Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd.u/s 3 of UGC Act 1956, Category A by MHRD
Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore-641 043, Tamil Nadu, India
Continuous Internal Assessment Test II – October 2024

SemesterIII

Class: II PG **Time: 2 Hours**

Branch: Mathematics **Max. Marks: 60**

23MMAC15–Differential Geometry

Course Outcomes:
CO1: Calculate the curvature and torsion of a curve.
CO2: Find the oscillating surface and oscillating curve at any point of a given curve.
CO3: Calculate the first and second fundamental forms of surface.
CO4: Solve the problems related to Gaussian curvature, the mean curvature, the curvature lines, the asymptotic lines.
CO5: Identify the appropriate approach to solve the problems on geodesics of a surface.

PART-A

Choose the correct answer **6x1=6**

1. Two space curves are said to be congruent if: CO3K1 A) They have the same length
B) They coincide with one another by rigid motions
C) They have the same curvature at every point
D) They lie on the same surface

2. The necessary and sufficient condition for a curve to be a helix is that CO3K2
A) the curve lies entirely in a plane
B) the ratio of the curvature to torsion is constant at all points
C) the ratio of the curvature to circular arc is constant
D) the curve can be represented as a projection of a sine wave

3. The equation of the normal *N* at a point *P* on the surface$r=(u,v)$, where *R* is a position vector is CO4K1
A) $R= r + a (r_1 * r_2)$
B) $R= r - a (r_1 * r_2)$
C) $R= r / a (r_1 * r_2)$
D) $R= r * a (r_1 * r_2)$

4. If $a=(\lambda,\mu)$ is tangential vector at *P* on a surface, then its magnitude is CO4K2
A) $|a| = (E\lambda^2 + 2G\lambda\mu + F\mu^2)^{\frac{1}{2}}$
B) $|a| = (E\lambda^2 + 2F\lambda\mu + G\mu^2)^{\frac{1}{2}}$
C) $|a| = (\lambda^2 + 2F\lambda\mu + G\mu^2)^{\frac{1}{2}}$
D) $|a| = (E\lambda^2 + 2F\lambda\mu + \mu^2)^{\frac{1}{2}}$

5. The parametriccurves are ________ if and only if F=0. CO5K1
A) normal
B)isometric
C)orthogonal
D)regular

6. A geodesic on a surface *S* is defined as: CO5K1
A) A curve that minimizes the area.
B) A curve that can be described by linear equations
C) A curve with zero curvature over S
D) A curve possesses stationary length for small variation over S

PART-B

Answer ALL questions **3x6=18**

7. a.Find the intrinsic equation of the curve $r=\left(a e^{u} \cos u, a e^{u} \sin u, b e^{u}\right)$. CO3K4

(or)

7. b. Prove that the projection C_1 of a general helix C on a plane perpendicular to its axis has its principal normal parallel to the corresponding principal normal of the helix and its correspondingcurvature is given by $\kappa=\kappa_1 \sin^2 \alpha$. CO3K4

8. a. Calculate the first fundamental coefficients and the area of the anchor ring corresponding to the domain $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$. CO4K4

(or)

8. b For a right helicoid given by $(u \cos v, u \sin v, a v)$, determine (r_1, r_2, N) at a point on the surface and the direction of the parametric curves. Find the direction making angle $\frac{\pi}{2}$ at a point on the surface with parametric curve v = constant. CO4K3

9. a. Prove that every family of curves on a surface possesses a family of orthogonal trajectories.CO5K2

(or)

9. b. If θ is the angle between the two curves given by the doublefamily at aCO5K3 $P\,du^2 + 2Q\,du\,dv + R\,dv^2$ point (u,v) on the surfaces, prove that

$$\tan\theta = \frac{2H\left(Q^2 - PR\right)^{1/2}}{ER - 2FQ + GP}$$

PART-C

Answer ALL questions **3x12=36**

10. a. Find the equations of the curve whose curvature and torsion are constant. CO3K3

(or)

10. b. Prove the necessary and sufficient condition for a curve to be helix is that the ratio of the curvature to torsion is constant at all points. CO3K4

11. a. Prove that the metric is invariant under a parametric transformation. CO4K4

(or)

11. b. If (l', m') are the direction coefficients of a line which makes an angle $\frac{\pi}{2}$ with the line whose direction coefficients are (l, m), then prove that CO4K4

$$l' = -\frac{1}{H}(Fl + Gm), \quad m' = \frac{1}{H}(El + Fm)$$

12. a. Find the surface of revolution which is isometric with the region of the right helicoid.

(or)CO5K3

12. b. Prove a necessary and sufficient condition for a curve $u=u(t), v=v(t)$ on a surface $r=r(u,v)$ to be a geodesic is that $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$ CO5K4

where

$$U = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}}\right) - \frac{\partial T}{\partial u} = \frac{1}{2T}\frac{dT}{dt}\cdot\frac{\partial T}{\partial \dot{u}}$$

$$V = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{v}}\right) - \frac{\partial T}{\partial v} = \frac{1}{2T}\frac{dT}{dt}\cdot\frac{\partial T}{\partial \dot{v}}$$

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