# Chapter VI

# CHAPTER VI

# GENERALISED INTERVAL – VALUED FUZZY SOFT SETS

**Definition: 6.1**

Let U = $\{x_1, x_2, ...., x_n\}$ be the universal set of elements and E = $\{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $\widetilde{F} : E \rightarrow \widetilde{F}(U)$ and $\mu$ be a fuzzy set of E, *i.e.* $\mu : E \rightarrow [0, 1]$, where $\widetilde{F}(U)$ is the set of all interval-valued fuzzy subsets on U. Let $\widetilde{F}_\mu : E \rightarrow F(U) \times [0, 1]$ be a function defined as follows:

$$\widetilde{F}_\mu(e) = (\widetilde{F}(e), \mu(e))$$

Then $\widetilde{F}_\mu$ is called **Generalised Interval-Valued Fuzzy Soft Set (GIVFSS)** over the soft universe (U, E). For each parameter $e_i$, $\widetilde{F}_\mu(e_i) = (\widetilde{F}(e_i)(x), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $\widetilde{F}(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$. So we can write $\widetilde{F}_\mu(e_i)$ as follows:

$$\widetilde{F}_\mu(e_i) = (\{ x_1 / \widetilde{F}(e_i)(x_1), x_2 / \widetilde{F}(e_i)(x_2), ................, x_n / \widetilde{F}(e_i)(x_n) \}, \mu(e_i))$$

**Example: 6.2**

Let U = $\{x_1, x_2, x_3\}$ be a set of universe, E = $\{e_1, e_2, e_3\}$ a set of parameters and let $\mu : E \rightarrow [0, 1]$. Define a function $\widetilde{F}_\mu : E \rightarrow F(U) \times [0, 1]$ as follows:

$$\widetilde{F}_\mu(e_1) = (\{ x_1 / [0.3, 0.6], x_2 / [0.7, 0.8], x_3 / [0.5, 0.8] \}, 0.6)$$

$$\widetilde{F}_\mu(e_2) = (\{ x_1 / [0.1, 0.4], x_2 / [0.0, 0.3], x_3 / [0.1, 0.5] \}, 0.5)$$

$$\widetilde{F}_\mu(e_3) = (\{ x_1 / [0.7, 0.8], x_2 / [0.1, 0.2], x_3 / [0.0, 0.4] \}, 0.3)$$

Then $\widetilde{F}_\mu$ is a GIVFSS over (U, E).

In matrix notation we write, $\widetilde{F}_\mu = \begin{pmatrix} [0.3, 0.6] & [0.7, 0.8] & [0.5, 0.8] & 0.6 \\ [0.1, 0.4] & [0.0, 0.3] & [0.1, 0.5] & 0.5 \\ [0.7, 0.8] & [0.1, 0.2] & [0.0, 0.4] & 0.3 \end{pmatrix}$

**Definition: 6.3**

Let $\widetilde{F}_\mu$ and $\widetilde{G}_\delta$ be two GIVFSSs over (U, E). $\widetilde{F}_\mu$ is called a **Generalised Interval-Valued Fuzzy Soft Subsets** of $\widetilde{G}_\delta$ and we write $\widetilde{F}_\mu \subseteq \widetilde{G}_\delta$ if:

1) $\mu(e)$ is a fuzzy subset of $\delta(e)$ $\forall e \in E$.
2) $\widetilde{F}(e)$ is an interval-valued fuzzy subset of $\widetilde{G}(e)$ $\forall e \in E$.

