CHAPTER I

NEUTROSOPHIC SOFT SETS

Definition:1.1

Let U be an universe of discourse then the **neutrosophic set** A is an object having the form $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in U\}$ where the functions $T_A, I_A, F_A : U \to \left]^{-}0,1^{+}\right[$ define respectively the degree of truth-membership ,the degree of indeterminacy membership and the degree of falsity-membership of the elements to the set A with the condition

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$$

The neutrosophic set takes the value from real standard (or) non-standard subsets of $\left]^{-}0,1^{+}\right[$so instead of $\left]^{-}0,1^{+}\right[$ the interval $[0,1]$ to use for technical applications .

The value of neutrosophic sets are taken from the non–standard unit interval $\left]^{-}0,1^{+}\right[$the non-standard finite number $1^{+} = 1 + \delta$ where '1' is the standard part and δ is its non-standard part and $^{-}0 = 0 - \delta$ where '0' is the standard part and δ is its non-standard part.

Example:1.2

Take the universe of discourse $U = \{x_1, x_2, x_3\}$ where x_1 characterizes the capability, x_2 characterizes the trust worthiness and x_3

characterizes the prices of the objects. It may further assumed that the values of x_1, x_2 and x_3 are in [0, 1].The degree of truth membership, the degree of indeterminacy membership and degree of falsity membership to explain the characteristics of the objects. Suppose A is a neutrosopthic set of x such that,

$$A = \{< x_1, 0.4, 0.5, 0.3 >, < x_2, 0.7, 0.2, 0.4 >, < x_3, 0.8, 0.3, 0.4 >\},$$

Where the degree of truth membership of capability is 0.4, the degree of indeterminacy membership of capability is 0.5 and the degree of falsity membership of capability is 0.3 etc.

Definition:1.3

Let U be an initial universe set and E be a set of parameters .Let $P(U)$ denotes the power set of U. Consider a non-empty set A, $A \subset E$. A pair (F,A) is called **soft set** over U, where F is a mapping given by $F : A \to P(U)$. It is also denoted by F_A.

Definition:1.4

Let U be an initial universe set and $A \subset E$ be a set of parameters. Let $N(U)$ denotes the set of all neutrosophic sets of U. The pair (F,A) is called **neutrosophic soft set** (in short NSS) over U where F is a mapping given by

$$F : A \to N(U)$$

and is denoted as $\widetilde{F}_A$

Example:1.5

Let U be the set of houses under consideration and E be the set of parameters. Let $E = \{e_1 - \text{beautiful}, e_2 - \text{wooden}, \; e_3 - \text{costly}, \; e_4 -$ moderate, $e_5 -$ cheap, $e_6 -$ green surroundings$\}$. Suppose that there are four houses in the universe U given by, $U = \{h_1, h_2, h_3\}$ and set of parameters

$A=\{e_1,e_2,e_3,e_4\}$. The NSS $\widetilde{F}_A$ describes the “attractiveness of houses”

$$F(\text{beautiful})=\{<h_1,0.5,0.6,0.3>,<h_2,0.4,0.7,0.6>,<h_3,0.6,0.2,0.3>\}$$

$$F(\text{wooden})=\{<h_1,0.6,0.3,0.5>,<h_2,0.7,0.4,0.3>,<h_3,0.8,0.1,0.2>\}$$

$$F(\text{costly})=\{<h_1,0.7,0.4,0.3>,<h_2,0.6,0.7,0.2>,<h_3,0.7,0.2,0.5>\}$$

$$F(\text{moderate})=\{<h_1,0.8,0.6,0.4>,<h_2,0.7,0.9,0.6>,<h_3,0.7,0.6,0.4>\}$$

Definition:1.6

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic soft sets over the common universe U. $\widetilde{F}_A$ is said to be **neutrosophic soft subset** of $\widetilde{G}_B$ if and only if,

1. $A \subset B$
2. $\widetilde{F}(e)$ is a neutrosophic subset of $\widetilde{G}(e)$

or $T_{\widetilde{F}(e)}(x) \le T_{\widetilde{G}(e)}(x), I_{\widetilde{F}(e)}(x) \le I_{\widetilde{G}(e)}(x), F_{\widetilde{F}(e)}(x) \ge F_{\widetilde{G}(e)}(x),\ \forall e \in A, \forall x \in U$.

This relationship is denoted by $\widetilde{F}_A \subseteq \widetilde{G}_B$.

$\widetilde{F}_A$ is said to be **neutrosophic soft super set** of $\widetilde{G}_B$ if $\widetilde{G}_B$ is a neutrosophic soft subset of $\widetilde{F}_A$. This relationship is denoted by $\widetilde{F}_A \supseteq \widetilde{G}_B$.

Definition:1.7

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two neutrosophic soft sets over the common universe U are said to be **neutrosophic soft equal** if $\widetilde{F}_A$ is neutrosophic soft sub set of $\widetilde{G}_B$ and $\widetilde{G}_B$ is a neutrosophic soft subset of $\widetilde{F}_A$. This relationship is denoted by $\widetilde{F}_A = \widetilde{G}_B$

Definition:1.8

Let $E=\{e_1,e_2,\ldots\ldots,e_n\}$ be a set of parameters. The **NOT set** of E is denoted by $\neg E$ and is $\neg E=\{\neg e_1,\neg e_2,\ldots,\neg e_n\}$ defined by where $\neg e_i =$ not $e_i, \forall i$.

