

Avinashilingam Institute for Home Science and Higher Education For Women
(Deemed to be University), Coimbatore – 641 043
Semester Examination-November 2018
I Semester

Class : I UG
 Major : Mathematics

Time : 3 Hours
 Max. Marks: 100

18BMAC03 – Differential Calculus
Part – A

Circle the correct answer

(10 x 1 = 10)

- 1) The n^{th} derivative of e^{7x} .
 a) $7 e^{7x}$ b) e^{7x} c) $7^n e^{7x}$ d) $n e^{7x}$
- 2) If $y = (2x - 3)^n$, then $y_n =$ _____
 a) $n2^n$ b) $n!2^n$ c) $n3^n$ d) $n!3^n$
- 3) If z is a function of u and v where u and v are functions of x and y , then $\frac{\partial z}{\partial x}$ is-
 a) $\frac{\partial z}{\partial u} \frac{\partial u}{\partial x}$ b) $\frac{\partial z}{\partial u} \frac{\partial u}{\partial y}$ c) $\frac{\partial z}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial x}$ d) $\frac{\partial z}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial y}$
- 4) If $x = e^u \cos v$, then $\frac{\partial^2 x}{\partial v^2} =$ _____
 a) $-e^u \cos v$ b) $e^u \cos v$ c) $e^u \sin v$ d) $-e^u \sin v$
- 5) The condition for $\frac{\partial^2 f}{\partial x^2}$ at (a, b) for $f(x, y)$ to attain a minimum at (a, b) is
 a) $\frac{\partial^2 f}{\partial x^2} < 0$ b) $\frac{\partial^2 f}{\partial x^2} > 0$ c) $\frac{\partial^2 f}{\partial x^2} = 0$ d) None
- 6) The condition for $\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$ at (a, b) for saddle point is
 a) $\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0$ b) $\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 < 0$
 c) $\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = 0$ d) None
- 7) The envelope of the family of straight lines $y = mx + \frac{a}{m}$ where m is the parameter is
 a) $y = 4ax$ b) $y^2 = 4a$ c) $y^2 = 4ax^2$ d) $y^2 = 4ax$
- 8) Curvature of a circle of radius r is
 a) r^2 b) $1/r^2$ c) r d) $1/r$
- 9) The radius of curvature when the curve is given in polar co-ordinates is
 a) $\frac{\left\{r^2 + \left(\frac{dr}{d\theta}\right)^2\right\}^{3/2}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 - r \frac{d^2 r}{d\theta^2}}$ b) $\frac{\left\{r^2 - \left(\frac{dr}{d\theta}\right)^2\right\}^{3/2}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 - r \frac{d^2 r}{d\theta^2}}$
 c) $\frac{\left\{r^2 + \left(\frac{dr}{d\theta}\right)^2\right\}^{3/2}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 + r \frac{d^2 r}{d\theta^2}}$ d) $\frac{\left\{r^2 - \left(\frac{dr}{d\theta}\right)^2\right\}^{3/2}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 + r \frac{d^2 r}{d\theta^2}}$
- 10) $p - r$ equation of a curve is given by
 a) $\frac{1}{p^2} = \frac{1}{r^2} - \frac{1}{r^4} \left(\frac{dr}{d\theta}\right)^2$ b) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta}\right)^2$
 c) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta}\right)^2$ d) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta}\right)^2$

Part – B**(5 x 6 = 30)****Answer all questions:**

11. a) Find y_n where $y = \frac{3}{(x+1)(2x-1)}$
or
 11. b) Find y_n where $y = \cos x \cos 2x \cos 3x$
12. a) If z is a function of x and y and if $x = e^u \sin v$, $y = e^u \cos v$, then prove that

$$\frac{\partial z}{\partial u} = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$$

or
 12. b) If u is a function of x and y and if $x = r \cos \theta$, $y = r \sin \theta$, then prove that

$$\frac{\partial u}{\partial \theta} = -y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y}$$
13. a) Find the maxima and minima of $f(x, y) = x^2 - xy + y^2 - 2x + y$
or
 13. b) Find the maxima and minima of $f(x, y) = 3x^2 + y^2 + 12x + 36$
14. a) Find the envelope of the family of circles $(x - a)^2 + y^2 = 2a$ where 'a' is the parameter
or
 14. b) What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $(1, 1)$?
15. a) Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ is $\frac{a^n r^{-n+1}}{n+1}$.
or
 15. b) Prove that the p - r equation of the cardioids $r = a(1 - \cos \theta)$ is $p^2 = \frac{r^3}{2a}$

Part – C**(5 x 12 = 60)****Answer all questions**

16. a) Prove that if $y = \sin(m \sin^{-1} x)$, then $(1 - x^2)y_2 - xy_1 + m^2 y = 0$
or
 16. b) If $xy = ax^2 + \frac{b}{x}$, then prove that $x^2 \frac{d^2 y}{dx^2} + 2 \left(x \frac{dy}{dx} - y \right) = 0$
17. a) If $z = f(x, y)$ and if $x = u^2 - v^2$, $y = 2uv$, then prove that

$$u \frac{\partial z}{\partial u} - v \frac{\partial z}{\partial v} = 2(u^2 + v^2) \frac{\partial z}{\partial x}$$

or
 17. b) If z is a function of x and y and $x = e^u + e^v$, $y = e^{-u} - e^v$, then prove that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$
18. a) Find the maxima and minima of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$
or
 18. b) Find the extreme values of $f(x, y) = x^3 y^2 (6 - x - y)$
19. a) Find the envelope of the family of straight lines $y + tx = 2at + at^3$, where t is the parameter.
or
 19. b) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin \theta)$
 $y = a(1 - \cos \theta)$ is $4a \cos \frac{\theta}{2}$
20. a) Prove that the radius of curvature of the curve $r^2 = a^2 \sin 2\theta$ is $\rho = \frac{a^2}{3r}$
or
 20. b) Show that the p - r equation of the parabola $\frac{2a}{r} = 1 - \cos \theta$ is $p^2 = ar$
