Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – May 2025
II Semester

Class : I P.G. **Time: 3 Hours**
Major : Mathematics **Max. Marks: 100**

23MMAC07 Advanced Algebra II

Course Outcomes:
CO1: Reduce the geometric problem to an algebraic problem.
CO2: Calculate the group of all automorphisms of a given field.
CO3: Find eigen values and eigen vectors of linear transformations.
CO4: Test orthogonality of given vectors.
CO5: Construct finite fields corresponding to a prime number.

Part A **10 x 1 = 10**
Choose the Correct Answer

1. G(K,F) is a subgroup of the group of all ____ of K CO1K1
   a. subfield b. finite extension c. automorphism d. normal extension

2. A subgroup of a solvable group is CO1K1
   a. subgroup b. a field c. normal subgroup d. solvable

3. If $T \in A(v)$ then $\lambda \in F$ is called a ___ of T if $\lambda - T$ is singular. CO2K1
   a. eigen value b. minimal polynomial c. eigen vector d. linearly independent

4. $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}^2 =$ ______
   a. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ b. $\begin{pmatrix} 0 & 0 \\ -2 & 2 \end{pmatrix}$ c. $\begin{pmatrix} 2 & 0 \\ 2 & 2 \end{pmatrix}$ d. $\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ CO2K1

5. The integers $n_1, n_2, \ldots\ldots, n_r$ are called the ____ of T. CO3K1
   a. dimension b. nilpotent c. invariants d. cyclic

6. If $T \in A(v)$ is nilpotent then k is called the index of nilpotence of T if $T^k = 0$ but CO3K1
   a. $T^{k-1} \leq 0$ b. $T^{k-1} \geq 0$ c. $T^{k-1} = 0$ d. $T^{k-1} \neq 0$

7. If F has characteristic 0. Then $A = \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix}$ satisfies CO4K1
   a. $A^p = 1$ b. $A^p = 0$ c. $A^p = \alpha$ d. $A^p = n$

8. If $\dim_F(v) = n$, then the characteristic polynomial of T, $P_r(x)$, is the product of its CO4K1
   a. minimal polynomial b. invariant c. elementary divisors d. dimension

9. Any two finite fields having the same number of elements are CO5K1
   a. isomorphic b. finite c. finite division d. primitive root

10. Any generator of cyclic group under multiplication is called a ____ of p. CO5K1
    a. Eulerian criterion b. automorphism c. finite division ring d. primitive root

Part B **5 x 6 = 30**
Answer ALL questions
Each answer should not exceed 400 words or two pages

11.a. If K is a finite extension of F, then prove that $G(K,F)$ is a finite group and its order, $o\big(G(K,F)\big)$ satisfies $o\big(G(K,F)\big) \leq [K:F]$. CO1K3

(or)

11.b. Prove that G is solvable if and only if $G^{(k)} = (e)$ for some integer k. CO1K3

12.a. If V is finite-dimensional over F then prove that for $S, T \in A(V)$.
(i) $r(ST) \leq r(T)$ (ii) $r(TS) \leq r(T)$ (iii) $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$. CO2K3

(or)

12.b. Prove that if V is n- dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, \dots, v_n$ and the matrix $m_2(T)$ in the basis $w_1, \dots\dots, w_n$ of V over F, then there is an element $C \in F_n$ such that$m_2(T) = Cm_1(T)C^{-1}$. In fact, if S is the linear transformation of V defined by $v_i S = w_i$ for $i = 1,2, \dots., n$ then C can be chosen to be $m_1(S)$. CO2K3

13.a. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F. CO3K2

(or)

13.b. Prove that two nilpotent linear transformations are similar if and only if they have the same invariants. CO3K2

14.a. Suppose that $V = V_1 \oplus V_2$, where V_1 and V_2 are subspaces of V invariant under T.
Let T_1 and T_2 be the linear transformations induced by T on V_1andV_2, respectively. If the minimal polynomial of T_1 over$F$ is $p_1(x)$ while that of T_2 is $p_2(x)$, then prove that the minimal polynomial for T over F is the least common multiple of $p_1(x)$ and $p_2(x)$. CO4K4

(or)

14.b. Suppose the two matrices A, B in F_n are similar in K_n where K is an extension of F. Then prove that A and B are already similar in F_n. CO4K4

15.a. Prove that for every prime number p and every positive integer m there exists a field having p^melements. CO5K4

(or)

15.b. State and Prove Jacobson theorem. CO5K4

Part C 5 x 12 = 60

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. Let K be the splitting field of $f(x)$ in $F[x]$ and let $p(x)$ be an irreducible factor of $f(x)$in $F[x]$. If the roots of $p(x)$ are $\alpha_1, \dots\dots, \alpha_r$, then prove that for each i there exists an automorphism σ_i in $G(K, F)$ such that $\sigma_i(\alpha_1) = \alpha_i$. CO1K3

(or)

16.b. If $p(x) \in F[x]$ is solvable by radicals over F, then prove that the Galois group over F of $p(x)$ is a solvable group. CO1K3

17.a. If A is an algebra, with unit element, over F, then prove that A is isomorphic to a Sub algebra of $A(V)$ for some vector space V over F. CO2K4

(or)

17.b. If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that λ is a root of the minimal polynomial of T. In particular, T only has a finite number of characteristic roots in F. CO2K4

18.a. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular. CO3K4

(or)

18.b. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$. CO3K4

19.a. Prove that for each $i = 1,2, \ldots ., k, V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus, \ldots . \oplus V_k$. The minimal polynomial of T_i is $q_i(x)^{l_i}$. CO4K4

(or)

19.b. Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors. CO4K4

20.a. Let G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by at most n elements of G, for every integer n. Then prove that G is a cyclic group. CO5K5

(or)

20.b. State and prove Wedder burn theorem. CO5K5

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