Example:1.9

In example 1.5, $\neg A=\{$not beautiful, not wooden, not costly, not moderate$\}$

Definition:1.10

The **complement of neutrosophic soft set** $\widetilde{F}_A$ is denoted by $\widetilde{F}_A^C$ and is defined by $\widetilde{F}_A^C = (\widetilde{F}^C, \neg A)$ where $\widetilde{F}_A^C$ is a mapping given by $\widetilde{F}_A^C : \neg A \to N(U)$ and

$\widetilde{F}_A^C(e) =$ neutrosophic soft complement with

$$T_{\widetilde{F}^C(e)}(x) = F_{\widetilde{F}(e)}(x),\ I_{\widetilde{F}^C(e)}(x) = I_{\widetilde{F}(e)}(x) \text{ and}$$

$$F_{\widetilde{F}^C(e)}(x) = T_{\widetilde{F}(e)}(x), \forall x \in U, \forall e \in \neg A$$

Example:1.11

In example 1.5, the neutrosophic soft complement $\widetilde{F}_A^C$ describes the "not attractiveness of the houses",

$F(\text{not beautiful}) = \{< h_1, 0.3, 0.6, 0.5 >, < h_2, 0.6, 0.7, 0.4 >, < h_3, 0.3, 0.2, 0.6 >\}$

$F(\text{not wooden}) = \{< h_1, 0.5, 0.3, 0.6 >, < h_2, 0.3, 0.4, 0.7 >, < h_3, 0.2, 0.1, 0.8 >\}$

$F(\text{not costly}) = \{< h_1, 0.3, 0.4, 0.7 >, < h_2, 0.2, 0.7, 0.6 >, < h_3, 0.5, 0.2, 0.7 >\}$

$F(\text{not moderate}) = \{< h_1, 0.4, 0.6, 0.8 >, < h_2, 0.6, 0.9, 0.7 >, < h_3, 0.4, 0.6, 0.7 >\}$

Definition:1.12

A neutrosophic soft set $\widetilde{F}_A$ over U is said to be **empty neutrosophic soft set** with respect to the parameter A if $T_{\widetilde{F}(e)}(x) = 0, I_{\widetilde{F}(e)}(x) = 0, F_{\widetilde{F}(e)}(x) = 0 \forall e \in A,\ \forall x \in U$. It is denoted by $\widetilde{0}_A$.

$$\therefore T_{\widetilde{0}(e)}(x) = 0, I_{\widetilde{0}(e)}(x) = 0, F_{\widetilde{0}(e)}(x) = 0$$

Example:1.13

Let $U = \{h_1, h_2, h_3, h_4\}$ be the set of four houses and A={beautiful, wooden} be the set of parameters that characterizes the houses. Consider the NSS $\widetilde{F}_A$

F(beautiful)=$\{< h_1, 0,0,0 >, < h_2, 0,0,0 >, < h_3, 0,0,0 >, < h_4, 0,0,0 >\}$

F(wooden)=$\{< h_1, 0,0,0 >, < h_2, 0,0,0 >, < h_3, 0,0,0 >, < h_4, 0,0,0 >\}$.

Here $\widetilde{F}_A$ is null neutrosophic soft set.

Definition:1.14

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two NSSs over the common universe U. Then **neutrosophic soft union** of $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by '$\widetilde{F}_A \cup \widetilde{G}_B$' is defined by $\widetilde{F}_A \cup \widetilde{G}_B = \widetilde{K}_D$, where $D = A \cup B$ and the truth membership, indeterminacy membership and falsity membership of $\widetilde{K}_D$ are as follows:

$$T_{\widetilde{K}(e)}(x) = \max(T_{\widetilde{F}(e)}(x), T_{\widetilde{G}(e)}(x)), I_{\widetilde{K}(e)}(x) = \frac{(I_{\widetilde{F}(e)}(x) + I_{\widetilde{G}(e)}(x))}{2}$$

$$F_{\widetilde{K}(e)}(x) = \min(F_{\widetilde{F}(e)}(x), F_{\widetilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U$$

Definition:1.15

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two NSSs over the common universe U. Then **neutrosophic soft intersection** of $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by '$\widetilde{F}_A \cap \widetilde{G}_B$'is defined by $\widetilde{F}_A \cap \widetilde{G}_B = \widetilde{K}_D$, where $D = A \cap B$ and the truth membership, indeterminacy membership and falsity membership of $\widetilde{K}_D$ are as follows:

$$T_{\widetilde{K}(e)}(x) = \min(T_{\widetilde{F}(e)}(x), T_{\widetilde{G}(e)}(x)), I_{\widetilde{K}(e)}(x) = \frac{(I_{\widetilde{F}(e)}(x) + I_{\widetilde{G}(e)}(x))}{2}$$

$$F_{\widetilde{K}(e)}(x) = \max(F_{\widetilde{F}(e)}(x), F_{\widetilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U.$$

Theorem:1.16

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two NSSs over the common universe U. Then

1) $\widetilde{F}_A \cup \widetilde{F}_A = \widetilde{F}_A$ and $\widetilde{F}_A \cap \widetilde{F}_A = \widetilde{F}_A$

2) $\widetilde{F}_A \cup \widetilde{G}_B = \widetilde{G}_B \cup \widetilde{F}_A$ and $\widetilde{F}_A \cap \widetilde{G}_B = \widetilde{G}_B \cap \widetilde{F}_A$

3) $\widetilde{F}_A \cup \widetilde{0}_A = \widetilde{F}_A$ and $\widetilde{F}_A \cap \widetilde{0}_A = \widetilde{0}_A$

4) $(\widetilde{F}_A^C)^C = \widetilde{F}_A$

Theorem:1.17

Let $\widetilde{F}_A$, $\widetilde{G}_B$ and $\widetilde{K}_D$ be three NSSs over the common universe U. Then