**Example: 6.4**

Let U = $\{x_1, x_2, x_3\}$ be a set of three cars and let E = $\{e_1, e_2, e_3\}$ be a set of parameters where $e_1 =$cheap, $e_2 =$expensive, $e_3 =$red. Let $\widetilde{F}_\mu$ be a GIVFSS over (U, E) defined as follows:

$\widetilde{F}_\mu(e_1) = (\ \{\ x_1/[0.1, 0.3]\ , x_2/[0.5, 0.7]\ , x_3/[0.3, 0.5]\ \}\ ,\ 0.4\ )$

$\widetilde{F}_\mu(e_2) = (\ \{\ x_1/[0.0, 0.3]\ , x_2/[0.0, 0.2]\ , x_3/[0.1, 0.3]\ \}\ ,\ 0.4\ )$

$\widetilde{F}_\mu(e_3) = (\ \{\ x_1/[0.5, 0.6]\ , x_2/[0.1, 0.1]\ , x_3/[0.1, 0.3]\ \}\ ,\ 0.1\ )$

Let $\widetilde{G}_\delta$ be another GIVFSS over (U, E) defined as follows:

$\widetilde{G}_\delta(e_1) = (\ \{\ x_1/[0.3, 0.6]\ , x_2/[0.7, 0.8]\ , x_3/[0.5, 0.8]\ \}\ ,\ 0.6\ )$

$\widetilde{G}_\delta(e_2) = (\ \{\ x_1/[0.2, 0.4]\ , x_2/[0.2, 0.3]\ , x_3/[0.3, 0.5]\ \}\ ,\ 0.5\ )$

$\widetilde{G}_\delta(e_3) = (\ \{\ x_1/[0.7, 0.8]\ , x_2/[0.2, 0.4]\ , x_3/[0.2, 0.5]\ \}\ ,\ 0.3\ )$

It is clear that $\widetilde{F}_\mu$ is a GIVFS subset of $\widetilde{G}_\delta$.

**Definition: 6.5**

Two GIVFSSs $\widetilde{F}_\mu$ and $\widetilde{G}_\delta$ over (U, E) are said to be **equal** and we write $\widetilde{F}_\mu = \widetilde{G}_\delta$ if $\widetilde{F}_\mu$ is a GIVFS subset of $\widetilde{G}_\delta$ and $\widetilde{G}_\delta$ is a GIVFS subset of $\widetilde{F}_\mu$. In other words, $\widetilde{F}_\mu = \widetilde{G}_\delta$ if the following conditions are satisfied:

1) $\mu(e)$ is equal to $\delta(e)$ $\forall e \in E$
2) $\widetilde{F}(e)$ is equal to $\widetilde{G}(e)$ $\forall e \in E$

**Definition: 6.6**

A GIVFSS is called a **Generalised Null Interval-Valued Fuzzy Soft Set**, denoted by $\widetilde{\phi}_\mu$ if $\widetilde{\phi}_\mu : E \to F(U) \times [0, 1]$ such that

$$\widetilde{\phi}_\mu(e) = \left(\widetilde{F}(e)(x)\,,\ \mu(e)\right)$$

Where $\widetilde{F}(e) = [0, 0] = [0]$ and $\mu(e) = 0$ $\forall e \in E$.

**Definition: 6.7**

A GIVFSS is called a **Generalised Absolute Interval-Valued Fuzzy Soft Set**, denoted by $\widetilde{A}_\mu$ if $\widetilde{A}_\mu : E \to F(U) \times [0, 1]$ such that

$$\widetilde{A}_\mu(e) = \left(\widetilde{F}(e)(x)\,,\ \mu(e)\right)$$

Where $\widetilde{F}(e) = [1, 1] = [1]$ and $\mu(e) = 1$ $\forall e \in E$.

**Definition: 6.8**

Let $\widetilde{F}_\mu$ be a GIVFSS over (U, E). Then the **Complement** of $\widetilde{F}_\mu$, denoted by $\widetilde{F}_\mu^C$ and is defined by $\widetilde{F}_\mu^C = \widetilde{G}_\delta$, such that $\delta(e) = c\left(\mu(e)\right)$ and $\widetilde{G}(e) = \widetilde{c}\left(\widetilde{F}(e)\right)$ $\forall e \in E$, where $c$ is a fuzzy complement and $\widetilde{c}$ is an interval-valued fuzzy complement.

