Chapter 9

Fuzzy λ_g^δ-closed sets in Fuzzy Topological Spaces

9.1 Introduction

The study of fuzzy sets was initiated by Zadeh (1965). Since the generalization of the usual notion of a set into a fuzzy set by Zadeh in his classic paper of 1965, many abstract structures were generalized using fuzzy sets. Fuzzy topological spaces were introduced by Chang (1968). Azad (1981) introduced the concept of fuzzy regular open sets and fuzzy regular closed sets in fuzzy topological spaces. Petricevic (1991) introduced the concept of fuzzy δ-open sets and fuzzy δ-closed sets in fuzzy topological spaces. In 2004, Georgiou presented the notion of (Λ, δ)-closed sets in general topology. Thereafter this notion grasped higher significance due its nature of being partially δ-open and partially δ-closed. This work consists of an extension of (Λ, δ)-closed sets to fuzzy topological spaces and the study of fuzzy λ_g^δ-closed sets in fuzzy topological spaces.

9.2 Fuzzy Λ_δ-sets and Fuzzy V_δ-sets

Definition 9.2.1. A fuzzy subset $F\Lambda_\delta(A)$ of a fuzzy topological space $(X, \mathcal{F})$ is defined as $F\Lambda_\delta(A) = \wedge\{D \in F\delta O(X, \mathcal{F}) \mid A \leq D\}$.

Note : $F\Lambda_\delta(A)$ need not be neither fuzzy δ-open nor fuzzy δ-closed.

Definition 9.2.2. A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***$F\Lambda_\delta(A)$-set*** if $F\Lambda_\delta(A) = A$.

Theorem 9.2.3. *For fuzzy subsets A, B and $A_i (i \in I = [0,1])$ of a fuzzy topological space $(X, \mathcal{F})$, the following are true.*

(i) $A \leq F\Lambda_\delta(A)$.

(ii) $F\Lambda_\delta(F\Lambda_\delta(A)) = F\Lambda_\delta(A)$.

(iii) *If* $A \leq B$ *then* $F\Lambda_\delta(A) \leq F\Lambda_\delta(B)$.

(iv) $F\Lambda_\delta(\bigwedge_{i\in I} A_i) \leq \bigwedge_{i\in I} F\Lambda_\delta(A_i)$.

(v) $F\Lambda_\delta(\bigvee_{i\in I} A_i) = \bigvee_{i\in I} F\Lambda_\delta(A_i)$.

Proof. (i) , (ii), (iii) follow from Definition 9.2.1

(iv) Suppose that $x \notin \bigwedge_{i\in I}\{F\Lambda_\delta(A_i)\}$ then there exists $i_0 \in I$ such that $x \notin F\Lambda_\delta(A_{i_0})$. This implies that there exists a fuzzy δ-open set D such that $x \notin D$ and $A_{i_0} \leq D$. Since $\bigwedge_{i\in I} A_i \leq A_{i_0} \leq D$ and $x \notin D$, we have $x \notin F\Lambda_\delta(\bigwedge A_i)$.

(v) From (i) and (iii), $A_i \leq F\Lambda_\delta(A_i) \leq F\Lambda_\delta(\bigvee_{i\in I} A_i)$, for each $i \in I$. This implies $\bigvee_{i\in I}(F\Lambda_\delta(A_i)) \leq F\Lambda_\delta(\bigvee_{i\in I} A_i)$. Conversely, suppose that $x \notin (F\Lambda_\delta(A_i))$. Then $x \notin \bigvee_{i\in I} F\Lambda_\delta(A_i)$, for each $i \in I$. This implies that there exists $S_i \in F\delta O(X, \mathcal{F})$ such that $A_i \leq S_i$ and $x \notin S_i$, for every $i \in I$. Since $\bigvee_{i\in I} A_i \leq \bigvee_{i\in I} S_i$ and $\bigvee_{i\in I} S_i$ is a fuzzy δ-open set not containing x. Hence $x \notin F\Lambda_\delta(\bigvee_{i\in I} A_i)$. Thus $F\Lambda_\delta(\bigvee_{i\in I} A_i) \leq \bigvee_{i\in I}\{F\Lambda_\delta(A_i)\}$.

□

Remark 9.2.4. The following Example shows that the converse of Theorem 9.2.3(iv) is not true even in infinite case.

Example 9.2.5. Let $X = \{a, b\}$ and $\mathcal{F} = \{0, 1, (0.2_a, 0.5_b), (0.5_a, 0.8_b)\}$. Then $F\delta O(X, \mathcal{F}) = \{0, 1, (0.2_a, 0.5_b)\}$. Let $A_1 = (0.1_a, 0.6_b)$ and $A_2 = (0.3_a, 0.3_b)$. Then

$A_1 \wedge A_2 = (0.1_a, 0.3_b)$. Also, $F\Lambda_\delta(A_1) = F\Lambda_\delta(A_2) = 1$ and $F\Lambda_\delta(A_1 \wedge A_2) = (0.2_a, 0.5_b)$ which implies $F\Lambda_\delta(A_1 \wedge A_2) \not\geq F\Lambda_\delta(A_1) \wedge F\Lambda_\delta(A_2)$.

Corollary 9.2.6. (i) $F\Lambda_\delta(A)$ is a $F\Lambda_\delta$-set.

(ii) If A is a fuzzy δ-open set, then A is a $F\Lambda_\delta$-set.

Proof. (i) Follows from (ii) of Theorem 9.2.3 and Definition 9.2.2.

(ii) Follows from Definitions 9.2.1 and 9.2.2.

□

Theorem 9.2.7. *In a fuzzy topological space $(X, \mathcal{F})$, the following are true.*

(i) Arbitrary Intersection of $F\Lambda_\delta$-sets is a $F\Lambda_\delta$-set.

(ii) Arbitrary Union of $F\Lambda_\delta$-sets is a $F\Lambda_\delta$-set.