1) $\widetilde{F}_A \cup (\widetilde{G}_B \cup \widetilde{K}_D) = (\widetilde{F}_A \cup \widetilde{G}_B) \cup \widetilde{K}_D$

2) $\widetilde{F}_A \cap (\widetilde{G}_B \cap \widetilde{K}_D) = (\widetilde{F}_A \cap \widetilde{G}_B) \cap \widetilde{K}_D$

3) $\widetilde{F}_A \cup (\widetilde{G}_B \cap \widetilde{K}_D) = (\widetilde{F}_A \cup \widetilde{G}_B) \cap (\widetilde{F}_A \cup \widetilde{K}_D)$

4) $\widetilde{F}_A \cap (\widetilde{G}_B \cup \widetilde{K}_D) = (\widetilde{F}_A \cap \widetilde{G}_B) \cup (\widetilde{F}_A \cap \widetilde{K}_D)$

Definition:1.18

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two NSSs over the common universe *U*. Then **'AND'** operation on $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by '$\widetilde{F}_A$ AND $\widetilde{G}_B$' is defined as $\widetilde{F}_A \wedge \widetilde{G}_B$ where the truth membership, indeterminacy membership and falsity membership of $(\widetilde{K}, A \times B)$ are as follows:

$$T_{\widetilde{K}(\alpha,\beta)}(x) = \min(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)),\ I_{\widetilde{K}(\alpha,\beta)}(x) = \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}$$

$$F_{\widetilde{K}(\alpha,\beta)}(x) = \max(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)), \forall \alpha \in A, \forall \beta \in B, \forall x \in U.$$

Definition:1.19

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two NSSs over the common universe *U*. Then **'OR'** operation on $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by '$\widetilde{F}_A$ OR $\widetilde{G}_B$' is defined as $\widetilde{F}_A \vee \widetilde{G}_B = (\widetilde{H}, A \times B)$, where the truth membership, indeterminacy membership and falsity membership of $(\widetilde{H}, A \times B)$ are as follows:

$$T_{\widetilde{H}(\alpha,\beta)}(x) = \max(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)),\ I_{\widetilde{H}(\alpha,\beta)}(x) = \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}$$

$$F_{\widetilde{H}(\alpha,\beta)}(x) = \min(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)), \forall \alpha \in A, \forall \beta \in B, \forall x \in U.$$

Theorem:1.20

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two NSS's over the common universe *U*. Then

1) $(\widetilde{F}_A \vee \widetilde{G}_B)^C = \widetilde{F}_A^C \wedge \widetilde{G}_B^C$

2) $(\widetilde{F}_A \wedge \widetilde{G}_B)^C = \widetilde{F}_A^C \vee \widetilde{G}_B^C$

Proof:

Let $\widetilde{F}_A = \{< x, T_{\widetilde{F}_A}(x), I_{\widetilde{F}_A}(x), F_{\widetilde{F}_A}(x) > / x \in U\}$ and $\widetilde{G}_B = \{< x, T_{\widetilde{G}_B}(x), I_{\widetilde{G}_B}(x), F_{\widetilde{G}_B}(x) > / x \in U\}$ be two NSSs over the common universe U. Also let $(\widetilde{H}, A \times B) = \widetilde{F}_A \vee \widetilde{G}_B$ where,

$$\widetilde{H}(\alpha,\beta) = \{< x, \max(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)), \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}, \min(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)) >, \forall x \in U\}$$

$$(\widetilde{F}_A \vee \widetilde{G}_B)^C = (\widetilde{H}, A \times B)^C = \{< x, \min(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)), \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2},$$

$$\max(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)) >, \forall x \in U\} \rightarrow \text{(i)}$$

$$\widetilde{F}_A^C \wedge \widetilde{G}_B^C = \{< x, \min(F_{\widetilde{F}^C(\alpha)}(x), F_{\widetilde{G}^C(\beta)}(x)), \frac{(I_{\widetilde{F}^C(\alpha)}(x) + I_{\widetilde{G}^C(\beta)}(x))}{2}, \max(T_{\widetilde{F}^C(\alpha)}(x), T_{\widetilde{G}^C(\beta)}(x)) >, \forall x \in U\}$$

$$= \{< x, \max(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)), \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}, \min(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)) >, \forall x \in U\}^C$$

$$= \{< x, \min(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)), \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}, \max(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)) >, \forall x \in U\}$$

$\rightarrow$(ii)

from (i) and (ii), $(\widetilde{F}_A \vee \widetilde{G}_B)^C = \widetilde{F}_A^C \wedge \widetilde{G}_B^C$

2) Let $(\widetilde{K}, A \times B) = \widetilde{F}_A \wedge \widetilde{G}_B$ where,

$$\widetilde{K}(\alpha,\beta) = \{< x, \min(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)), \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}, \max(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)) >, \forall x \in U\}$$

$$(\widetilde{F}_A \wedge \widetilde{G}_B)^C = (\widetilde{K}, A \times B)^C = \{< x, \max(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)), \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}, \min(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)) >, \forall x \in U\}$$

$$\rightarrow \text{(iii)}$$

$$\widetilde{F}_A^C \vee \widetilde{G}_B^C = \{< x, \max(F_{\widetilde{F}^C(\alpha)}(x), F_{\widetilde{G}^C(\beta)}(x)), \frac{(I_{\widetilde{F}^C(\alpha)}(x) + I_{\widetilde{G}^C(\beta)}(x))}{2}, \min(T_{\widetilde{F}^C(\alpha)}(x), T_{\widetilde{G}^C(\beta)}(x)) >, \forall x \in U\}$$

$$= \{< x, \min(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)), \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}, \max(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)) >, \forall x \in U\}^C$$

$$= \{< x, \max(F_{\widetilde{F}(\alpha)}(x), F_{\widetilde{G}(\beta)}(x)), \frac{(I_{\widetilde{F}(\alpha)}(x) + I_{\widetilde{G}(\beta)}(x))}{2}, \min(T_{\widetilde{F}(\alpha)}(x), T_{\widetilde{G}(\beta)}(x)) >, \forall x \in U\}$$

