,d_S) to another (T,d_T) and assume $p \in S$, then f is continuous at p, iff for every sequence $\{x_n\}$ in S convergent to p, the sequence $\{f(x_n)\}$ in T converges to $f(p)$ **CO4K4**

12. a. Prove that in Euclidean space \mathbb{R}^k every Cauchy sequence is convergent. **CO3K4**

(or)

12. b. State and prove continuity of composite function. **CO5K4**

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