Chapter V

Soft Compact Spaces

CHAPTER – V
SOFT COMPACT SPACES

Definition : 5.1

A family ψ of soft sets in $SS(U)_A$ is a **cover** of a soft set (F, A) in $SS(U)_A$ if $(F, A) \tilde{\subseteq} \tilde{\cup}\{(F_i, A) : (F_i, A) \in \psi, i \in I\}$. It is a **soft open cover** if each member of ψ is a soft open set. A sub cover of ψ is a sub family of ψ which is also a cover.

Definition : 5.2

A family ψ of soft sets has the **finite intersection property** if the intersection of the members of each finite sub family of ψ is not null soft set.

Definition : 5.3

A soft topological space (U, τ, A) is **soft compact** if each soft open cover of $\tilde{A}$ has a finite sub cover.

Theorem : 5.4

A soft topological space is compact if and only if each family of soft closed sets with the finite intersection property has a non null intersection.

Proof

If ψ is a family of soft sets in a soft topological space (U, τ, A), then ψ is a cover of U_A if and only if one of the following conditions holds :

1) $\tilde{\bigcup}_{i \in I} \{(F_i, A) : (F_i, A) \in \psi\} = \tilde{A}$;

2) $\{\tilde{\bigcup}_{i \in I} \{(F_i, A) : (F_i, A) \in \psi\}\}^c = (\tilde{A})^c = \Phi_A$;

3) $\tilde{\bigcap}_{i \in I} \{(F_i, A)^c : (F_i, A) \in \psi\} = \Phi_A$.

Hence the soft topological space (U, τ, A) is compact if and only if each family of soft open sets over U such that no finite sub family covers $\tilde{A}$, fails to be a cover, and this is true if and only if each family of soft closed sets which has the finite intersection property has a non null intersection.

Theorem : 5.5

Let (X, τ) be a soft Hausdorff space. If F_A is soft compact on X, then F_A is soft closed.

Proof

We must show that F_A^c is soft open. Let $x \tilde{\in} F_A^c$. So for each $e \in E$, $x \in F_A^c(e) = X \setminus F_A(e)$ and $x \notin F_A(e)$. Then for all $y \in F_A(e)$, $x \neq y$. Since (X, τ) is soft Hausdorff space there exists $(G_B)_y, (H_C)_y \in \tau$ such that $x \tilde{\in} (G_B)_y$, $y \tilde{\in} (H_C)_y$ and $(G_B)_y \tilde{\cap} (H_C)_y = \Phi$.

This implies, for all $e \in E$, $x \in (G_B)_y(e)$, $y \in (H_C)_y(e)$ and $(G_B)_y(e) \cap (H_C)_y(e) = \Phi$. Then $F_A(e) \subset (H_C)_y(e)$, so $F_A \tilde{\subseteq} (H_C)_y$. The family $\mathcal{C} = \{(H_C)_y : y \tilde{\in} F_A\}$ is a soft open cover of F_A. Since F_A is soft compact, F_A has a finite sub cover. So, $F_A \tilde{\subseteq} \bigcup_{i=1}^{n} (H_C)_{yi}$. Then $\bigcup_{i=1}^{n} (H_C)_{yi}$ and $\bigcap_{i=1}^{n} (G_B)_{yi} \in \tau$ and $(\bigcup_{i=1}^{n} (H_C)_{yi}) \tilde{\cap} (\bigcap_{i=1}^{n} (G_B)_{yi}) = \Phi$. Since $x \tilde{\in} (G_B)_y$, then $x \tilde{\in} (G_B)_{yi} \tilde{\subseteq} (\bigcup_{i=1}^{n} (H_C)_{yi})^c \tilde{\subseteq} F_A^c$. Hence F_A^c is soft open.

Theorem : 5.6

Let f_{pu} be a soft pu-continuous function carrying the compact soft topological space (U, τ, A) onto the soft topological space (V, τ^*, B). Then (V, τ^*, B) is compact.

Proof

Let $\psi = \{(G_i, B) : i \in I\}$ be a cover of V_B by soft open sets. Then since f_{pu} is soft pu-continuous, the family of all soft sets of the form $f_{pu}^{-1}(G_i, B)$, for $(G_i, B) \in \psi$, is a soft open cover of $\widetilde{A}$ which has a finite sub cover. However, since f_{pu} is surjective, then it is easily seen that $f_{pu}(f_{pu}^{-1}(G, B)) = (G, B)$ for any soft set (G, B) over V. Thus, the family of images of members of the sub cover is a finite sub family of ψ which covers V_B. Consequently, (V, τ^*, B) is compact.