CHAPTER – III

Bipolar Spherical Fuzzy MST

3.1 Introduction

Graph theory is used for modelling real world network systems such as: water, construction, pattern designs, transport, electricity, internet, etc. In this chapter, we propose an algorithm for finding minimum spanning tree of an undirected bipolar graph where the edge lengths are represented by bipolar spherical number. To construct the minimum spanning tree of undirected bipolar spherical fuzzy graph, a new algorithm and score function based on matrix approach has been introduced.

3.2 Bipolar Spherical Fuzzy Minimum Spanning Tree Algorithm

In this section, we have defined score function of bipolar spherical fuzzy set and presented bipolar spherical fuzzy minimum spanning tree algorithm.

Definition 3.2.1: Let A be a bipolar spherical fuzzy set, we define a new score function as follows:

$$S(A) = \frac{1}{6}\left\{T^P + 1 - I^P + 1 - F^P + 1 + T^N - I^N - F^N\right\}$$

In the following, we propose Bipolar Spherical Fuzzy Minimum Spanning Tree algorithm [BSFMST]

Step (1): Input bipolar spherical fuzzy adjacency matrix A.

Step (2): Interpret the bipolar spherical fuzzy matrix into score matrix S_{ij} by using score.

Step (3): Find the score matrix $S(A)$ either row-wise or column-wise to find the cost of the corresponding edge e_{ij} in $S(A)$ that is the minimum entries in S_{ij} .

Step (4): Set $S_{ij} = 0$ if the edge e_{ij} of selected S_{ij} construct a cycle with the previous marked elements of the score matrix $S(A)$ else mark S_{ij} .

Step (5): Repeat Step (3) & Step (4) until all (n-1) entries of the matrix of $S(A)$ are either marked to zero or all the non-zero entries are marked.

Step (6): Compute minimum cost spanning tree of the graph G by construct the tree T including only the marked elements from the score matrix $S(A)$.

Step (7): End

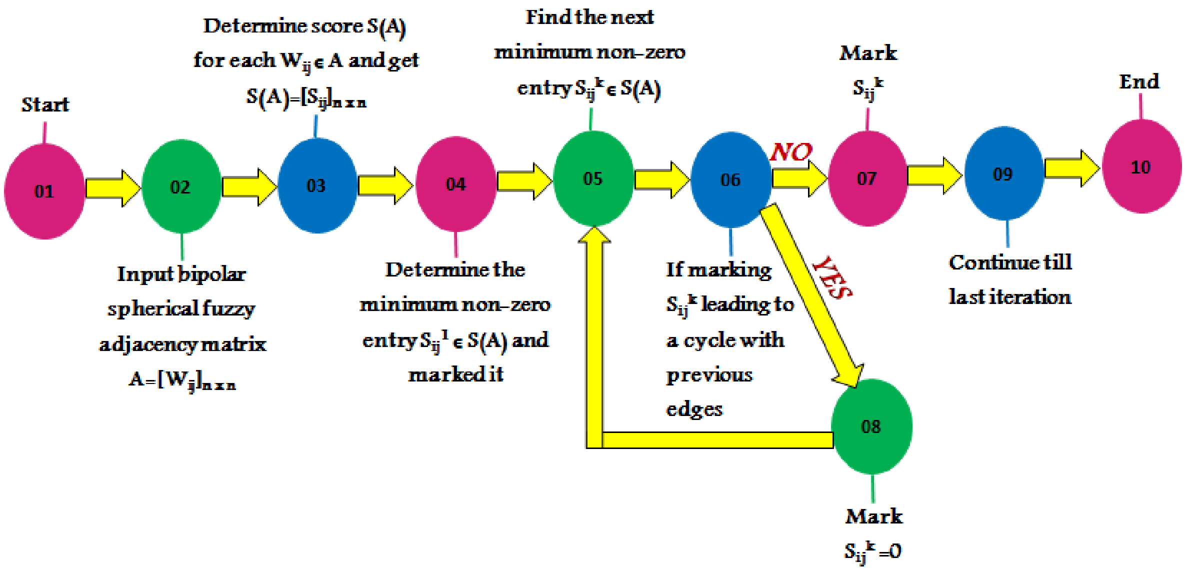

Fig. 3.1 Flow chart for proposed algorithm

3.3 Numerical Example

In this section, we describe a minimum spanning tree problem and discuss it on a graph.

