

INTRODUCTION

Most of the real life problems have various uncertainties. Traditional mathematical tools are unable to solve uncertain problems. A number of theories have been proposed for dealing with uncertainties in an efficient way.

In 1965, Zadeh introduced the concept of Fuzzy set theory which provides us with an intuitively pleasing method of representing one form of uncertainty. In 1999, Molodtsov initiated a novel concept of Soft set theory, which is completely a new approach for modeling vagueness and uncertainty. In 2001, Maji et al. initiated the concept of Fuzzy Soft sets.

“Given an initial universe set U and the set of parameters E , a Soft set U is a pair (F, A) where $A \subseteq E$ and F is a mapping given by $F : A \rightarrow P(U)$. (Here $P(U)$ is the power set of U)”

“Given a non-empty set X , a Fuzzy set μ of X is a mapping given by $\mu : X \rightarrow [0,1]$ ”.

“Given an initial universe set U and the set of parameter E , a Fuzzy soft set U is a pair (\tilde{F}, A) where $A \subseteq E$ and \tilde{F} is a mapping given by $\tilde{F} : A \rightarrow FP(U)$. (Here $FP(U)$ is the set of all fuzzy subsets of U)”

The algebraic structures of set theories dealing with uncertainties have been studied by many mathematicians. Rosenfeld (1971) proposed the concept of Fuzzy groups. Aktas and Cagman (2007) introduced the notion of Soft groups. Aygunoglu and Aygun (2009) introduced the concept of Fuzzy soft groups. Muhammad Irfan Ali (2009) introduced the concept of Soft semigroups and Munazza Naz et al. (2013) introduced the concept of Fuzzy Soft semigroups.

Studies on Soft semigroups and Fuzzy Soft semigroups are carried out by several researchers.

- 1) Soft semigroups, Muhammad Irfan Ali. (2009)
- 2) Soft Regular semigroups, Muhammad Irfan Ali. (2009)
- 3) Soft Ordered semigroups, Jun, Y.B., Lee, K.J. and Khan, A. (2010)
- 4) Soft LA-semigroups, Muhammad Aslam., Shabir. and Asif Mehmood. (2010)
- 5) Soft ternary semigroups, Muhammad Shabir. and Ali Ahmad. (2011)
- 6) Soft M-semigroups, Vijayabalaji,S. and Shakila, V. (2013)
- 7) Uni Soft semigroups, Chang Su Kim., Jeong Gi Kang. and Joo Sup Kim.(2013)
- 8) Soft Neutrosophic semigroups, Mumtaz Ali et al., (2014)
- 9) Soft Neutrosophic LA-semigroups, Florentin Smarandache et al., (2014)
- 10)Int-Soft semigroups, Song, S.Z., Kim, H.S. and Jun, Y.B. (2014)
- 11)Uni-Soft ternary semigroups, Abid Khan. and Muhammad Sarwar. (2015)
- 12)Intuitionistic Fuzzy Soft semigroups, Jiehuazhou., Yuewu Li. and Yunqiang Yin. (2011)
- 13)Fuzzy Soft semigroups, Munazza Naz et al. (2013)
- 14)Intuitionistic Fuzzy Soft ordered ternary semigroups, Muhammad Akram. and Naveed Yaqoob. (2013)

15) Fuzzy Soft Γ -semigroups, Muhammad Akram., Kavikumar, J. and Azme Bin Khamis. (2014)

16) Bipolar Fuzzy Soft Γ -semigroups, Muhammad Akram., Jacob Kavikumar. and Azme Bin Khamis. (2014)

17) Fuzzy Soft LA-semigroups, Saranya, S. and Kalaichelvi, A. (2015).

The main aim of this thesis is to study Soft sets, Soft semigroups, Fuzzy Soft sets and Fuzzy soft semigroups.