$\rightarrow$(iv)

from (iii) and (iv), $(\widetilde{F}_A \wedge \widetilde{G}_B)^C = \widetilde{F}_A^C \vee \widetilde{G}_B^C$

Definition:1.21

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two NSSs over the common universe *U*. Then the **Cartesian product** of $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by ‘$\widetilde{F}_A \hat{\times} \widetilde{G}_B = \widetilde{H}_D$’is defined by

$\widetilde{H}_D = \{((\alpha, \beta), f_{\widetilde{H}_D}(\alpha, \beta)) : (\alpha, \beta) \in A \times B\}$, where the truth membership, indeterminacy membership and falsity membership of $\widetilde{H}_D$ are as follows:

$$T_{f_{\widetilde{H}_D(\alpha,\beta)}}(x) = \min(T_{f_{\widetilde{F}_A(\alpha)}}(x), T_{f_{\widetilde{G}_B(\beta)}}(x)), I_{f_{\widetilde{H}_D(\alpha,\beta)}}(x) = \frac{(I_{f_{\widetilde{F}_A(\alpha)}}(x), I_{f_{\widetilde{G}_B(\beta)}}(x))}{2}$$

$$F_{f_{\widetilde{H}_D(\alpha,\beta)}}(x) = \max(F_{f_{\widetilde{F}_A(\alpha)}}(x), F_{f_{\widetilde{G}_B(\beta)}}(x)) \forall \alpha, \beta \in A \times B, \forall x \in U.$$

Example:1.22

Let $U = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_1, e_2, e_3\}$ and $B = \{e_3, e_6\}$ be two subsets of *E*. Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two NSSs over the common universe *U*. Suppose that

$$\widetilde{F}_A = (e_1, \{< h_1, (0.7, 0.6, 0.7) >, < h_2, (0.4, 0.2, 0.8) >, < h_3, (0.9, 0.1, 0.5) >, < h_4, (0.4, 0.7, 0.7) >\}),$$

$$= (e_2, \{< h_1, (0.5, 0.7, 0.8) >, < h_2, (0.5, 0.9, 0.3) >, < h_3, (0.5, 0.6, 0.8) >, < h_4, (0.5, 0.8, 0.5) >\}),$$

$$= (e_3, \{< h_1, (0.8,0.6,0.9) >, < h_2, (0.5,0.9,0.9) >, < h_3, (0.7,0.5,0.4) >, < h_4, (0.3,0.5,0.6) >\}).$$

$$\widetilde{G}_B = (e_3, \{< h_1, (0.8,0.9,0.6) >, < h_2, (0.7,0.8,0.8) >, < h_3, (0.5,0.6,0.4) >, < h_4, (0.3,0.3,0.6) >\}),$$

$$= (e_6, \{< h_1, (0.8,0.4,0.6) >, < h_2, (0.6,0.2,0.8) >, < h_3, (0.6,0.4,0.6) >, < h_4, (0.5,0.7,0.4) >\}).$$

The Cartesian product of $\widetilde{F}_A$ and $\widetilde{G}_B$ is obtained as follows

$$\begin{aligned}
\widetilde{F}_A \hat{\times} \widetilde{G}_B = \{&((e_1, e_3), \{< h_1, (0.7,0.75,0.7) >, < h_2, (0.4,0.5,0.8) >, < h_3, (0.5,0.35,0.5) >, < h_4, (0.3,0.5,0.7) >\} \\
&((e_1, e_6), \{< h_1, (0.7,0.5,0.7) >, < h_2, (0.4,0.2,0.8) >, < h_3, (0.6,0.25,0.6) >, < h_4, (0.4,0.7,0.7) >\}), \\
&((e_2, e_3), \{< h_1, (0.5,0.8,0.8) >, < h_2, (0.5,0.85,0.8) >, < h_3, (0.5,0.6,0.8) >, < h_4, (0.5,0.55,0.6) >\}), \\
&((e_2, e_6), \{< h_1, (0.5,0.55,0.8) >, < h_2, (0.5,0.55,0.8) >, < h_3, (0.5,0.5,0.8) >, < h_4, (0.5,0.75,0.5) >\}), \\
&((e_3, e_3), \{< h_1, (0.8,0.75,0.9) >, < h_2, (0.5,0.85,0.9) >, < h_3, (0.5,0.55,0.4) >, < h_4, (0.3,0.6,0.4) >\}), \\
&((e_3, e_6), \{< h_1, (0.8,0.5,0.9) >, < h_2, (0.5,0.55,0.9) >, < h_3, (0.6,0.45,0.6) >, < h_4, (0.3,0.6,0.6) >\}).
\end{aligned}$$