Proof. (i) Let A_i, where $i \in I$ be $F\Lambda_\delta$-sets.

$$
\begin{aligned}
F\Lambda_\delta(\bigwedge_{i\in I} A_i) &= F\Lambda_\delta(A_1 \wedge A_2 \wedge \ldots \wedge A_n \wedge \ldots) \\
&\leq F\Lambda_\delta(A_1) \wedge F\Lambda_\delta(A_2) \wedge \ldots \wedge F\Lambda_\delta(A_n) \wedge \ldots \\
&\qquad \text{(By Theorem 9.2.3 (iv))} \\
&= A_1 \wedge A_2 \wedge \ldots \wedge A_n \wedge \ldots \\
&\qquad \text{(Since each } A_i \text{ is a } F\Lambda_\delta\text{-set)} \\
&= \bigwedge_{i\in I} A_i
\end{aligned}
$$

Also by Theorem 9.2.3 (i), $\bigwedge_{i\in I} A_i \leq F\Lambda_\delta(\bigwedge_{i\in I} A_i)$. Hence arbitrary intersection of $F\Lambda_\delta$-sets is a $F\Lambda_\delta$-set.

(ii) Follows directly from Theorem 9.2.3(v).

□

Definition 9.2.8. A fuzzy subset $F\vee_\delta(A)$ of a fuzzy topological space $(X, \mathcal{F})$ is defined as $FV_\delta(A) = \vee\{C \in F\delta C(X, \mathcal{F}) \mid C \leq A\}$.

Definition 9.2.9. A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***FV_δ-set*** if $FV_\delta(A) = A$.

Theorem 9.2.10. *For fuzzy subsets A, B and $A_i (i \in I = [0,1])$ of a fuzzy topological space $(X, \mathcal{F})$, the following are true.*

(i) $FV_\delta(A) \leq A$.

(ii) $FV_\delta(FV_\delta(A)) = FV_\delta(A)$.

(iii) *If* $A \leq B$ *then* $FV_\delta(A) \leq FV_\delta(B)$.

(iv) $FV_\delta(\bigwedge_{i \in I}\{A_i\}) = \bigwedge_{i \in I}\{FV_\delta(A_i)\}$.

(v) $FV_\delta(\bigvee_{i \in I} A_i) \geq \bigvee_{i \in I}\{FV_\delta(A_i)\}$.

(vi) *If* A *is a fuzzy* δ*-closed set then* A *is a* FV_δ*-set.*

(vii) $F\Lambda_\delta(1-A) = 1 - FV_\delta(A)$ *and* $FV_\delta(1-A) = 1 - F\Lambda_\delta(A)$.

Proof. (i) to (vi) Similar to Theorem 9.2.3 and Corollary 9.2.11.

(vii)
$$\begin{aligned} 1 - FV_\delta(A) &= 1 - \vee\{C \mid C \in F\delta C(X,\mathcal{F}) \text{ and } C \leq A\} \\ &= \wedge\{1 - C \mid 1 - C \in F\delta O(X,\mathcal{F}) \text{ and } 1 - C \geq 1 - A\} \\ &= \wedge\{D \mid D \in F\delta O(X,\mathcal{F}) \text{ and } 1 - A \leq D\} \\ &= F\Lambda_\delta(1-A). \end{aligned}$$

Similarly, the other equality can be proved. □

Corollary 9.2.11. $FV_\delta(A)$ is a FV_δ-set.

Proof. Follows from (ii) of Theorem 9.2.10 and Definition 9.2.9. □

Definition 9.2.12. A function $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ is called a ***fuzzy*** $\boldsymbol{\Lambda_\delta}$***-continuous*** (briefly $F\Lambda_\delta$-continuous) function if the inverse image of every fuzzy closed set in $(Y, \mathcal{G})$ is a fuzzy Λ_δ-set in $(X, \mathcal{F})$.

Theorem 9.2.13. *For a map* $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$*, the following are equivalent.*

(i) f *is* $F\Lambda_\delta$*-continuous;*

(ii) Inverse image of every fuzzy open set in $(Y, \mathcal{G})$ *is fuzzy* V_δ*-set in* $(X, \mathcal{F})$*.*

Proof. Follows from Theorem 9.2.10(vii). □

9.3 Fuzzy (Λ, δ)-closed sets in fuzzy topological spaces

Definition 9.3.1. A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***fuzzy*** (Λ, δ)***-closed*** (briefly $F(\Lambda, \delta)$-closed) set if $A = K \wedge L$, where K is a $F\Lambda_\delta$-set and L is a fuzzy δ-closed set in $(X, \mathcal{F})$.The family of all fuzzy (Λ, δ)-closed sets in $(X, \mathcal{F})$ is denoted by $F(\Lambda, \delta)C(X, \mathcal{F})$.

Theorem 9.3.2. *The following are equivalent for a fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$.*

(i) A is $F(\Lambda, \delta)$-closed in $(X, \mathcal{F})$;

(ii) $A = K \wedge cl_\delta(A)$, where K is a $F\Lambda_\delta$-set of $(X, \mathcal{F})$;

(iii) $A = F\Lambda_\delta(A) \wedge cl_\delta(A)$;

(iv) $A = F\Lambda_\delta(A) \wedge L$, where L is a fuzzy δ-closed set of $(X, \mathcal{F})$.

Proof. (i) $\Rightarrow$ (ii) Let $A = K \wedge L$, where K is a $F\Lambda_\delta$-set and L is a fuzzy δ-closed set. Now, $A \leq L \Rightarrow cl_\delta(A) \leq L$. Also, $A \leq K \wedge cl_\delta(A) \leq K \wedge L = A$. Therefore $A = K \wedge cl_\delta(A)$.

(ii) $\Rightarrow$ (iii) Let $A = K \wedge cl_\delta(A)$, where K is a $F\Lambda_\delta$-set. Now, $A \leq K \Rightarrow F\Lambda_\delta(A) \leq F\Lambda_\delta(K) = K \Rightarrow F\Lambda_\delta(A) \leq K$. Therefore $A \leq F\Lambda_\delta(A) \wedge cl_\delta(A) \leq K \wedge cl_\delta(A) = A$. Hence $A = F\Lambda_\delta(A) \wedge cl_\delta(A)$.

(iii) $\Rightarrow$ (iv) Let $A = F\Lambda_\delta(A) \wedge cl_\delta(A)$ and put $cl_\delta(A) = L$. Hence $A = F\Lambda_\delta(A) \wedge L$, where L is a fuzzy δ-closed set.

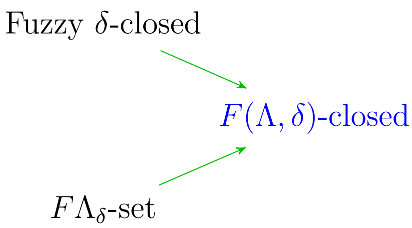
(iv) $\Rightarrow$ (i) Follows from Definition 9.3.1 and Corollary 9.2.11.