**Example: 6.9**

Consider a GIVFSS $\widetilde{F}_\mu$ over (U, E) as in Example 6.2

$$\widetilde{F}_\mu = \begin{pmatrix} [0.3, 0.6] & [0.7, 0.8] & [0.5, 0.8] & 0.6 \\ [0.1, 0.4] & [0.0, 0.3] & [0.1, 0.5] & 0.5 \\ [0.7, 0.8] & [0.1, 0.2] & [0.0, 0.4] & 0.3 \end{pmatrix}$$

By using the basic fuzzy complement for $\mu(e)$ and interval-valued fuzzy complement for $\widetilde{F}(e)$ we have $\widetilde{F}_\mu^C = \widetilde{G}_\delta$ where

$$\widetilde{G}_\delta = \begin{pmatrix} [0.4, 0.7] & [0.2, 0.3] & [0.2, 0.5] & 0.4 \\ [0.6, 0.9] & [0.7, 1.0] & [0.5, 0.9] & 0.5 \\ [0.2, 0.3] & [0.8, 0.9] & [0.6, 1.0] & 0.7 \end{pmatrix}$$

**Theorem: 6.10**

Let $\widetilde{F}_\mu$ be a GIVFSS over (U, E). Then the following holds: $\left(\widetilde{F}_\mu^C\right)^C = \widetilde{F}_\mu$.

**Proof:**

Since $\widetilde{F}_\mu^C = \widetilde{G}_\delta$ then $\left(\widetilde{F}_\mu^C\right)^C = \widetilde{G}_\delta^C$ but from Definition 6.1 $\widetilde{G}_\delta = \left(\widetilde{c}\left(\widetilde{F}(e)\right), c\left(\mu(e)\right)\right)$ then

$$\widetilde{G}_\delta^C = \left(\widetilde{c}\left(\widetilde{c}\left(\widetilde{F}(e)\right)\right), c\left(c\left(\mu(e)\right)\right)\right)$$

$$= \left(\widetilde{F}(e), \mu(e)\right)$$

$$= \widetilde{F}_\mu$$

**Definition: 6.11**

**Union** of two GIVFSSs $\left(\widetilde{F}_\mu, A\right)$ and $\left(\widetilde{G}_\delta, B\right)$, denoted by $\widetilde{F}_\mu \cup \widetilde{G}_\delta$, is a GIVFSS $\left(\widetilde{H}_\upsilon, C\right)$ where $C = A \cup B$ and $\widetilde{H}_\upsilon : E \to F(U) \times [0, 1]$ is defined by

$\widetilde{H}_{\upsilon}(e) = \left(\widetilde{H}(e)\, ,\, \upsilon(e)\right)$ such that $\widetilde{H}(e) = \widetilde{F}(e) \cup \widetilde{G}(e)$ and $\upsilon(e) = s(\,\mu(e)\, ,\, \delta(e))$ where $s$ is a $s$ - norm and $\widetilde{H}(e) = \left\lfloor \sup\left(\mu^{-}_{\widetilde{F}(e)}\, ,\, \mu^{-}_{\widetilde{G}(e)}\right),\ \sup\left(\mu^{+}_{\widetilde{F}(e)}\, ,\, \mu^{+}_{\widetilde{G}(e)}\right)\right\rfloor$

**Example: 6.12**

Consider GIVFSS $\widetilde{F}_{\mu}$ and $\widetilde{G}_{\delta}$ as in Example 6.4. By using interval-valued fuzzy union and basic fuzzy union we have $\widetilde{F}_{\mu} \cup \widetilde{G}_{\delta} = \widetilde{H}_{\upsilon}$, where

$$\begin{aligned}\widetilde{H}_{\upsilon}(e_1) = (\ \{\, & x_1/[\sup(0.1,0.3)\, ,\, \sup(0.3,0.6)]\, ,\, x_2/[\sup(0.5,0.7)\, ,\, \sup(0.7,0.8)] \\ & x_3/[\sup(0.3,0.5)\, ,\, \sup(0.5,0.8)]\,\}\ ,\ \max(0.4,0.6)\ )\end{aligned}$$

$$= \left(\left\{x_1/[0.3,\ 0.6]\, ,\, x_2/[0.7,\ 0.8]\, ,\, x_3/[0.5,\ 0.8]\right\},\ 0.6\right)$$

