Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination –August 2020
VI Semester

Class : III UG **Time : 2 Hours**
Major : Special Education and Mathematics **Max. Marks : 50**

15BSMC13 Abstract Algebra II

Part A 10 x 1 = 10

Choose the Correct Answer

1. Let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R, then
   a. $d(a) = d(ab)$ b. $d(a) < d(ab)$
   c. $d(a) > d(ab)$ d. $d(b) = d(ab)$

2. In a Euclidean ring R, a and b in R are said to be relatively prime if their greatest common divisor is $a|an$.
   a. unit of N b. zero of N
   c. unit of R d. zero of R

3. $F^{(1)}$ is _______ to $F^{(n)}$ for > 1 .
   a. isomorphic b. not isomorphic
   c. homomorphic d. equal

4. $F^{(n)}$ is isomorphic to $F^{(m)}$ if and only if
   a. $n = m$ b. $n < m$
   c. $m < n$ d. $m \neq n$

5. If V is a finite-dimensional vector space and W is a subspace of V such that $\dim V = \dim W$, then
   a. $W \subset V$ b. $V \subset W$
   c. $V = W$ d. $V \neq W$

6. If $\dim_F V = n$ then $\dim_F (Hom(V, V)$ is
   a. n b. n^2
   c. $2n$ d. 1

7. Which of the following is not true for the sub sets S and T of a Vector Space V.
   a. $L(S \cup T) = L(S) + L(T)$ b. $L(L(S)) = S$
   c. $L(S \cap T) = L(S) \cap L(T)$ d. $L(S) = L(T) for\ S \leq T$

8. For a subspace W of a vector space V, the annihilator of W is a subspace of
   a. V b. W
   c. $\hat{V}$ d. $\hat{W}$

9. If $u, v \in V$ then u is said to be orthogonal to v if
   a. $(u, v) = 0$ b. $(u, u) = 0$
   c. $(v, v) = 0$ d. $(u, v) = 1$

10. $||v|| =$
    a. $\sqrt{(v, v)}$ b. (v, v)
    c. $(v, v)^2$ d. (v, v^2)

Part B $\quad 3 \times 6 = 18$

Answer any **Three** questions

Each answer should not exceed 400 words or two pages

11. Show that Euclidean ring possess a unit element.

12. Let R be a Euclidean ring. Suppose that for $a, b, c \in R$, $a|bc$ but $(a, b) = 1$. Then prove that $a|c$

13. If V is a vector space over F. Then prove the following
    (i) $\alpha 0 = 0$ for $\alpha \in F$
    (ii) $0v = 0$ for $v \in V$
    (iii) $(-\alpha)v = -(\alpha v)$ for $\alpha \in F, v \in V$
    (iv) If $v \neq 0$, then $\alpha v = 0$ implies that $\alpha = 0$.

14. Prove that if V is a vector space over F and if W is a subspace of V, then $V/_W$ is a vector space over F.

15. Show that if $s \leq v$ then the linear span of S is a subspace of V.

16. Prove that if V is finite-dimensional over F then any two bases of V have the same number of elements.

17. Prove that if $\dim_F V = m$ then $\dim_F Hom(V, F) = m$.

18. Show that $A(A(W)) = W$, where $A(W)$ denotes the annihilator of W.

19. Prove that $W^{\perp}$ is a subspace of V.

20. If V is a finite-dimensional inner product space and W is subspace of V, then prove that $(W^{\perp})^{\perp} = W$.

Part C $\quad 2 \times 11 = 22$

Answer any **Two** questions

Each answer should not exceed 800 words or four pages

21. Let R be a Euclidean ring and let A be an ideal of R. Then prove that there exists an element $a_o \in A$ such that A consists exactly of all $a_0 x$ as x range over R.

22. State and prove Unique factorization theorem.

23. Prove that if T is a homomorphism of U onto V with kernel W, then V is isomorphic to $V/_W$.

24. If V is the internal direct sum of $U_1, U_2, \dots, U_n$, then prove that V is isomorphic to the external sum of $U_1, U_2, \dots, U_n$.

25. Prove that if $v_1, v_2, \dots v_n$ is a basis of V over F and if $w_1, \dots, w_m$ in V are linearly independent over F, then $m \leq n$.

26. If V is a finite dimensional vector space and if W is a subspace of V, then prove that W is also finite-dimensional, $\dim W \leq \dim V$ and $\dim V/W = \dim V - \dim W$.

27. Show that if V is finite-dimensional vector space and $v \neq 0 \in V$, then there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

28. Prove that if V and W are of dimensions m and n, respectively, over F, then $Hom(V, W)$ is of dimension mn over F.

29. State and prove Schwarz inequality.

30. Prove that every finite dimensional inner product space has an orthonormal set as a basis.

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