□

Theorem 9.3.3. *Every fuzzy δ-closed(resp. $F\Lambda_\delta$-) set is a $F(\Lambda, \delta)$-closed set but not conversely.*

Proof. Follows from Definition 9.3.1 and the fact that 1 is $F(\Lambda, \delta)$-closed(resp. fuzzy δ-closed). □

Figure 9.1:



Example 9.3.4. Let $X = \{a, b\}$ and $\mathcal{F} = \{0, 1, (0.2_a, 0.5_b), (0.5_a, 0.8_b)\}$. Then $(0.2_a, 0.5_b)$ is $F(\Lambda, \delta)$-closed but not fuzzy δ-closed and $(0.8_a, 0.5_b)$ is $F(\Lambda, \delta)$-closed but not a $F\Lambda_\delta$-set.

Theorem 9.3.5. *Every fuzzy δ-dense (Thangaraj, 2015) set which is also $F(\Lambda, \delta)$-closed is a $F\Lambda_\delta$-set.*

Proof. Let $(X, \mathcal{F})$ be a fuzzy topological space and A be a fuzzy δ-dense as well as $F\Lambda_\delta$-closed in $(X, \mathcal{F})$. Then by Theorem 9.3.2(ii), $A = K \wedge cl_\delta(A)$, where K is a $F\Lambda_\delta$-set. Since A is fuzzy δ-dense, $cl_\delta(A) = 1$ (Thangaraj(2015), Definition 4.8) and hence $A = K$, where K is a $F\Lambda_\delta$-set. □

Theorem 9.3.6. *Let $(X, \mathcal{F})$ be a fuzzy topological spaces. If A is fuzzy open then $cl(A)$ is $F(\Lambda, \delta)$-closed.*

Proof. If A is fuzzy open then $cl(A)$ is fuzzy regular closed[**?**] and therefore fuzzy δ-closed. Further, the proof follows from Theorem 9.3.5. □

Definition 9.3.7. A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***fuzzy* (Λ, δ)*-open*** (briefly $F(\Lambda, \delta)$-open) set if $A = K \vee L$, where K is a $F\vee_\delta$-set and L is a fuzzy δ-open set.

Equivalently, the complement of a fuzzy (Λ, δ)-closed set is called fuzzy (Λ, δ)-open. The family of all fuzzy (Λ, δ)-open sets in $(X, \mathcal{F})$ is denoted by $F(\Lambda, \delta)O(X, \mathcal{F})$.

Theorem 9.3.8. *Let $(X, \mathcal{F})$ be a fuzzy topological space. Then*

(i) Arbitrary intersection of $F(\Lambda, \delta)$-closed sets is $F(\Lambda, \delta)$-closed in $(X, \mathcal{F})$.

(ii) *Arbitrary union of $F(\Lambda, \delta)$-open sets is $F(\Lambda, \delta)$-open in $(X, \mathcal{F})$.*

Proof. (i) Let A_i be a $F(\Lambda, \delta)$-closed set, for each $i \in I$. Then $A_i = K_i \wedge L_i$, where K_i is a $F\wedge_\delta$-set and L_i is a fuzzy δ-closed set, for each $i \in I$. Now $\bigwedge_{i\in I} A_i = \bigwedge_{i\in I}(K_i \wedge L_i) = (\bigwedge_{i\in I} K_i) \wedge (\bigwedge_{i\in I} L_i)$. Since any intersection of $F\Lambda_\delta$-sets is a $F\Lambda_\delta$-set [By Theorem 9.2.7] and fuzzy δ-closed sets is fuzzy δ-closed, A_i is a $F(\Lambda, \delta)$-closed set.

(ii) Let A_i be a $F(\Lambda, \delta)$-open set for each $i \in I$. Then $X \setminus A_i$ is a $F(\Lambda, \delta)$-closed set for each $i \in I$. $X \setminus \bigvee_{i\in I} A_i = \bigwedge_{i\in I}(X \setminus A_i)$. Therefore by (i), $\bigvee_{i\in I} A_i$ is $F(\Lambda, \delta)$-open.

□

Theorem 9.3.9. *The following are equivalent for a fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$.*

(i) *A is $F(\Lambda, \delta)$-open;*

(ii) *$A = K \vee int_\delta(A)$, where K is a FV_δ-set;*

(iii) *$A = FV_\delta(A) \vee int_\delta(A)$;*

(iv) *$A = FV_\delta(A) \vee L$, where L is a fuzzy δ-open set.*

Proof. Similar to Theorem 9.3.2. □

Definition 9.3.10. ***Fuzzy (Λ, δ)-closure*** (briefly $\boldsymbol{F(\Lambda, \delta)cl(A)}$) of a fuzzy subset A is defined as $F(\Lambda, \delta)cl(A) = \wedge\{D \in F(\Lambda, \delta)C(X, \mathcal{F}) \mid A \leq D\}$.

Theorem 9.3.11. *For fuzzy subsets A and B of a fuzzy topological space $(X, \mathcal{F})$, the following conditions are true.*

(i) *$A \leq F(\Lambda, \delta)cl(A)$.*

(ii) *If $A \leq B$, then $F(\Lambda, \delta)cl(A) \leq F(\Lambda, \delta)cl(B)$.*

(iii) *$F(\Lambda, \delta)cl(\mathbf{0}) = \mathbf{0}$ and $F(\Lambda, \delta)cl(\mathbf{1}) = \mathbf{1}$.*

(iv) *$F(\Lambda, \delta)cl(A)$ is a fuzzy (Λ, δ)-closed set.*

(v) *A is fuzzy (Λ, δ)-closed iff $F(\Lambda, \delta)cl(A) = A$.*

Proof. Straight forward. □

Definition 9.3.12. A function $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ is said to be ***fuzzy* (Λ, δ)*-continuous*** (briefly $F(\Lambda, \delta)$-continuous) function if $f^{-1}(B)$ is a $F(\Lambda, \delta)$-closed in X for each fuzzy closed set B in Y.