Definition:1.23

Let $\widetilde{F}_{A_1}, \widetilde{F}_{A_2}, \ldots\ldots\ldots\ldots, \widetilde{F}_{A_n}$ be *n* neutrosophic soft sets over the common universe *U*. Then the **Cartesian product of *n* neutrosophic soft sets** $\widetilde{F}_{A_1}, \widetilde{F}_{A_2}, \ldots\ldots\ldots\ldots, \widetilde{F}_{A_n}$ is denoted by $\widetilde{F}_{A_1} \times \widetilde{F}_{A_2} \times \ldots\ldots\ldots\ldots \times \widetilde{F}_{A_n} = \widetilde{F}_{\hat{\times} A_n}$ is defined by

$$\widetilde{F}_{\hat{\times} A_n} = \{((\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots, \alpha_n), f_{\widetilde{F}_{\hat{\times} A_n}}(\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots, \alpha_n)) : (\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots, \alpha_n) \in A_1 \times A_2 \times \ldots\ldots \times A_n\}$$

where the truth membership, indeterminacy membership and falsity membership of $\widetilde{F}_{\hat{\hat{\times}} A_n}$ are as follows:

$$T_{f_{\widetilde{F}\hat{\times} A_n (\alpha_1, \alpha_2, \ldots\ldots, \alpha_n)}}(x) = \min(T_{f_{\widetilde{F}_{A1(\alpha_1)}}}(x), T_{f_{\widetilde{F}_{A_2(\alpha_2)}}}(x), \ldots\ldots\ldots, T_{f_{\widetilde{F}_{A_n(\alpha_n)}}})$$

$$I_{f_{\widetilde{F}\hat{\times} A_n (\alpha_1, \alpha_2, \ldots\ldots, \alpha_n)}}(x) = \frac{(I_{f_{\widetilde{F}_{A1(\alpha_1)}}}(x) + I_{f_{\widetilde{F}_{A2(\alpha_2)}}}(x) + \ldots\ldots\ldots + I_{f_{\widetilde{F}_{A_n(\alpha_n)}}})}{n}$$

$$F_{f_{\widetilde{F}\hat{\times} A_n (\alpha_1, \alpha_2, \ldots\ldots, \alpha_n)}}(x) = \max(F_{f_{\widetilde{F}_{A1(\alpha_1)}}}(x), F_{f_{\widetilde{F}_{A2(\alpha_2)}}}(x), \ldots\ldots\ldots, F_{f_{\widetilde{F}_{A_n(\alpha_n)}}}) \forall \alpha, \beta \in A \times B, \forall x \in U.$$

Definition:1.24

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be NSSs over the common universe *U*. Then **neutrosophic soft relation** from $\widetilde{F}_A$ to $\widetilde{G}_B$ is a neutrtosophic soft sub set of $\widetilde{F}_A \times \widetilde{G}_B$. In other words neutrosophic soft relation from $\widetilde{F}_A$ to $\widetilde{G}_B$ is of the form $(\widetilde{R}, D)$, where $(D \subseteq A \times B)$ and

$\widetilde{R}(\alpha,\beta) \subseteq \widetilde{F}_A \times \widetilde{G}_B \forall (\alpha,\beta) \in D$

Example:1.25

In example 1.22, the neutrosophic soft relation $\widetilde{R}$ from $\widetilde{F}_A$ to $\widetilde{G}_B$ as follows,

$$\begin{aligned}
\widetilde{R} = \{&((e_1,e_3),\{<h_1,(0.7,0.75,0.7)>,<h_2,(0.4,0.5,0.8)>,<h_3,(0.5,0.35,0.5)>,<h_4,(0.3,0.5,0.7)>\}),\\
&((e_2,e_3),\{<h_1,(0.5,0.8,0.8)>,<h_2,(0.5,0.85,0.8)>,<h_3,(0.5,0.6,0.8)>,<h_4,(0.5,0.55,0.6)>\}),\\
&((e_2,e_6),\{<h_1,(0.5,0.55,0.8)>,<h_2,(0.5,0.55,0.8)>,<h_3,(0.5,0.5,0.8)>,<h_4,(0.5,0.75,0.5)>\}),\\
&((e_3,e_3),\{<h_1,(0.8,0.75,0.9)>,<h_2,(0.5,0.85,0.9)>,<h_3,(0.5,0.55,0.4)>,<h_4,(0.3,0.6,0.4)>\}).
\end{aligned}$$

Definition:1.26

Let $\widetilde{R}$ be a neutrosophic soft relation from $\widetilde{F}_A$ to $\widetilde{G}_B$ then **inverse neutrosophic soft relation** $\widetilde{R}^{-1}$ is defined as

$$\widetilde{R}^{-1}(\alpha,\beta) = \widetilde{R}(\beta,\alpha), \forall (\alpha,\beta) \in A \times B.$$

Example:1.27

In 1.25, a neutrosophic soft relation $\widetilde{R}^{-1}$, from $\widetilde{G}_B$ to $\widetilde{F}_A$ as follows.

$$\begin{aligned}
\widetilde{R}^{-1} = \{&((e_3,e_1),\{<h_1,(0.7,0.75,0.7)>,<h_2,(0.4,0.5,0.8)>,<h_3,(0.5,0.35,0.5)>,<h_4,(0.3,0.5,0.7)>\}),\\
&((e_3,e_2),\{<h_1,(0.5,0.8,0.8)>,<h_2,(0.5,0.85,0.8)>,<h_3,(0.5,0.6,0.8)>,<h_4,(0.5,0.55,0.6)>\}),\\
&((e_6,e_2),\{<h_1,(0.5,0.55,0.8)>,<h_2,(0.5,0.55,0.8)>,<h_3,(0.5,0.5,0.8)>,<h_4,(0.5,0.75,0.5)>\}),\\
&((e_3,e_3),\{<h_1,(0.8,0.75,0.9)>,<h_2,(0.5,0.85,0.9)>,<h_3,(0.5,0.55,0.4)>,<h_4,(0.3,0.6,0.4)>\}).
\end{aligned}$$

Theorem:1.28

If $\widetilde{R}$ be a neutrosophic soft relation from $\widetilde{F}_A$ to $\widetilde{G}_B$ then $\widetilde{R}^{-1}$ is a neutrosophic soft relation from $\widetilde{G}_B$ to $\widetilde{F}_A$.