Example 3.3.1

Assume the graph $G=(V,E)$ where V be the vertices and E be the edge of the graph. Here we have 9 vertices and 18 edges. Erection of the minimum cost spanning tree are discussed as follows

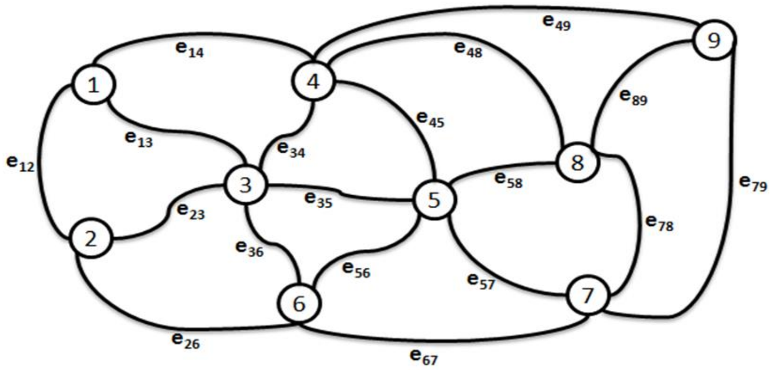

Fig. 3.2: Undirected Graph *G=(V,E)* with 9 vertices and 18 edges

E	Edge length
e_{12}	$\{0.4,0.8,0.3,-0.2,-0.4,-0.5\}$
e_{13}	$\{0.9,0.7,0.8,-0.4,-0.5,-0.8\}$
e_{14}	$\{0.3,0.5,0.5,-0.7,-0.5,-0.2\}$
e_{23}	$\{0.9,0.1,0.5,-0.8,-0.6,-0.3\}$
e_{26}	$\{0.6,0.7,0.5,-0.5,-0.4,-0.3\}$
e_{34}	$\{0.3,0.4,0.2,-0.7,-0.6,-0.3\}$
e_{35}	$\{0.8,0.9,0.6,-0.6,-0.4,-0.3\}$
e_{36}	$\{0.4,0.5,0.1,-0.3,-0.2,-0.4\}$
e_{45}	$\{0.3,0.4,0.7,-0.6,-0.3,-0.6\}$
e_{48}	$\{0.6,0.5,0.3,-0.5,-0.4,-0.8\}$
e_{49}	$\{0.3,0.4,0.2,-0.5,-0.6,-0.8\}$
e_{56}	$\{0.8,0.5,0.2,-0.5,-0.3,-0.8\}$
e_{57}	$\{0.1,0.1,0.8,-0.3,-0.4,-0.5\}$
e_{58}	$\{0.3,0.4,0.6,-0.4,-0.7,-0.6\}$
e_{67}	$\{0.5,0.4,0.3,-0.4,-0.5,-0.9\}$
e_{78}	$\{0.6,0.7,0.8,-0.6,-0.4,-0.2\}$

e_{79}	$\{0.4, 0.1, 0.3, -0.2, -0.5, -0.7\}$
e_{89}	$\{0.1, 0.9, 0.5, -0.5, -0.2, -0.8\}$

The bipolar spherical fuzzy adjacency matrix A is given below

$$S(A) = \begin{pmatrix} 0 & e_{12} & e_{13} & e_{14} & 0 & 0 & 0 & 0 & 0 \\ e_{12} & 0 & e_{23} & 0 & 0 & e_{26} & 0 & 0 & 0 \\ e_{13} & e_{23} & 0 & e_{34} & e_{35} & e_{36} & 0 & 0 & 0 \\ e_{14} & 0 & e_{34} & 0 & e_{45} & 0 & 0 & e_{48} & e_{49} \\ 0 & 0 & e_{35} & e_{45} & 0 & e_{56} & e_{57} & e_{58} & 0 \\ 0 & e_{26} & e_{36} & 0 & e_{56} & 0 & e_{67} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{57} & e_{67} & 0 & e_{78} & e_{79} \\ 0 & 0 & 0 & e_{48} & e_{58} & 0 & e_{78} & 0 & e_{89} \\ 0 & 0 & 0 & e_{49} & 0 & 0 & e_{79} & e_{89} & 0 \end{pmatrix}$$