Theorem 9.3.13. *If a fuzzy function $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ is said to be fuzzy (Λ, δ)-continuous then for each fuzzy set A in X, $f(F(\Lambda, \delta)cl(A)) \leq cl(f(A))$.*

Proof. $cl(f(A))$ is fuzzy closed in Y. By hypothesis, $f^{-1}(cl(f(A)))$ is $F(\Lambda, \delta)$-closed in X. Now, $f(A) \leq cl(f(A)) \Rightarrow A \leq f^{-1}(f(A)) \leq f^{-1}(cl(f(A))) \Rightarrow F(\Lambda, \delta)cl(A) \leq F(\Lambda, \delta)cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A))) \Rightarrow f(F(\Lambda, \delta)cl(A)) \leq cl(f(A))$. □

Proposition 9.3.14. Every fuzzy super-continuous(resp. $F\Lambda_\delta$-continuous) function is $F(\Lambda, \delta)$-continuous but not conversely.

Proof. Follows from Theorem 9.3.3. □

Example 9.3.15. Let $X = Y = \{a, b\}$ and $\mathcal{F} = \mathcal{G} = \{0, 1, (0.2_a, 0.5_b), (0.5_a, 0.8_b)\}$. Define $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ as follows

$$f(a, b) = \begin{cases} (a, b), & \text{if } a = 0.2 \text{ and } b = 0.5 \\ (b, a), & \text{otherwise.} \end{cases}$$

Then $f^{-1}\{(0.2_a, 0.5_b)\} = (0.2_a, 0.5_b)$ is $F(\Lambda, \delta)$-open but not fuzzy δ-open. Hence f is $F(\Lambda, \delta)$-continuous but not fuzzy super-continuous.

Example 9.3.16. Let $X = Y = \{a, b\}$ and $\mathcal{F} = \mathcal{G} = \{0, 1, (0.2_a, 0.5_b), (0.5_a, 0.8_b)\}$. Define $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ as follows

$$f(a, b) = \begin{cases} (b, a), & \text{if } a = 0.5 \text{ and } b = 0.8 \\ 1, & \text{otherwise.} \end{cases}$$

Then $f^{-1}\{(0.5_a, 0.8_b)\} = (0.8_a, 0.5_b)$ is $F(\Lambda, \delta)$-open but not a $F\Lambda_\delta$-set. Hence f is $F(\Lambda, \delta)$-continuous but not $F\Lambda_\delta$-continuous.

9.4 Fuzzy λ_g^δ-closed sets

Definition 9.4.1. A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***fuzzy λ_g^δ-closed set*** (briefly $F\lambda_g^\delta$-closed set) if $cl_\delta(A) \leq U$, whenever $A \leq U$ and U is a fuzzy (Λ, δ)-open set. The family of all fuzzy λ_g^δ-closed sets is denoted by $F\lambda_g^\delta C(X, \mathcal{F})$.

Example 9.4.2. Let $X = [0, 1]$ and $\mathcal{F} = \{0, 1, A, B\}$ where,

$$A = \begin{cases} 0.5, & \text{if } x = 2/3 \\ 0, & \text{otherwise.} \end{cases}$$

and

$$B = \begin{cases} 0.8, & \text{if } x = 2/3 \\ 1, & \text{otherwise.} \end{cases}$$

Let $C = \begin{cases} 0.3, & \text{if } x = 2/3 \\ 1, & \text{otherwise.} \end{cases}$
Then C is $F\lambda_g^\delta$-closed in $(X, \mathcal{F})$.

Proposition 9.4.3. Every Fuzzy δ-closed set is a $F\lambda_g^\delta$-closed set but not conversely.

Proof. Let A be a fuzzy δ-closed set in a fuzzy topological space $(X, \mathcal{F})$. Let U be a fuzzy (Λ, δ)-open set such that $A \leq U$. Since A is fuzzy δ-closed, $cl_\delta(A) = A \leq U$ and thus A is $F\lambda_g^\delta$-closed. □

Example 9.4.4. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then C is $F\lambda_g^\delta$-closed but not δ-closed.

Proposition 9.4.5. Every fuzzy regular closed set is a $F\lambda_g^\delta$-closed set but not conversely.

Proof. Follows from the fact that every fuzzy regular closed set is a fuzzy δ-closed set and Proposition 9.5. □

Example 9.4.6. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then C is $F\lambda_g^\delta$-closed but not fuzzy regular closed.

Remark 9.4.7. Fuzzy closed set and $F\lambda_g^\delta$-closed set are independent of each other.

Example 9.4.8. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then C is $F\lambda_g^\delta$-closed but not fuzzy closed.

Example 9.4.9. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then

$$A^c = \begin{cases} 0.2, & \text{if } x = 2/3 \\ 0, & \text{otherwise.} \end{cases}$$

is fuzzy closed but not $F\lambda_g^\delta$-closed.

Remark 9.4.10. Fuzzy (Λ, δ)-closed set and $F\lambda_g^\delta$-closed set are independent of each other.

Example 9.4.11. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then C is $F\lambda_g^\delta$-closed but not fuzzy (Λ, δ)-closed.

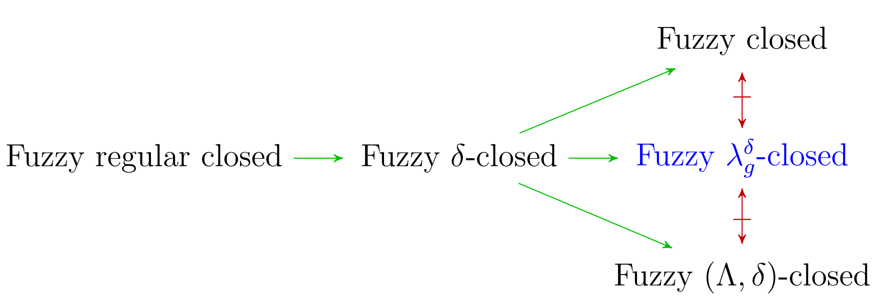
Example 9.4.12. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then

$$A = \begin{cases} 0.5, & \text{if } x = 2/3 \\ 0, & \text{otherwise.} \end{cases}$$

is fuzzy (Λ, δ)-closed but not $F\lambda_g^\delta$-closed.

Remark 9.4.13. The following Figure represents the relation between the various fuzzy closed sets discussed above.