Proof:

$$\widetilde{R}^{-1}(\alpha,\beta) = \widetilde{R}(\beta,\alpha), \forall (\alpha,\beta) \in A \times B.$$

$$= f_{\widetilde{G}_B}(\beta) \cap f_{\widetilde{F}_A}(\alpha)$$

$$= f_{\widetilde{F}_A}(\alpha) \cap f_{\widetilde{G}_B}(\beta), \forall (\alpha, \beta) \in A \times B$$

hence $\widetilde{R}^{-1}$ is a neutrosophic soft relation from $\widetilde{G}_B$ to $\widetilde{F}_A$.

Theorem:1.29

Let $\widetilde{R}_1$ and $\widetilde{R}_2$ be two neutrosophic soft relations. Then

1) $(\widetilde{R}_1^{-1})^{-1} = \widetilde{R}_1$
2) $\widetilde{R}_1 \subseteq \widetilde{R}_2 \Rightarrow \widetilde{R}_1^{-1} \subseteq \widetilde{R}_2^{-1}$

Proof:

1) $(\widetilde{R}_1^{-1})^{-1}(\alpha, \beta) = \widetilde{R}_1^{-1}(\beta, \alpha)$

$$= R_1(\alpha, \beta)$$

2) $R_1(\alpha, \beta) \subseteq R_2(\alpha, \beta)$

$$\Rightarrow R_1^{-1}(\beta, \alpha) \subseteq R_2^{-1}(\beta, \alpha)$$

$$\Rightarrow R_1^{-1} \subseteq R_2^{-1}$$

Definition:1.30

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be NSS's over the common universe U. Let $\widetilde{R}$ be a soft relation from $\widetilde{F}_A$ to $\widetilde{G}_B$. Then **domain** $D(\widetilde{R})$ and **Range** $R(\widetilde{R})$ of $\widetilde{R}$ respectively is defined as

$$D(\widetilde{R}) = \{(\alpha, f_{\widetilde{F}_A}(\alpha)) \in \widetilde{F}_A : \widetilde{R}(\alpha, \beta) \in \widetilde{R}\}$$

$$R(\widetilde{R}) = \{(\beta, f_{\widetilde{G}''_B}(\beta)) \in \widetilde{G}_B : \widetilde{R}(\alpha, \beta) \in \widetilde{R}\}$$

Example:1.31

In example 1.25, the $D(\widetilde{R})$ and $R(\widetilde{R})$ is as follows

$D(\widetilde{R}) = \{(e_1, \{< h_1, (0.7, 0.6, 0.7) >, < h_2, (0.4, 0.2, 0.8) >, < h_3, (0.9, 0.1, 0.5) >, < h_4, (0.4, 0.7, 0.7) >\})$
$(e_2, \{< h_1, (0.5, 0.7, 0.8) >, < h_2, (0.5, 0.9, 0.3) >, < h_3, (0.5, 0.6, 0.8) >, < h_4, (0.5, 0.8, 0.5) >\})$

$(e_3, \{< h_1, (0.8, 0.6, 0.9) >, < h_2, (0.5, 0.9, 0.9) >, < h_3, (0.7, 0.5, 0.4) >, < h_4, (0.3, 0.5, 0.6) >\})$

$R(\widetilde{R}) = \{(e_3, \{< h_1, (0.8, 0.9, 0.6) >, < h_2, (0.7, 0.8, 0.8) >, < h_3, (0.5, 0.6, 0.4) >, < h_4, (0.3, 0.3, 0.6) >\})$
$(e_6, \{< h_1, (0.8, 0.4, 0.6) >, < h_2, (0.6, 0.2, 0.8) >, < h_3, (0.6, 0.4, 0.6) >, < h_4, (0.5, 0.7, 0.4) >\})$

Theorem:1.32

Let $\widetilde{R}_1$ and $\widetilde{R}_2$ be two neutrosophic soft relations. Then

1) $\widetilde{R}_1 \subseteq \widetilde{R}_2 \Rightarrow R(\widetilde{R}_1) \subseteq R(\widetilde{R}_2)$
2) $\widetilde{R}_1 \subseteq \widetilde{R}_2 \Rightarrow D(\widetilde{R}_1) \subseteq D(\widetilde{R}_2)$

Definition:1.33

The **composition of two neutrosophic soft relations** $\widetilde{R}_1$ and $\widetilde{R}_2$ is defined by

$(\widetilde{R}_1 \circ \widetilde{R}_2)(\alpha, \gamma) = \widetilde{R}_1(\alpha, \beta) \cap \widetilde{R}_2(\beta, \gamma)$ where

$\widetilde{R}_1$ is a neutrosophic soft relation from $\widetilde{F}_A$ to $\widetilde{G}_B$ and $\widetilde{R}_2$ is a neutrosophic soft relation from $\widetilde{G}_B$ to $\widetilde{H}_D$.