Similarly we get

$$\widetilde{H}_{\upsilon}(e_2) = \left(\left\{x_1/[0.2,\ 0.4]\, ,\, x_2/[0.2,\ 0.3]\, ,\, x_3/[0.3,\ 0.5]\right\},\ 0.5\right)$$

$$\widetilde{H}_{\upsilon}(e_3) = \left(\left\{x_1/[0.7,\ 0.8]\, ,\, x_2/[0.2,\ 0.4]\, ,\, x_3/[0.2,\ 0.5]\right\},\ 0.3\right)$$

In matrix notation we write

$$\widetilde{H}_{\upsilon}(e) = \begin{pmatrix} [0.3,\ 0.6] & [0.7,\ 0.8] & [0.5,\ 0.8] & 0.6 \\ [0.2,\ 0.4] & [0.2,\ 0.3] & [0.3,\ 0.5] & 0.5 \\ [0.7,\ 0.8] & [0.2,\ 0.4] & [0.2,\ 0.5] & 0.3 \end{pmatrix}$$

**Theorem: 6.13**

Let $\widetilde{F}_{\mu}$, $\widetilde{G}_{\delta}$ and $\widetilde{H}_{\upsilon}$ be any three GIVFSSs. Then the following results hold:

1) $\widetilde{F}_{\mu} \cup \widetilde{G}_{\delta} = \widetilde{G}_{\delta} \cup \widetilde{F}_{\mu}$

2) $\widetilde{F}_{\mu} \cup (\widetilde{G}_{\delta} \cup \widetilde{H}_{\upsilon}) = (\widetilde{F}_{\mu} \cup \widetilde{G}_{\delta}) \cup \widetilde{H}_{\upsilon}$

3) $F_{\mu} \cup F_{\mu} \subseteq F_{\mu}$

4) $\widetilde{F}_\mu \cup \widetilde{A}_\mu = \widetilde{A}_\mu$

5) $\widetilde{F}_\mu \cup \widetilde{\phi}_\mu = \widetilde{F}_\mu$

**Proof:**

1) $\widetilde{F}_\mu \cup \widetilde{G}_\delta = \widetilde{H}_\upsilon$

2) From Definition 6.11 we have $\widetilde{H}_\upsilon(e) = \left(\widetilde{H}(e)\,,\, \upsilon(e)\right)$ such that $\widetilde{H}(e) = \widetilde{F}(e) \cup \widetilde{G}(e)$ and $\upsilon(e) = s(\,\mu(e)\,,\, \delta(e))$.

   But $\widetilde{H}(e) = \widetilde{F}(e) \cup \widetilde{G}(e) = \widetilde{G}(e) \cup \widetilde{F}(e)$ ( since union of interval-valued fuzzy sets is commutative ) and $\upsilon(e) = s(\,\mu(e)\,,\, \delta(e)) = s(\,\delta(e)\,,\, \mu(e))$ ( since $s$ - norm is commutative ) , then $\widetilde{G}_\delta \cup \widetilde{F}_\mu = \widetilde{H}_\upsilon$.

3) The proof is straightforward from Definition 6.11.
4) The proof is straightforward from Definition 6.11.
5) The proof is straightforward from Definition 6.11.
6) The proof is straightforward from Definition 6.11.