Figure 9.2:



Theorem 9.4.15. *If A is $F\lambda_g^\delta$-closed and $A \leq B \leq cl_\delta(A)$ then B is also a $F\lambda_g^\delta$-closed set.*

Proof. Let A be $F\lambda_g^\delta$-closed in a fuzzy topological space $(X, \mathcal{F})$ with $A \leq B \leq cl_\delta(A)$ and let $B \leq U$, where U is a fuzzy (Λ, δ)-open set in $(X, \mathcal{F})$. Since $A \leq B \leq cl_\delta(A)$, $cl_\delta(A) = cl_\delta(B)$ and $A \leq U$. Therefore $cl_\delta(B) = cl_\delta(A) \leq U$, since A is $F\lambda_g^\delta$-closed. This proves that B is $F\lambda_g^\delta$-closed. □

Theorem 9.4.16. *If A is fuzzy (Λ, δ)-open as well as $F\lambda_g^\delta$-closed then A is fuzzy δ-closed.*

Proof. Let A be $F\lambda_g^\delta$-closed in a fuzzy topological space $(X, \mathcal{F})$. Since $A \leq A$ and A is fuzzy (Λ, δ)-open as well as $F\lambda_g^\delta$-closed, $cl_\delta(A) \leq A$ which proves A is fuzzy δ-closed. □

Theorem 9.4.17. *A fuzzy subset A is $F\lambda_g^\delta$-closed iff $A\bar{q}B \Rightarrow cl_\delta(A)\bar{q}B$, for every fuzzy (Λ, δ)-closed set B of $(X, \mathcal{F})$.*

Proof. *Necessity:* Let A be $F\lambda_g^\delta$-closed and $A\bar{q}B$, for every fuzzy (Λ, δ)-closed set B of $(X, \mathcal{F})$. $A\bar{q}B \Rightarrow A \leq B^c$, where B^c is fuzzy (Λ, δ)-open in $(X, \mathcal{F})$. Since A is $F\lambda_g^\delta$-closed, $cl_\delta(A) \leq B^c \Rightarrow cl_\delta(A)\bar{q}B$.

Sufficiency: Let $A\bar{q}B \Rightarrow cl_\delta(A)\bar{q}B$, for every fuzzy (Λ, δ)-closed set B of $(X, \mathcal{F})$ and let $A \leq B$, where B is fuzzy (Λ, δ)-open in $(X, \mathcal{F})$. Then by the definition on quasi-coincidence, $A\bar{q}B^c \Rightarrow cl_\delta(A)\bar{q}B^c \Rightarrow cl_\delta(A) \leq B^c$. This proves that A is $F\lambda_g^\delta$-closed. □

Theorem 9.4.18. *If A is $F\lambda_g^\delta$-closed in $(X, \mathcal{F})$ and x_r is a fuzzy point of $(X, \mathcal{F})$ such that $x_r q cl_\delta(A)$ then $cl_\delta(x_r)qA$.*

Proof. Let $x_r q cl_\delta(A)$, where A is $F\lambda_g^\delta$-closed in $(X, \mathcal{F})$. Suppose $cl_\delta(x_r)\bar{q}A$ then $A \leq (cl_\delta(x_r))^c$. Since $(cl_\delta(x_r))^c$ is fuzzy (Λ, δ)-open and A is $F\lambda_g^\delta$-closed, $cl_\delta(A) \leq (cl_\delta(x_r))^c \leq$

$((x_r))^c \Rightarrow x_r \leq (cl_\delta(A))^c \Rightarrow x_r \bar{q} cl_\delta(A)$, a contradiction. Thus $cl_\delta(x_r)qA$. □

9.5 $F\lambda_g^\delta$-open sets

Definition 9.5.1. A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***fuzzy*** λ_g^δ***-open set*** (briefly $F\lambda_g^\delta$-open set) if $U \leq int_\delta(A)$, whenever $U \leq A$ and U is a fuzzy (Λ, δ)-closed set. The family of all fuzzy λ_g^δ-open sets is denoted by $F\lambda_g^\delta O(X, \mathcal{F})$.

Equivalently, the complement of a $F\lambda_g^\delta$-closed set is called as a $F\lambda_g^\delta$-open set.

Example 9.5.2. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then

$$1 - A = \begin{cases} 0.5, & \text{if } x = 2/3 \\ 1, & \text{otherwise.} \end{cases}$$

is a fuzzy (Λ, δ)-open set.

Proposition 9.5.3. Every fuzzy δ-open set is a $F\lambda_g^\delta$-open set but not conversely.

Proof. Let A be a fuzzy δ-open set in $(X, \mathcal{F})$. Then $1 - A$ is a fuzzy δ-closed set in $(X, \mathcal{F})$. By Proposition $1 - A$ is a $F\lambda_g^\delta$-closed set and hence A is $F\lambda_g^\delta$-open. □

Example 9.5.4. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then $1 - C$ is $F\lambda_g^\delta$-open but not δ-open.

Proposition 9.5.5. Every fuzzy regular open set is a $F\lambda_g^\delta$-open set but not conversely.

Proof. Let A be a fuzzy regular open set in $(X, \mathcal{F})$. Then $1 - A$ is a fuzzy regular closed set in $(X, \mathcal{F})$. By Proposition $1-A$ is a $F\lambda_g^\delta$-closed set and hence A is $F\lambda_g^\delta$-open. □

Example 9.5.6. Let $X, \mathcal{F}, A, B$ and C be defined as in Example 9.4.2. Then $1 - C$ is $F\lambda_g^\delta$-open but not regular open.

Theorem 9.5.8. *If A is a $F\lambda_g^\delta$-open set in $(X, \mathcal{F})$ such that $int_\delta(A) \leq B \leq A$ then B is $F\lambda_g^\delta$-open.*

Proof. Let A be a $F\lambda_g^\delta$-open set in $(X, \mathcal{F})$ such that $int_\delta \leq B \leq A$. Then $1 - A$ is a $F\lambda_g^\delta$-closed set in $(X, \mathcal{F})$ with $1 - A \leq 1 - B \leq cl_\delta(1 - A)$. This implies $1 - B$ is a $F\lambda_g^\delta$-closed set in $(X, \mathcal{F})$ and hence B is $F\lambda_g^\delta$-open in $(X, \mathcal{F})$. □

Definition 9.5.9. A fuzzy set A is called ***fuzzy λ_g^δ-q-neighborhood of a point x_r*** if there exists $F\lambda_g^\delta$-open set U such that $x_{rq}U \leq A$.