Theorem:1.34

If $\widetilde{R}_1$ and $\widetilde{R}_2$ are two neutrosophic soft relations from $\widetilde{F}_A$ to $\widetilde{G}_B$, then

$(\widetilde{R}_1 \circ \widetilde{R}_2)^{-1} = \widetilde{R}_2^{-1} \circ \widetilde{R}_1^{-1}$

Proof:

$$\begin{aligned}
((\widetilde{R}_1 \circ \widetilde{R}_2)(\alpha, \gamma))^{-1} &= \widetilde{R}_1 \circ \widetilde{R}_2(\gamma, \alpha) \\
&= \widetilde{R}_1(\gamma, \beta) \cap \widetilde{R}_2(\beta, \alpha) \\
&= \widetilde{R}_2(\beta, \alpha) \cap \widetilde{R}_1(\gamma, \beta) \\
&= \widetilde{R}_2^{-1}(\alpha, \beta) \cap \widetilde{R}_1^{-1}(\beta, \gamma) \\
&= \widetilde{R}_2^{-1} \circ \widetilde{R}_1^{-1}
\end{aligned}$$

Definition:1.35

Let $\widetilde{R}$ be a neutrosophic soft relation from $\widetilde{F}_A$ to $\widetilde{F}_A$

1) Its **neutrosophic soft reflexive** relation if $\widetilde{R}(\alpha,\alpha) \subseteq \widetilde{R}(\alpha,\alpha), \forall \alpha \in A$
2) Its **neutrosophic soft symmetric** relation if $\widetilde{R}(\alpha,\beta) = \widetilde{R}(\beta,\alpha), \forall \alpha,\beta \in A$
3) Its **neutrosophic soft transitive** relation if $\widetilde{R} \circ \widetilde{R} \subseteq \widetilde{R}$
4) Its **neutrosophic soft equivalence** relation if, it is symmetric, transitive and reflexive.

Theorem:1.36

Let $\widetilde{R}$ be a neutrosophic soft relation from $\widetilde{F}_A$ to $\widetilde{F}_A$

1) If $\widetilde{R}$ is symmetric if and only if $\widetilde{R}^{-1}$ is symmetric
2) If $\widetilde{R}$ is symmetric if and only if $\widetilde{R}^{-1} = \widetilde{R}$
3) If $\widetilde{R}_1$ and $\widetilde{R}_2$ are symmetric relations on $\widetilde{F}_A$ then $\widetilde{R}_1 \circ \widetilde{R}_2$ is symmetric relation on $\widetilde{F}_A$ if and only if $\widetilde{R}_1 \circ \widetilde{R}_2 = \widetilde{R}_2 \circ \widetilde{R}_1$

Proof:

1) Assume that $\widetilde{R}$ is symmetric. Then,

$$\begin{aligned} \widetilde{R}^{-1}(\alpha,\beta) &= \widetilde{R}(\beta,\alpha) \\ &= \widetilde{R}(\alpha,\beta) \\ &= \widetilde{R}^{-1}(\beta,\alpha) \end{aligned}$$

So, $\widetilde{R}^{-1}$ is symmetric.

Conversely, assume that $\widetilde{R}^{-1}$ is symmetric. Then,

$$\begin{aligned} \widetilde{R}(\alpha,\beta) &= \widetilde{R}(\beta,\alpha) \\ &= \widetilde{R}^{-1}(\alpha,\beta) \\ &= \widetilde{R}(\beta,\alpha) \end{aligned}$$

So, $\widetilde{R}$ is symmetric.

Similarly one can prove (2) and (3).

Note:1.37

If $\widetilde{R}$ is symmetric, then $\widetilde{R}^{n}$ is symmetric for all positive integer n, where $\widetilde{R}^{n} = \widetilde{R} \circ \widetilde{R} \circ ...... \circ \widetilde{R}$ n times

Theorem:1.38

Let $\widetilde{R}$ be an neutrosophic soft relation from $\widetilde{F}_A$ to $\widetilde{F}_A$

1) If $\widetilde{R}$ is transitive, then $\widetilde{R}^{-1}$ is also transitive.
2) If $\widetilde{R}$ is transitive, then $\widetilde{R} \circ \widetilde{R}$ is also transitive.
3) If $\widetilde{R}$ is reflexive, then $\widetilde{R}^{-1}$is also reflexive.
4) If $\widetilde{R}$ is symmetric and transitive, then $\widetilde{R}$is also reflexive.

Proof:

$$
\begin{aligned}
1)\widetilde{R}^{-1}(\alpha,\beta) &= \widetilde{R}(\beta,\alpha) \\
&\subseteq \widetilde{R} \circ R(\beta,\alpha) \\
&= \widetilde{R}(\beta,\gamma) \cap \widetilde{R}(\gamma,\alpha) \\
&= \widetilde{R}(\gamma,\alpha) \cap \widetilde{R}(\beta,\gamma) \\
&= \widetilde{R}^{-1}(\alpha,\gamma) \cap \widetilde{R}^{-1}(\gamma,\beta) \\
&= R^{-1} \circ R^{-1}(\alpha,\beta)
\end{aligned}
$$

hence, $R^{-1} \circ R^{-1} \subseteq R^{-1}$.

so, $\widetilde{R}^{-1}$is also transitive.

Similarly, one can prove (2),(3) and (4).

