



# Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category 'A' by MHRD (now MoE)

Re-accredited with an 'A++' Grade by NAAC CGPA 3.65/4, Category I by UGC

Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test I- August 2025

Semester V

Class : III UG

Time : 2 Hours

Major : Mathematics

Max. Marks: 60

## 23BMAC10-Advanced Algebra

### Course Outcomes:

CO1: understand the basic concepts of group actions and their applications.

CO2: recognize and use the Sylow theorems to characterize certain finite groups.

CO3: know the fundamental concepts in ring theory such as the concepts of ideals, quotient rings, integral domains and fields.

CO4: learn in detail about polynomial rings.

CO5: grasp the fundamental properties of finite field extensions and classification of finite fields.

### PART A

6x1 = 6

#### Choose the Correct Answer

- For  $g$  in  $G$  and for  $x$  in  $X$ ,  $stab(x) =$  CO1K2  
a.  $x stab(x) x^{-1}$     b.  $g stab(x) g^{-1}$     c.  $x stab(x)$     d.  $g stab(x)$
- The cardinality of the orbit of  $x$ ,  $|Gx| =$  CO1K2  
a.  $|G||x|$     b.  $|G|$     c.  $|G||stab(x)|$     d.  $|G|/|x|$
- For a group of order 72, any subgroup of order 8 is a \_\_\_\_\_. CO2K1  
a. Sylow 8-subgroup of  $G$     b. Sylow 9-subgroup of  $G$   
c. Sylow 3-subgroup of  $G$     d. Sylow 2-subgroup of  $G$
- A Sylow  $p$ -subgroup of a finite group  $G$  is a normal subgroup of  $G$  if and only if it is \_\_\_\_\_. CO2K1  
a. the only Sylow  $p$ -subgroup of  $G$     b. any one of Sylow  $p$ -subgroup of  $G$   
c. the largest Sylow  $p$ -subgroup of  $G$     d. the smallest Sylow  $p$ -subgroup of  $G$
- The set  $2\mathbb{Z}$  of even integers under ordinary addition and multiplication is a \_\_\_\_\_. CO3K1  
a. ring    b. commutative ring    c. ring with unity    d. commutative ring with unity
- For the subring  $\{0, 2, 4\}$  of the ring  $\mathbb{Z}_6$ , the unity is \_\_\_\_\_. CO3K1  
a. 0    b. 1    c. 2    d. 4

### Part B

3x6=18

#### Answer ALL questions

- a. State and prove structure theorem for orbits. CO1K3  
(or)
- b. State and prove orbit counting theorem CO1K3
- a. Determine the number of normal subgroups of a group  $G$  of order 30. CO2K4  
(or)
- b. If  $G$  is a group of order  $pq$ , where  $p$  and  $q$  are primes,  $p < q$  and  $p$  does not divide  $q-1$ , then show that  $G$  is cyclic CO2K4
- a. State and prove subring test. CO3K3  
(or)
- b. Prove that a finite integral domain is a field. CO3K3

### Part C

3x12= 36

#### Answer ALL questions

- a. How many distinct necklaces of 8 beads can be made by stringing together beads of two colours? CO1K4  
(or)
- b. Prove that  $A_n$  is simple for  $n \geq 5$ . CO1K4
- a. State and prove Sylow's first theorem. CO2K3  
(or)
- b. State and prove Sylow's second theorem. CO2K4
- a. State and prove (i) characteristic of a ring with unity CO3K3  
(ii) characteristic of an integral domain.  
(or)
- b. State and prove existence of factor rings. CO3K4