Theorem 9.5.10. *Let $(X, \mathcal{F})$ be a fuzzy topological space. Then A is $F\lambda_g^\delta$-open iff for each point x_r with $x_{rq}A$, A is a fuzzy λ_g^δ-q-neighborhood of a point x_r.*

Proof. *Necessity:* Let A be a $F\lambda_g^\delta$-open set. Let x_r be a fuzzy point such that $x_{rq}A$. Then $r + A(x) > 1$. Suppose there is no $F\lambda_g^\delta$-open set U such that $x_{rq}U \leq A$. That is, $x_r\bar{q}B$, for all $U \leq A$ and U is $F\lambda_g^\delta$-open. Then $r + U(x) \leq 1 \Rightarrow r + A(x) \leq 1$ [Since $A \leq A$ and A is $F\lambda_g^\delta$-open], which is a contradiction.

Sufficiency: Let $G = \{B \in F\lambda_g^\delta O(X, \mathcal{F}) \mid B < A\} \neq A$. Then there exists x such that $G(x) < A(x)$. Let $\vartheta = 1 \setminus G(x) > 0$. Take $\varrho = x_\vartheta$. Then $A(x) + \vartheta > G(x) + \vartheta = 1$. Therefore $x_{\vartheta q}A \Rightarrow x_{\vartheta q}U$, for every $U \in F\lambda_g^\delta O(X, \mathcal{F})$, by criteria. Since any member $B \in F\lambda_g^\delta O(X, \mathcal{F})$ which is $< A$ is contained in G, $B \leq G$. This implies $B(x) + \vartheta \leq G(x) + \vartheta = 1 \Rightarrow x_\vartheta\bar{q}B$, a contradiction. □

9.6 $F\lambda_g^\delta$-closure and $F\lambda_g^\delta$-interior

Definition 9.6.1. The ***Fuzzy*** λ_g^δ***-closure*** (briefly $F\lambda_g^\delta$-closure) of a fuzzy subset A in a fuzzy topological space $(X, \mathcal{F})$ is defined as

$$F\lambda_g^\delta cl(A) = \wedge\{P : A \leq P \text{ and } P \in F\lambda_g^\delta C(X, \mathcal{F})\}$$

and denoted by $F\lambda_g^\delta cl(A)$.

Proposition 9.6.2. Let A be a fuzzy subset and x_r be a fuzzy point of a fuzzy topological space $(X, \mathcal{F})$. Then $x_r \in F\lambda_g^\delta cl(A)$ iff $J \wedge A \neq 0$ for each $J \in F\lambda_g^\delta O(X, \mathcal{F})$ and $x_r \in J$.

Proof. *Necessity:* Suppose if there exists a $F\lambda_g^\delta$-open set J such that $x_r \in J$ and $A \wedge J = 0$. Then $1 - J$ is a $F\lambda_g^\delta$-closed set with $A \leq 1 - J$ and $x_r \notin 1 - J$. By Definition 9.6.1, $F\lambda_g^\delta cl(A) \leq 1 - J$. This implies $x_r \notin F\lambda_g^\delta cl(A)$. *Sufficiency:* Suppose if $x_r \notin F\lambda_g^\delta cl(A)$. Then there exists a $F\lambda_g^\delta$-closed set D such that $A \leq D$ and $x_r \notin D$. This implies $1 - D$ is a $F\lambda_g^\delta$-open set with $x_r \in 1 - D$ and $(1 - D) \wedge A = 0$. Taking $J = 1 - D$, we arrive at a contradiction. □

Proposition 9.6.3. For any two fuzzy sets A and B of a fuzzy topological space $(X, \mathcal{F})$, the following properties are true.

(i) $F\lambda_g^\delta cl(0) = 0$ and $F\lambda_g^\delta cl(1) = 1$;

(ii) If $A \leq B$, then $F\lambda_g^\delta cl(A) \leq F\lambda_g^\delta cl(B)$;

(iii) $A \leq F\lambda_g^\delta cl(A)$;

(iv) $F\lambda_g^\delta cl(F\lambda_g^\delta cl(A)) = F\lambda_g^\delta cl(A)$;

(v) $F\lambda_g^\delta cl(A \vee B) = F\lambda_g^\delta cl(A) \vee F\lambda_g^\delta cl(B)$;

(vi) $F\lambda_g^\delta cl(A \wedge B) \leq F\lambda_g^\delta cl(A) \wedge F\lambda_g^\delta cl(B)$.

Proof. (i), (ii), (iii) and (iv) follow from Definition 9.6.1.

(v) Since $A \leq A \vee B$ and $B \leq A \vee B$, by (ii) we get, $F\lambda_g^\delta cl(A) \leq F\lambda_g^\delta cl(A \vee B)$ and $F\lambda_g^\delta cl(B) \leq F\lambda_g^\delta cl(A \vee B)$. This implies $F\lambda_g^\delta cl(A) \vee F\lambda_g^\delta cl(B) \leq F\lambda_g^\delta cl(A \vee B)$. On the other hand, $A \leq F\lambda_g^\delta cl(A)$ and $B \leq F\lambda_g^\delta cl(B)$ giving $A \vee B \leq F\lambda_g^\delta cl(A) \vee F\lambda_g^\delta cl(B)$. By (ii) and (iv), we have $F\lambda_g^\delta cl(A \vee B) \leq F\lambda_g^\delta cl(A) \vee F\lambda_g^\delta cl(B)$.

(vi) Since $A \wedge B \leq A$ and $A \wedge B \leq B$, by (ii) we have, $F\lambda_g^\delta cl(A \wedge B) \leq F\lambda_g^\delta cl(A)$ and $F\lambda_g^\delta cl(A \wedge B) \leq F\lambda_g^\delta cl(B)$ giving $\lambda_g^\delta cl(A \wedge B) \leq F\lambda_g^\delta cl(A) \wedge F\lambda_g^\delta cl(B)$.

