

**Avinashilingam Institute for Home Science and Higher Education for Women**  
(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)  
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B  
Coimbatore - 641 043, Tamil Nadu, India

**Bachelor's Degree Examination - January 2021**  
**V Semester**

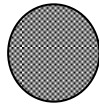



**Class : III UG**  
**Major : Mathematics and Spl. Education and Mathematics**

**Time : 3 Hours**  
**Max. Marks : 100**

**18BMAC15 / 18BSC12 Complex Analysis - I**

**PART A**  
**Choose the Correct Answer**

**10 x 1 = 10**

- If  $f(z)$  is differentiable at  $z_0$  then  $f(z)$  is \_\_\_\_\_ at  $z_0$ .  
a. analytic      b. continuous      c. connected      d. convex
- The function  $f(z) = \bar{z}$  is \_\_\_\_\_ anywhere  
a. continuous      b. not continuous  
c. differentiable      d. not differentiable
- A point at which a function ceases to be analytic is called a  
a. singular point      b. non-singular point  
c. regular point      d. non-regular point
- Which of the following is not an entire function?  
a.  $e^z$       b.  $\sin z$       c.  $\cos z$       d.  $\tan z$
- If  $u$  is harmonic and  $v$  is a harmonic conjugate of  $u$  then  
a.  $u$  is a harmonic conjugate of  $-v$       b.  $-u$  is a harmonic conjugate of  $v$   
c.  $u^2 + v^2$  is a harmonic conjugate of  $-v$       d.  $\frac{u}{v}$  is a harmonic and conjugate of  $+v$
- A function  $f(x, y)$  satisfying Laplace equation is called  
a. Analytic      b. Holomorphic      c. Harmonic      d. Non-harmonic
- Under the bilinear transformation  $w = \frac{az+b}{cz+d}$ , the point  $Z = -\frac{d}{c}$  goes to  $W =$   
a. 0      b.  $ab$   
c.  $\infty$       d.  $\frac{a}{b}$
- The fixed points of  $W = \frac{2zi+5}{z-4i}$  are \_\_\_\_\_.  
a. 2, 5      b.  $5i, i$   
c.  $5i, 1$       d.  $5, i$
- Which of the following is not a simply connected region?  
a.       b.       c.       d. 
- The value of the integral of  $1/z$  along a semicircular arc from  $-1$  to  $1$  in the clockwise direction will be  
a. zero      b.  $-\pi i$       c.  $\pi i$       d.  $2\pi i$

**Part B****5 x 6 = 30****Answer ALL questions****Each answer should not exceed 400 words or two pages**

11.a. Show that the function  $f(z) = xy + iy$  is continuous everywhere but not differentiable anywhere.

(or)

11.b. If  $f(z) = \cos z$ , find  $f'(z)$

12.a. Show that the function  $f(z) = |z|^2$  is differentiable at  $z = 0$  but not analytic at  $z = 0$ .

(or)

12.b. If  $f(z) = u + iv$  is analytic, then show that two families of curves  $u(x, y) = c_1$  and  $v(x, y) = c_2$  where  $c_1$  and  $c_2$  are parameters are orthogonal.

13.a. If  $e^z = u + iv$ , then show that  $u$  and  $v$  are harmonic functions.

(or)

13.b. Construct the analytic function  $f(z)$ , given that  $u(x, y) = x^3 - 3xy^2 + x$ .

14.a. Prove that the cross ratio is preserved by a bilinear transformation.

(or)

14.b. Find the bilinear transformation that maps the points  $\infty, i, 0$  onto  $0, i, \infty$  respectively.

15.a. Find the integral of  $f(z) = z^2$  along the parabolic arc  $y = x^2$  from  $(0,0)$  to  $(1,1)$ .

(or)

15.b. State and prove the Cauchy's integral theorem.

**Part C****5 x 12 = 60****Answer ALL questions****Each answer should not exceed 800 words or four pages**

16.a. State and prove the necessary conditions for differentiability of a function in terms of its real and imaginary parts.

(or)

16.b. Show that the function  $f(z) = \sqrt{|xy|}$  satisfies the C-R equations at  $z = 0$  but not differentiable at  $z = 0$

17.a. Show that  $f(z) = \frac{1}{z}$  is analytic everywhere except at  $z = 0$  and find its derivative.

(or)

17.b. Suppose  $f(z)$  is analytic in a domain. Prove that

- i. if its real part is a constant
- ii. if its imaginary part is a constant
- iii. if its modulus is a constant then  $f(z)$  is a constant

18.a. If  $f(z)$  is analytic, then show that

$$(i) \nabla^2 |Re f(z)|^2 = 2 |f'(z)|^2; \quad (ii) \nabla^2 |f(z)|^2 = 4 |f'(z)|^2.$$

(or)

18.b. Verify whether the function  $v(x, y) = e^x(x \cos y - y \sin y)$  is a harmonic function and find its conjugate harmonic.

19.a. Prove that the bilinear transformations which map the real axis onto the circle  $|w| = 1$  are of the form  $w = e^{i\lambda} \frac{z - z_1}{z - \bar{z}_2}$  where  $\lambda$  is a real number.

(or)

19.b. Evaluate  $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ , where  $C$  is the circle  $|z| = \frac{3}{2}$

20.a. Using Cauchy's integral formula to evaluate  $\int_C \frac{2z-1}{z^3-z} dz$ , where  $C$  is **(i)**  $|z| = 1/2$ ; **(ii)**  $|z| = 2$ .

(or)

20.b. State and prove Cauchy's integral formula for derivative.

\*\*\*\*\*