The plan of study is as follows:

- 1) Soft sets and Soft semigroups
- 2) Soft Regular semigroups
- 3) Soft Γ -semigroups
- 4) Soft ternary semigroups
- 5) Soft LA-semigroups
- 6) Fuzzy Soft sets and Fuzzy Soft semigroups
- 7) Fuzzy Soft LA-semigroups.

The First chapter is devoted to the study of Soft sets and Soft semigroups. In this chapter preliminary definitions, operations and properties regarding Soft sets and Soft semigroups are introduced with interesting examples. The definitions include Restricted Product, Extended Product, Extended Intersection, Restricted Intersection, Extended Union, Restricted Union, ' * '- operation, ' AND '- operation, ' OR '- operation of two Soft sets and Soft homomorphism of two Soft semigroups. Soft semi-prime ideals and Soft prime ideals over a semigroup are defined and some of their properties are studied.

The Second chapter deals with Soft Regular semigroups. In this chapter Regular semigroups are classified by considering its Soft right ideals and Soft left ideals. The important results proved here are given in Theorems 2.2, 2.3,

2.4 and 2.9. Finally an important characterization regarding Soft Regular semigroups is proved which is given in Theorem 2.10.

Soft Γ -semigroups are studied in Chapter III. In 1986, Sen and Saha introduced Γ -semigroups as a generalization of semigroups. In 2012, Thawhat Changphas and Boonyen Thongkam introduced Soft Γ -semigroups and investigated some properties. These properties of Soft Γ -semigroups are studied in detail in this chapter.

Soft ternary semigroups are studied in Chapter IV. The introduction of the mathematical literature of ternary algebraic systems dated back to 1932. In 1932, Lehmer investigated certain ternary algebraic systems called triplexes which turn out to be ternary groups. The notion of ternary semigroup was introduced by Banach. Soft ternary semigroups are introduced by Muhammad Shabir and Ali Ahmad in 2012. In this chapter the concept of Soft ternary semigroups, Soft left (right, lateral, quasi) ideals of ternary semigroups are defined and their properties are studied. Important properties are given in Theorems 4.7, 4.10, 4.15 and 4.23.

Chapter V deals with Soft LA-semigroups. In 1978, Mustaq and Yusuf introduced LA-semigroups. A groupoid S satisfying the left invertive law namely, $(ab)c = (cb)a$ for all $a, b, c \in S$ is called a left almost semigroup and is abbreviated as LA-semigroup. In 2010, Muhammad Aslam, Muhammad Shabir and Asif Mehmood introduced Soft LA-semigroups and established some important theorems. In this chapter the definition of Soft LA-semigroup is given and some of its characteristics are discussed.

Fuzzy Soft sets and Fuzzy Soft semigroups are studied in Chapter VI. Fundamental definitions, operations and properties regarding Fuzzy Soft sets are given. Fuzzy Soft sets over a semigroup are defined and some of their basic properties are studied. The operations of multiplication for two Fuzzy Soft sets over a semigroup are defined. Fuzzy Soft ideals are defined over a semigroup

and their properties are investigated. Interesting results proved in this chapter are given in Theorems 6.27, 6.29, 6.31, 6.32, 6.35, 6.36, 6.37 and 6.39.

The author of this thesis introduced the concept of Fuzzy Soft LA-semigroups and studied their basic properties. Chapter VII deals with this Fuzzy Soft LA-semigroup. The definition of Fuzzy Soft LA-semigroup is given and their properties are discussed in detail. Main results proved in this chapter are given in Theorems 7.5, 7.6, 7.7, 7.8, 7.10, 7.11, 7.12, 7.13, 7.14 and 7.15. Two interesting characterizations regarding Fuzzy Soft LA-semigroups are proved and are given in Theorems 7.4 and 7.9.

It is worth mentioning that the author of this thesis published an article related to Fuzzy Soft LA-semigroups as detailed below :

“Fuzzy Soft LA-semigroups – A new Approach”, International Journal of Innovative Research in Science, Engineering and Technology, Vol.4, No-5, May 2015, pp: 3329-3337 [59].