Thus, the score matrix using the score function

$$\begin{pmatrix} 0 & 0.5 & 0.55 & 0.383 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0.567 & 0 & 0 & 0.433 & 0 & 0 & 0 \\ 0.55 & 0.567 & 0 & 0.483 & 0.4 & 0.517 & 0 & 0 & 0 \\ 0.383 & 0 & 0.483 & 0 & 0.417 & 0 & 0 & 0.583 & 0.6 \\ 0 & 0 & 0.4 & 0.417 & 0 & 0.617 & 0.467 & 0.533 & 0 \\ 0 & 0.433 & 0.517 & 0 & 0.617 & 0 & 0.633 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.467 & 0.633 & 0 & 0.35 & 0.667 \\ 0 & 0 & 0 & 0.583 & 0.533 & 0 & 0.35 & 0 & 0.366 \\ 0 & 0 & 0 & 0.6 & 0 & 0 & 0.667 & 0.366 & 0 \end{pmatrix}$$

Fig. 3.3: Score Matrix

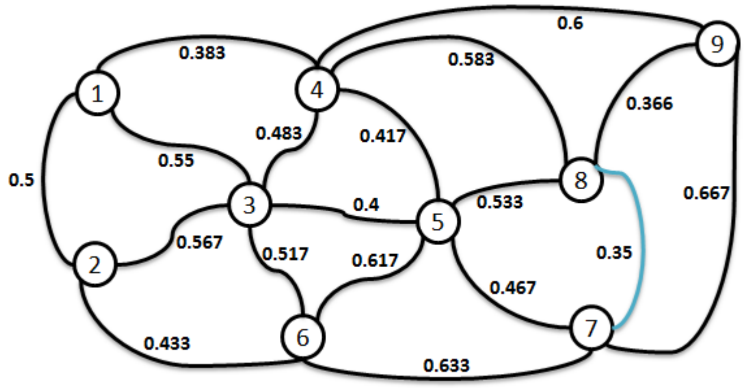

Fig. 3.4: The selected edge (7,8) in G

It is clearly observed that 0.35 selected from Fig. 3.3 is the least value and colored corresponding edge (7,8) is given in Fig. 3.4.

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Fig. 3.5: Score Matrix

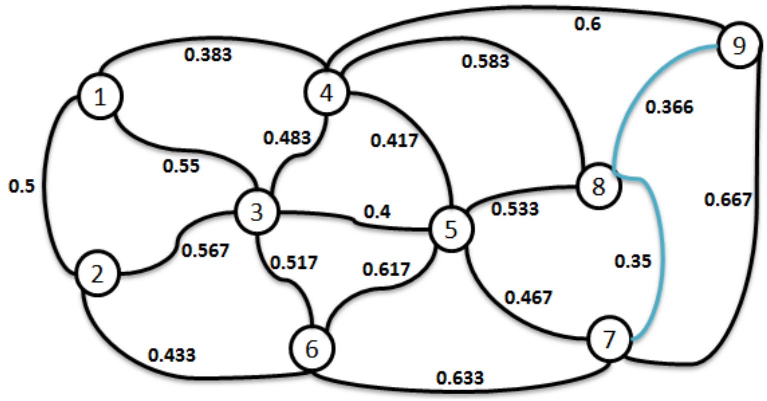

Fig. 3.6: The selected edge (8,9) in G

The next non-zero minimum entry from Fig. 3.5 & Fig. 3.6 is 0.366 is marked and the corresponding edge is (8,9) is highlighted.

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Fig. 3.7: Score Matrix

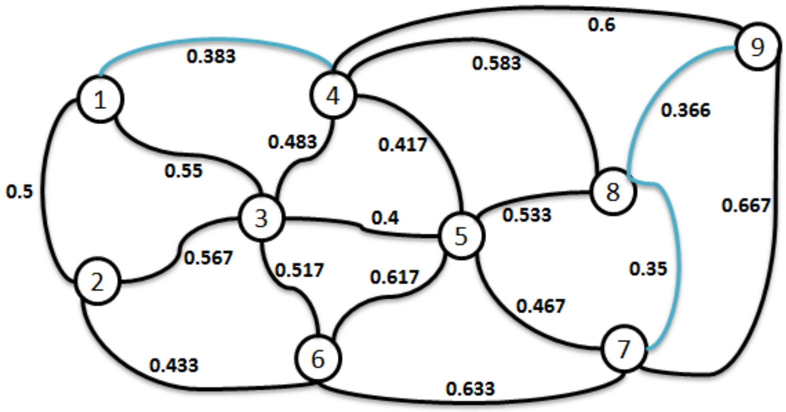

Fig. 3.8: The selected edge (1,4) in G

The next non-zero minimum entry from Fig. 3.7 & Fig. 3.8 is 0.383 is marked and the corresponding edge (1,4) is highlighted.