□

Theorem 9.6.4. *A fuzzy point $x_r \in F\lambda_g^\delta cl(A)$ iff every $F\lambda_g^\delta$-open set which is q-coincident with x_r is also q-coincident with A.*

Proof. Let $x_r \in F\lambda_g^\delta cl(A) \Leftrightarrow$ for every $F\lambda_g^\delta$-closed set $F > A$, $x_r \in F$ or $F(x) \geq t$ [Since $F\lambda_g^\delta cl(A)$ is the intersection of all $F\lambda_g^\delta$-closed sets containing $A \Leftrightarrow$ for every $F\lambda_g^\delta$-open set $B \leq A^c$, $B(x) \leq 1 \setminus r$. That is, for every $F\lambda_g^\delta$-open set B satisfying $B(x) \geq 1 - r$ and B is not contained in $1 \setminus A$. By Proposition 2.1 (Pu, Liu (1980)), B is not contained in $1 \setminus A \Leftrightarrow B$ is q-coincident with $1 \setminus (1 \setminus A) = A$. Therefore $x_r \in F\lambda_g^\delta cl(A)$ iff every $F\lambda_g^\delta$-open set which is q-coincident with x_r is also q-coincident with A. □

Definition 9.6.5. The ***Fuzzy λ_g^δ-interior*** (briefly $F\lambda_g^\delta$-interior) of a fuzzy subset A in a fuzzy topological space $(X, \mathcal{F})$ is defined as

$$F\lambda_g^\delta int(A) = \vee\{Q : Q \leq A \text{ and } Q \in F\lambda_g^\delta O(X, \mathcal{F})\}$$

and denoted by $F\lambda_g^\delta int(A)$.

Proposition 9.6.6. For any two fuzzy sets A and B of a fuzzy topological space $(X, \mathcal{F})$, the following properties are true.

(i) $F\lambda_g^\delta int(0) = 0$ and $F\lambda_g^\delta int(1) = 1$;

(ii) If $A \leq B$, then $F\lambda_g^\delta int(A) \leq F\lambda_g^\delta int(B)$;

(iii) $F\lambda_g^\delta int(A) \leq A$;

(iv) $F\lambda_g^\delta int(F\lambda_g^\delta int(A)) = F\lambda_g^\delta int(A)$;

(v) $F\lambda_g^\delta int(A \vee B) \geq F\lambda_g^\delta int(A) \vee F\lambda_g^\delta int(B)$;

(vi) $F\lambda_g^\delta int(A \wedge B) = F\lambda_g^\delta int(A) \wedge F\lambda_g^\delta int(B)$;

(vii)1 $- F\lambda_g^\delta int(A) = F\lambda_g^\delta cl(1-A)$.

Proof. (i) to (vi) is similar to previous proposition.

$$\begin{aligned}\text{(vii)}1 - F\lambda_g^\delta int(A) &= 1 - \vee\{Q : Q \leq A \text{ and } Q \in F\lambda_g^\delta O(X,\mathcal{F})\}\\ &= \wedge\{1-Q : 1-Q \geq 1-A \text{ and } 1-Q \in F\lambda_g^\delta C(X,\mathcal{F})\}\\ &= F\lambda_g^\delta cl(1-A).\end{aligned}$$
□

9.7 Somewhat Fuzzy λ_g^δ-continuous function

Definition 9.7.1. A fuzzy function $f : (X,\mathcal{F}) \to (Y,\mathcal{G})$ is called ***somewhat fuzzy*** λ_g^δ***-continuous*** if for $B \in \mathcal{G}$ and $f^{-1}(B) \neq 0$, there exists a $F\lambda_g^\delta$-open set in $(X,\mathcal{F})$ such that $A \neq 0$ and $A \leq f^{-1}(B)$. That is, $F\lambda_g^\delta int(f^{-1}(B)) \neq 0$.

Proposition 9.7.2. Every somewhat fuzzy δ-continuous function is a fuzzy somewhat λ_g^δ-continuous function.

Proof. Let $f : (X,\mathcal{F}) \to (Y,\mathcal{G})$ and B be a fuzzy open set in $(Y,\mathcal{G})$ and $f^{-1}(B) \neq 0$. Since f is somewhat fuzzy δ-continuous. there exists a fuzzy δ open set A in $(X,\mathcal{F})$ such that $A \neq 0$ and $A \leq f^{-1}(B)$. As every fuzzy δ-open set is a $F\lambda_g^\delta$-open set, there exists a fuzzy $F\lambda_g^\delta$-open set A in $(X,\mathcal{F})$ such that $A \neq 0$ and $A \leq f^{-1}(B)$. This proves f is somewhat fuzzy λ_g^δ-continuous. □

Proposition 9.7.4. If $f : (X,\mathcal{F}) \to (Y,\mathcal{G})$ is a somewhat fuzzy λ_g^δ-continuous function and $g : (Y,\mathcal{G}) \to (Z,\mathcal{H})$ is a fuzzy continuous function (resp. fuzzy δ-continuous function) then $g \circ f : (X,\mathcal{F}) \to (Z,\mathcal{H})$ is a somewhat fuzzy λ_g^δ-continuous function.
In other words, Composition of a somewhat fuzzy λ_g^δ-continuous function and a fuzzy continuous function (resp. fuzzy δ-continuous function) is a somewhat fuzzy λ_g^δ-continuous function.

Proof. Let C be a fuzzy open set in $(Z, \mathcal{H})$. Since g is fuzzy continuous(resp. fuzzy δ-continuous), $g^{-1}(C)$ is fuzzy open(resp. fuzzy δ-open) in $(Y, \mathcal{G})$. Since $g^{-1}(C)$ is fuzzy open(as fuzzy δ-open is fuzzy open), $g^{-1}(C) \neq 0$ and since f is somewhat fuzzy λ_g^δ-continuous, there exists a fuzzy λ_g^δ-open set A in $(X, \mathcal{F})$ such that $A \neq 0$ and $A \leq f^{-1}(g^{-1}(C))$. This proves $g \circ f$ is somewhat fuzzy λ_g^δ-continuous. □

Proposition 9.7.5. If f is a somewhat fuzzy λ_g^δ-continuous function then for a fuzzy closed set B of $(Y, \mathcal{G})$, such that $f^{-1}(B) \neq 1$, there exists a fuzzy λ_g^δ-closed set $A \neq 1$ of X such that $f^{-1}(B) \leq A$.