**Definition: 6.14**

Intersection of two GIVFSSs $\left(\widetilde{F}_\mu\,, A\right)$ and $\left(\widetilde{G}_\delta\,, B\right)$, denoted by $\widetilde{F}_\mu \cap \widetilde{G}_\delta$, is a GIVFSS $\left(\widetilde{H}_\upsilon\,, C\right)$ where $C = A \cap B$ and $\widetilde{H}_\upsilon : E \to F(U) \times [0,\, 1]$ is defined by $\widetilde{H}_\upsilon(e) = \left(\widetilde{H}(e)\,,\, \upsilon(e)\right)$ such that $\widetilde{H}(e) = \widetilde{F}(e) \cap \widetilde{G}(e)$ and $\upsilon(e) = t(\,\mu(e)\,,\, \delta(e))$ where $t$ is a $t$- norm and $\widetilde{H}(e) = \left[\inf\left(\mu^-_{\widetilde{F}(e)}\,,\, \mu^-_{\widetilde{G}(e)}\right)\,,\ \inf\left(\mu^+_{\widetilde{F}(e)}\,,\, \mu^+_{\widetilde{G}(e)}\right)\right]$.

**Example: 6.15**

Consider GIVFSS $\widetilde{F}_\mu$ and $\widetilde{G}_\delta$ as in Example 6.12. By using interval-valued fuzzy intersection and basic fuzzy intersecion we have $\widetilde{F}_\mu \cap \widetilde{G}_\delta = \widetilde{H}_\upsilon$, where

$$\widetilde{H}_{\upsilon}(e_1) = ( \ \{ \ x_1 /[\inf(0.1,0.3) \ , \ \inf(0.3,0.6)] \ , \ x_2 /[\inf(0.5,0.7) \ , \ \inf(0.7,0.8)] \\ x_3 /[\inf(0.3,0.5) \ , \ \inf(0.5,0.8)] \ \} \ , \ \min(0.4,0.6) \ )$$

$$= \left( \left\{ x_1 /[0.1, \ 0.3] \ , \ x_2 /[0.5, \ 0.7] \ , \ x_3 /[0.3, \ 0.5] \right\} , \ 0.4 \right)$$

Similarly we get

$$\widetilde{H}_{\upsilon}(e_2) = \left( \left\{ x_1 /[0.0, \ 0.3] \ , \ x_2 /[0.0, \ 0.2] \ , \ x_3 /[0.1, \ 0.3] \right\} , \ 0.4 \right)$$

$$\widetilde{H}_{\upsilon}(e_3) = \left( \left\{ x_1 /[0.5, \ 0.6] \ , \ x_2 /[0.1, \ 0.1] \ , \ x_3 /[0.1, \ 0.3] \right\} , \ 0.1 \right)$$

In matrix notation we write

$$\widetilde{H}_{\upsilon}(e) = \begin{pmatrix} [0.1, \ 0.3] & [0.5, \ 0.7] & [0.3, \ 0.5] & 0.4 \\ [0.0, \ 0.3] & [0.0, \ 0.2] & [0.1, \ 0.3] & 0.4 \\ [0.5, \ 0.6] & [0.1, \ 0.1] & [0.1, \ 0.3] & 0.1 \end{pmatrix}$$

**Theorem: 6.16**

Let $\widetilde{F}_{\mu}$, $\widetilde{G}_{\delta}$ and $\widetilde{H}_{\upsilon}$ be any three GIVFSSs. Then the following results hold:

1) $\widetilde{F}_{\mu} \cap \widetilde{G}_{\delta} = \widetilde{G}_{\delta} \cap \widetilde{F}_{\mu}$

2) $\widetilde{F}_{\mu} \cap (\widetilde{G}_{\delta} \cap \widetilde{H}_{\upsilon}) = (\widetilde{F}_{\mu} \cap \widetilde{G}_{\delta}) \cap \widetilde{H}_{\upsilon}$

3) $\widetilde{F}_{\mu} \cap \widetilde{F}_{\mu} \subseteq \widetilde{F}_{\mu}$

4) $\widetilde{F}_{\mu} \cap \widetilde{A}_{\mu} = \widetilde{F}_{\mu}$

5) $\widetilde{F}_{\mu} \cap \widetilde{\phi}_{\mu} = \widetilde{\phi}_{\mu}$

**Proof:**

1) $\widetilde{F}_{\mu} \cap \widetilde{G}_{\delta} = \widetilde{H}_{\upsilon}$

2) From Definition 6.14 we have $\widetilde{H}_{\upsilon}(e) = \left( \widetilde{H}(e) \ , \ \upsilon(e) \right)$ such that $\widetilde{H}(e) = \widetilde{F}(e) \cap \widetilde{G}(e)$ and $\upsilon(e) = t(\mu(e) \ , \ \delta(e))$.