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Fig. 3.9: Score Matrix

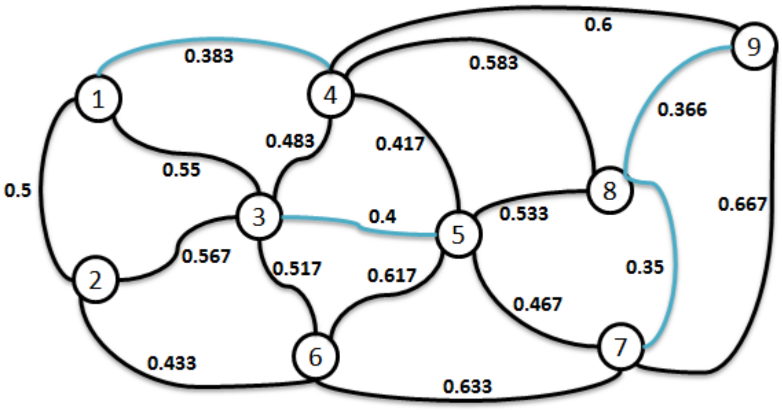

Fig. 3.10: The selected edge (3,5) in G

It is clearly observed from Fig. 3.9 that the next non-zero minimum entry is 0.4 and the corresponding edge (3,5) is highlighted in Fig. 3.10.

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Fig. 3.11: Score Matrix

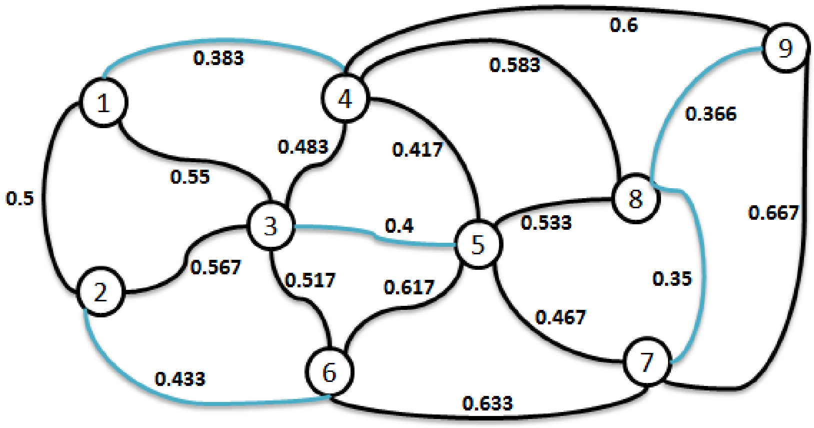

Fig. 3.12: The selected edge (2,6) in G

It is clearly observed from Fig. 3.11 that the next non-zero minimum entry is 0.417 is selected but while drawing edge it produces the cycle. So we reject and select the next least value 0.433 and colored corresponding edge (2,6) in Fig. 3.12.

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Fig. 3.13: Score Matrix

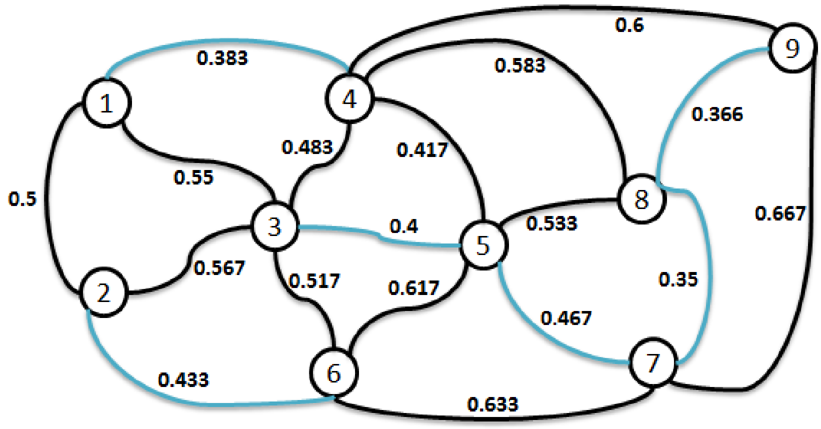

Fig. 3.14: The selected edge (5,7) in G

It is clearly observed from Fig. 3.13 that the next non-zero minimum entry 0.467 is selected and the corresponding edge (5,7) is highlighted in Fig. 3.14.