Definition:1.39

Let $\widetilde{R}$ be an **equivalence relation on neutrosophic soft relation** from $\widetilde{F}_A$ to $\widetilde{G}_B$ then equivalence class of $(\alpha, f_{\widetilde{F}_A}(\alpha))$ denoted by $[(\alpha, f_{\widetilde{F}_A}(\alpha))]_R$ is defined as

$$[(\alpha, f_{\widetilde{F}_A}(\alpha))]_R = \{(\beta, f_{\widetilde{F}_A}(\beta)) : \widetilde{R}(\alpha,\beta) \in \widetilde{R}\}$$

Example:1.40

In example 1.25, the equivalence class of $(\alpha, f_{\widetilde{F}_A}(\alpha))$ as follows

$$[(\alpha, f_{\widetilde{F}_A}(\alpha))]_R = \{(e_3, \{<h_1, (0.8,0.9,0.6)>, <h_2, (0.7,0.8,0.8)>, <h_3, (0.5,0.6,0.4)>, <h_4, (0.3,0.3,0.6)>\})\}$$

Theorem:1.41

Let $\widetilde{R}$ be an equivalence relation on neutrosophic soft relation from $\widetilde{F}_A$ to $\widetilde{F}_A$ for any $(\alpha, f_{\widetilde{F}_A}(\alpha)), (\beta, f_{\widetilde{F}_A}(\beta)) \in \widetilde{F}_A$, $\widetilde{R}(\alpha,\beta) \in \widetilde{R}$ if and only if $[(\alpha, f_{\widetilde{F}_A}(\alpha))]_R = [(\beta, f_{\widetilde{F}_A}(\beta))]_R$.

Proof:

Suppose $[(\alpha, f_{\widetilde{F}_A}(\alpha))]_R = [(\beta, f_{\widetilde{F}_A}(\beta))]_R$.

Since $\widetilde{R}$ is reflexive $\widetilde{R}(\beta,\beta) \in \widetilde{R}$.

hence $(\beta, f_{\widetilde{F}_A}(\beta)) \in [(\beta, f_{\widetilde{F}_A}(\beta))]_R = [(\alpha, f_{\widetilde{F}_A}(\alpha))]_R$ which gives $\widetilde{R}(\alpha,\beta) \in \widetilde{R}$.

Conversely, suppose $\widetilde{R}(\alpha,\beta) \in \widetilde{R}$.

Let $(\alpha, f_{\widetilde{F}_A}(\alpha)) \in [(\alpha, f_{\widetilde{F}_A}(\alpha))]_R$

then $\widetilde{R}(\alpha,\beta) \in \widetilde{R}$.

Using transitive property of $\widetilde{R}$ gives $(\alpha, f_{\widetilde{F}_A}(\alpha)) \in [(\beta, f_{\widetilde{F}_A}(\beta))]_R$

hence $(\alpha, f_{\widetilde{F}_A}(\alpha)) \subseteq (\beta, f_{\widetilde{F}_A}(\beta))$.

Using this similar argument $(\beta, f_{\widetilde{F}_A}(\beta)) \subseteq (\alpha, f_{\widetilde{F}_A}(\alpha))$

hence $(\alpha, f_{\widetilde{F}_A}(\alpha)) = (\beta, f_{\widetilde{F}_A}(\beta))$.

So, $[(\alpha, f_{\widetilde{F}_A}(\alpha))]_R = [(\beta, f_{\widetilde{F}_A}(\beta))]_R$

Definition:1.42

A collection of non-empty neutrosophic soft subsets $P = \{\widetilde{F}_{A_i} : i \in I\}$ of a neutrosophic soft set $\widetilde{F}_A$ is called a **partition** of $\widetilde{F}_A$

1) $\widetilde{F}_{A_i} \neq \phi$

2) $\widetilde{F}_A = \bigcup_i \widetilde{F}_{A_i}$

3) $\widetilde{F}_{A_i} \cap \widetilde{F}_{A_j} = \phi$. If $i \neq j \forall i, j \in I$, I be any arbitrary index set.

Here elements of the partition are called a block of $\widetilde{F}_A$. More over a NSS $\widetilde{F}_A$, we can define a neutrosophic soft relation on $\widetilde{F}_A$ by $\widetilde{R}(\alpha,\beta)$ if and only if $(\alpha, f_{\widetilde{F}_A}(\alpha))$ and $(\beta, f_{\widetilde{F}_A}(\beta))$ belong to the same block.

Theorem:1.43

Let $P = \{\widetilde{F}_{A_i} : i \in I\}$ be a partition of NSS $\widetilde{F}_A$ the neutrosophic soft relation $\widetilde{R}$ defined on $\widetilde{F}_A$ as $\widetilde{R}(\alpha,\beta)$ if and only if $(\alpha, f_{\widetilde{F}_A}(\alpha))$ and $(\beta, f_{\widetilde{F}_A}(\beta))$ are the elements of the same block is an equivalence relation.

Proof:

Reflexive: Let $(\alpha, f_{\widetilde{F}_A}(\alpha))$ be any element of $\widetilde{F}_A$. It is clear that $(\alpha, f_{\widetilde{F}_A}(\alpha))$ is in same block itself. Hence $\widetilde{R}(\alpha,\alpha) \in \widetilde{R}$.

Symmetric: If $\widetilde{R}(\alpha,\beta) \in \widetilde{R}$. then $(\alpha, f_{\widetilde{F}_A}(\alpha))$ and $(\beta, f_{\widetilde{F}_A}(\beta))$ are in the same block. Therefore $\widetilde{R}(\beta,\alpha) \in \widetilde{R}$.

Transitive: If $\widetilde{R}(\alpha,\beta) \in \widetilde{R}$ and $\widetilde{R}(\beta,\gamma) \in \widetilde{R}$ then $(\alpha, f_{\widetilde{F}_A}(\alpha))$, $(\beta, f_{\widetilde{F}_A}(\beta))$ and $(\gamma, f_{\widetilde{F}_A}(\gamma))$ must lie in the same block. Therefore $\widetilde{R}(\alpha,\gamma) \in \widetilde{R}$.