But $\widetilde{H}(e) = \widetilde{F}(e) \cap \widetilde{G}(e) = \widetilde{G}(e) \cap \widetilde{F}(e)$ ( since intersection of interval-valued fuzzy sets is commutative ) and $\upsilon(e) = t(\mu(e), \delta(e)) = t(\delta(e), \mu(e))$ ( since $t$-norm is commutative ), then $\widetilde{G}_\delta \cap \widetilde{F}_\mu = \widetilde{H}_\upsilon$.

3) The proof is straightforward from Definition 6.14.
4) The proof is straightforward from Definition 6.14.
5) The proof is straightforward from Definition 6.14.
6) The proof is straightforward from Definition 6.14.

**Theorem: 6.17**

Let $\widetilde{F}_\mu$ and $\widetilde{G}_\delta$ be any two GIVFSSs. Then the DeMorgan's Laws hold:

1) $\left(\widetilde{F}_\mu \cup \widetilde{G}_\delta\right)^C = \widetilde{F}_\delta^C \cap \widetilde{G}_\mu^C$

2) $\left(\widetilde{F}_\mu \cap \widetilde{G}_\delta\right)^C = \widetilde{F}_\delta^C \cup \widetilde{G}_\mu^C$

**Proof:**

1) $$\begin{aligned} \widetilde{F}_\mu^C \cap G_\delta^C &= \left( \left(\tilde{c}\left(F(e)\right), c\left(\mu(e)\right)\right) \cap \left(\tilde{c}\left(G(e)\right), c\left(\delta(e)\right)\right) \right) \\ &= \left( \left(\tilde{c}\left(F(e)\right) \cap \tilde{c}\left(G(e)\right)\right), \left(c\left(\mu(e)\right) \cap c\left(\delta(e)\right)\right) \right) \\ &= \left( \left(\left(F(e)\right) \cup \left(G(e)\right)\right)^{\tilde{c}}, \left(\mu(e) \cup \delta(e)\right)^C \right) \\ &= \left(\widetilde{F}_\mu \cup \widetilde{G}_\delta\right) \end{aligned}$$

2) The proof is similar to (1).

**Theorem: 6.18**

Let $\widetilde{F}_\mu$, $\widetilde{G}_\delta$ and $\widetilde{H}_\upsilon$ be any three GIVFSSs. Then the following results hold:

1) $\widetilde{F}_\mu \cup (\widetilde{G}_\delta \cap \widetilde{H}_\upsilon) = (\widetilde{F}_\mu \cup \widetilde{G}_\delta) \cap (\widetilde{F}_\mu \cup \widetilde{H}_\upsilon)$

2) $\widetilde{F}_\mu \cap (\widetilde{G}_\delta \cup \widetilde{H}_\upsilon) = (\widetilde{F}_\mu \cap \widetilde{G}_\delta) \cup (\widetilde{F}_\mu \cap \widetilde{H}_\upsilon)$

**Proof:**

1) For all $x \in E$,

$$\lambda_{F(x)\cup\left(G(x)\cap H(x)\right)}(x)$$

$$=\left[\ \sup\left(\lambda^{-}_{F(x)}(x)\,,\ \lambda^{-}_{G(x)\cap H(x)}(x)\right),\ \sup\left(\lambda^{+}_{F(x)}(x)\,,\ \lambda^{+}_{G(x)\cap H(x)}(x)\right)\right]$$