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Fig. 3.15: Score Matrix

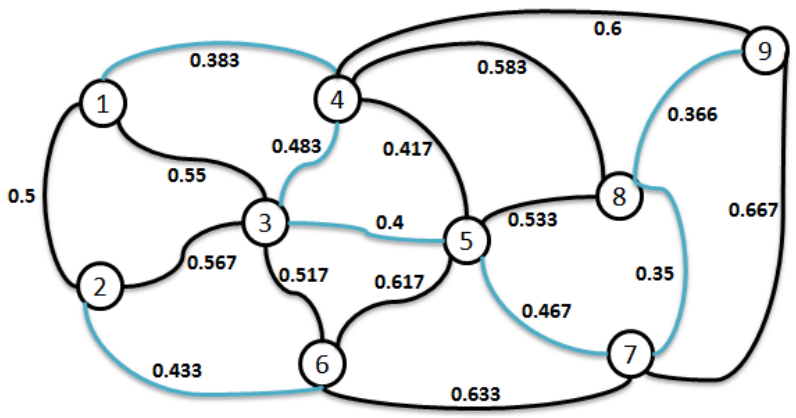

Fig. 3.16: The selected edge (3,4) in G

It is clearly observed from Fig. 3.15 that the next non-zero minimum entry 0.483 is selected and the corresponding edge (3,4) is highlighted in Fig. 3.16.

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Fig. 3.17: Score Matrix

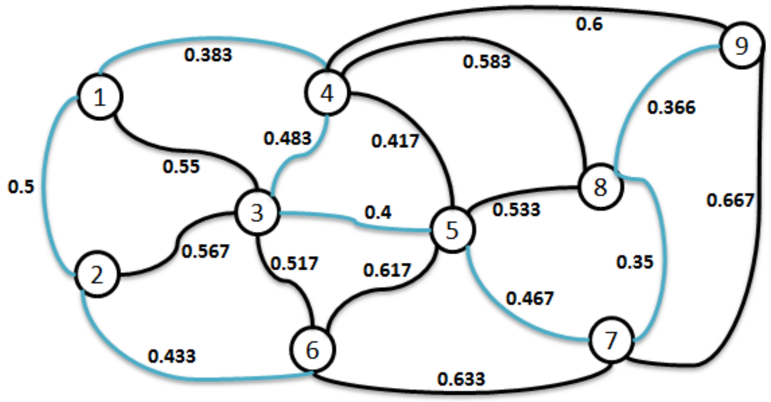

Fig. 3.18: The selected edge (1,2) in G

It is clearly observed from Fig. 3.17 that the next non-zero minimum entry 0.5 is selected and colored corresponding edge (1,2) in Fig. 3.18.

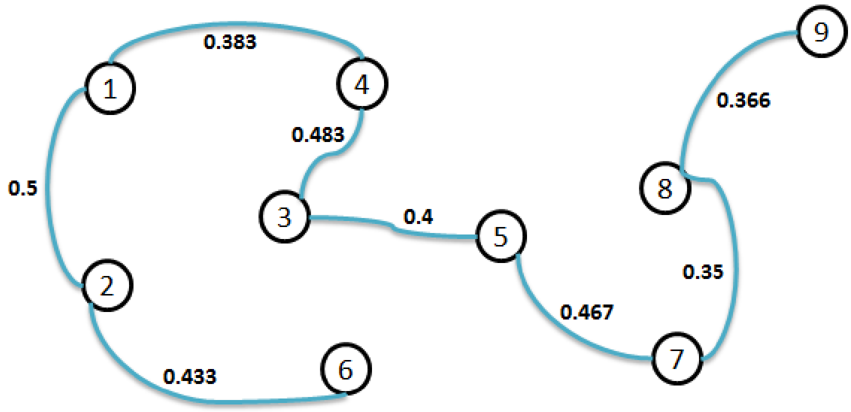

Fig. 3.19: The final path of minimum spanning tree

Using the above steps, the crisp minimum cost spanning tree is 3.382 and the final path of minimum spanning tree is $\{6,2\},\{2,1\},\{1,4\},\{4,3\},\{3,5\},\{5,7\},\{7,8\},\{8,9\}$.