Proof. Let B be a fuzzy closed set in $Y, \mathcal{G}$ such that $f^{-1}(B) \neq 1$. Then $1 \setminus B$ is a fuzzy open set such that$f^{-1}(1 \setminus B) = 1 \setminus f^{-1}(B) \neq 0$. Since f is somewhat fuzzy λ_g^δ-continuous, there exists a fuzzy δ-open set $A \neq 0$ in $(X, \mathcal{F})$ such that $A \leq 1 \setminus f^{-1}(B)$. Let $C = 1 \setminus A$. Then $C \neq 1$ is a fuzzy δ-closed set such that $f^{-1}(B) = 1 \setminus (1 \setminus f^{-1}(B) \leq 1 \setminus (A) = C$. □

Definition 9.7.6. A fuzzy set A of a fuzzy topological space $X, \mathcal{F}$ is called a ***fuzzy $\boldsymbol{\lambda_g^\delta}$-dense*** (briefly, $F\lambda_g^\delta$-dense) if there exists no fuzzy λ_g^δ-closed set B in $(X, \mathcal{F})$ such that $A < B < 1$. Equivalently, $F\lambda_g^\delta cl(A) = 1$.

Theorem 9.7.7. *Let $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ be a one-one and onto fuzzy function. Then the following are equivalent.*

(i) f is somewhat fuzzy λ_g^δ-continuous;

(ii) If A is a fuzzy closed set in $(Y, \mathcal{G})$ such that $f^{-1}(A) \neq 1$, then there exists a fuzzy λ_g^δ-closed set $B \neq 1$ in $(X, \mathcal{F})$ such that $B \geq f^{-1}(A)$;

(iii) If A is a fuzzy λ_g^δ-dense set in $(X, \mathcal{F})$ then $f(A)$ is a fuzzy λ_g^δ-dense set in $(Y, \mathcal{G})$.

Proof. (i) $\Rightarrow$ (ii) Let A be a fuzzy closed set in $(Y, \mathcal{G})$ such that $f^{-1}(A) \neq 1$. Now, $1 \setminus A \in \mathcal{G}$ and $f^{-1}(1 \setminus A) = 1 \setminus f^{-1}(A) \neq 0$ [as $f^{-1}(A) \neq 1$]. By hypothesis, there exists a fuzzy λ_g^δ-open set C in $(X, \mathcal{F})$ such that $C \leq f^{-1}(1 \setminus A)$. This implies $C \leq 1 - f^{-1}(A)$ and hence $f^{-1}(A) \leq 1 \setminus C$. We observe that $1 \setminus C$ is a fuzzy λ_g^δ-closed set in $(X, \mathcal{F})$. Taking $1 \setminus C = B$ we get, $B \geq f^{-1}(A)$.

(ii) $\Rightarrow$ (iii) Let A be a fuzzy λ_g^δ-dense set in $(X, \mathcal{F})$. Suppose if $f(A)$ not a fuzzy λ_g^δ-dense set in $(Y, \mathcal{G})$. Then there exists a non-zero fuzzy λ_g^δ-closed set B such that $f(A) < B < 1$. Since $B < 1$, $f^{-1}(B) \neq 1$. Therefore by hypothesis, there exists a fuzzy λ_g^δ-closed set $C \neq 1$ in $(X, \mathcal{F})$ such that $C \geq f^{-1}(B)$. Then $C \geq f^{-1}(B) \geq f^{-1}(f(A)) \geq A$. Thus there exists a fuzzy λ_g^δ-closed set C in $(X, \mathcal{F})$ such that $C \geq A$, which contradicts our assumption on A.

(iii) $\Rightarrow$ (i) Let A be a fuzzy open set in $(Y, \mathcal{G})$ and $f^{-1}(A) \neq 0$. Suppose that there exists no λ_g^δ-open set B in $(X, \mathcal{F})$ such that $B \neq 0$ and $B \leq f^{-1}(A)$. That is, $F\lambda_g^\delta int(f^{-1}(A) = 0 \Rightarrow 1 \setminus F\lambda_g^\delta int(f^{-1}(A) = 1 \Rightarrow F\lambda_g^\delta cl(1 \setminus f^{-1}(A)) = 1$. By hypothesis, $f(1 \setminus f^{-1}(A))$ is a fuzzy λ_g^δ-dense set in $(Y, \mathcal{G}) \Rightarrow F\lambda_g^\delta cl(1 \setminus f^{-1}(A)) = 1$ but $f(1 \setminus f^{-1}(A)) = f(f^{-1}(1 \setminus A)) \leq 1 \setminus A < 1$. Here $f(1 \setminus f^{-1}(A)) \leq 1 \setminus A$ which will give that $F\lambda_g^\delta cl(f(1 \setminus f^{-1}(A))) \leq F\lambda_g^\delta cl(1 \setminus A) < F\lambda_g^\delta cl(1) = 1$. Hence we must have $F\lambda_g^\delta int(f^{-1}(A)) \neq 0$. Therefore f is somewhat fuzzy δ-continuous.

□

Theorem 9.7.8. *Let $(X, \mathcal{F})$ and $(Y, \mathcal{G})$ be fuzzy topological spaces and $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ be a somewhat fuzzy δ-continuous, one-one and onto function. If $int_\delta(A) = 0$, for any fuzzy set $A \neq 0$ in $(X, \mathcal{F})$ then $F\lambda_g^\delta int(f(A)) = 0$ in $(Y, \mathcal{G})$.*

Proof. Let $A \neq 0$ be a fuzzy set in $X, \mathcal{F}$ such that $int_\delta(A) = 0$. Then $1 \setminus int_\delta(A) = 1 \Rightarrow cl_\delta(1 \setminus A) = 1$. $1 \setminus A$ is a fuzzy δ-dense set in $(X, \mathcal{F})$. Since f is somewhat fuzzy δ-continuous, $cl_\delta[f(1 \setminus A)] = 1$ [Thangarj, Proposition 4.11]. Since f is one-one, onto we have $f(1 \setminus A) = 1 \setminus f(A)$. This implies $cl_\delta[1 \setminus f(A)] = 1 \Rightarrow int_\delta(f(A)) = 0$. We have, $F\lambda_g^\delta int(f(A)) \leq int_\delta(f(A)) = 0 \Rightarrow F\lambda_g^\delta int(f(A)) = 0$ in $(Y, \mathcal{G})$. □

Theorem 9.7.9. *Let $(X, \mathcal{F})$ and $(Y, \mathcal{G})$ be fuzzy topological spaces and $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ be a somewhat fuzzy λ_g^δ-continuous, one-one and onto function. If $F\lambda_g^\delta int(A) = 0$, for any fuzzy set $A \neq 0$ in $(X, \mathcal{F})$ then $F\lambda_g^\delta int(f(A)) = 0$ in $(Y, \mathcal{G})$.*

Proof. Similar to Theorem 9.7.8 □