$$=\left[\ \sup\left(\lambda^{-}_{F(x)}(x)\,,\ \inf\left(\lambda^{-}_{G(x)}(x)\,,\ \lambda^{-}_{H(x)}(x)\right)\right),\right.$$
$$\left.\sup\left(\lambda^{+}_{F(x)}(x)\,,\ \inf\left(\lambda^{+}_{G(x)}(x)\,,\ \lambda^{+}_{H(x)}(x)\right)\right)\right]$$

$$=\left[\ \inf\left(\sup\left(\lambda^{-}_{F(x)}(x)\,,\ \lambda^{-}_{G(x)}(x)\right),\sup\left(\lambda^{-}_{F(x)}(x)\,,\ \lambda^{-}_{H(x)}(x)\right)\right),\right.$$
$$\left.\inf\left(\sup\left(\lambda^{+}_{F(x)}(x)\,,\ \lambda^{+}_{G(x)}(x)\right),\sup\left(\lambda^{+}_{F(x)}(x)\,,\ \lambda^{+}_{H(x)}(x)\right)\right)\right]$$

$$=\lambda_{\left(F(x)\cup G(x)\right)\cap\left(F(x)\cup H(x)\right)}(x)$$

and

$$\gamma_{\mu(x)\cup\left(\delta(x)\cap\upsilon(x)\right)}(x)$$

$$=\max\left\{\gamma_{\mu(x)}(x)\,,\ \gamma_{\delta(x)\cap\upsilon(x)}(x)\right\}$$

$$=\max\left\{\gamma_{\mu(x)}(x)\,,\ \min\left(\gamma_{\delta(x)}(x)\,,\ \gamma_{\upsilon(x)}(x)\right)\right\}$$

$$=\min\left\{\max\left(\gamma_{\mu(x)}(x)\,,\ \gamma_{\delta(x)}(x)\right),\ \max\left(\gamma_{\mu(x)}(x)\,,\ \gamma_{\upsilon(x)}(x)\right)\right\}$$

$$=\min\left\{\gamma_{\mu(x)\cup\delta(x)}(x)\,,\ \gamma_{\mu(x)\cup\upsilon(x)}(x)\right\}$$

$$=\gamma_{\left(\mu(x)\cup\delta(x)\right)\cap\left(\mu(x)\cup\upsilon(x)\right)}(x)$$

2) The proof is similar to (1).

**Definition: 6.19**

If $(F_\mu, A)$ and $(G_\delta, B)$ are two GIVFSSs then "$(F_\mu, A)$ **AND** $(G_\delta, B)$" denoted by $(F_\mu, A)\wedge(G_\delta, B)$ is defined by $(F_\mu, A)\wedge(G_\delta, B)=(H_\lambda, A\times B)$ where $H_\lambda(\alpha,\beta)=\big(\,H(\alpha,\beta)\,,\,\lambda(\alpha,\beta)\big)$ $\forall(\alpha,\beta)\in A\times B$, such that $H(\alpha,\beta)=F(\alpha)\cap G(\beta)$ and $\lambda(\alpha,\beta)=t\big(\,\mu(\alpha),\,\delta(\beta)\big)$, $\forall(\alpha,\beta)\in A\times B$, where $t$ is a $t$-norm.

**Definition: 6.20**

If $(F_\mu, A)$ and $(G_\delta, B)$ are two GIVFSSs then "$(F_\mu, A)$ **OR** $(G_\delta, B)$" denoted by $(F_\mu, A)\vee(G_\delta, B)$ is defined by $(F_\mu, A)\vee(G_\delta, B)=(H_\lambda, A\times B)$ where $H_\lambda(\alpha,\beta)=\big(\,H(\alpha,\beta)\,,\,\lambda(\alpha,\beta)\big)$ $\forall(\alpha,\beta)\in A\times B$, such that $H(\alpha,\beta)=F(\alpha)\cup G(\beta)$ and $\lambda(\alpha,\beta)=s\big(\,\mu(\alpha),\,\delta(\beta)\big)$, $\forall(\alpha,\beta)\in A\times B$, where $s$ is a $s$-norm.