β Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces

Saranya, M

(14PMA014)

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore - 641 043

In Partial Fulfilment of the Requirements for the Degree of

Master of Science in Mathematics

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Signature of the Supervisor

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INTRODUCTION

Intuitionistic fuzzy topology is considered to be an interesting as well as rapidly growing field in Mathematics. Before a decade only a handful number of research works were being undertaken in this concept but now intuitionistic fuzzy topology has become one among the most famous research areas in Mathematics. The origin of this particular concept is fuzzy sets. Ever since the establishment of fuzzy sets by Zadeh [1965], fuzzy has invaded almost all branches of Mathematics. Later the introduction of fuzzy topology by Chang [1967] was an annexation towards the hike of fuzzy sets and fuzzy topology. The perception of intuitionistic fuzzy sets by Atanassov [1986] was a breakthrough towards the evolution of intuitionistic fuzzy topology. Using the notion of intuitionistic fuzzy sets, Coker [1997] has constructed the basic concepts of intuitionistic fuzzy topological spaces. We are well aware of the existence of sets and fuzzy sets. Now let us proceed to know the necessity and importance of introducing intuitionistic fuzzy sets in the field of mathematics.

"Intuitionistic fuzzy sets" occupies our daily lives in the following way.

Consider a garment manufacturing company which manufactures about 2000 garments per day. Out of the 2000 manufactured garments 1750 are being disturbed to the dealers for sale. Then what about the remaining 250 garments?

Have they not been sold out?

Sets and fuzzy sets say "YES" they have not been sold. But this need not be case every time, because there may be customers who come directly to the garment company to make their purchase. Here comes "INTUITIONISTIC FUZZY SETS". Intuitionistic fuzzy sets contributes to the fact that the statement, "the remaining count of garments were not sold out" is not true. Rather it reveals that this count may also include the garments which were directly purchased by the customers at the company, the garments which were circulated to the external designer agencies for additional designing, the garments which were defective and hence forwarded for rework and finally the garments which were not sold out. For this accuracy of statistical values we have intuitionistic fuzzy sets.

In this thesis work a new class of intuitionistic fuzzy set namely "Intuitionistic fuzzy β generalized closed set" is being introduced. Further its corresponding open set, applications of intuitionistic fuzzy closed set, continuous mapping and contra continuous mapping are being introduced and their respective properties are discussed.

In Chapter I, the recent developments in intuitionistic fuzzy topology contributed by various authors are presented. This forms the basement for the remaining chapters of this thesis.

In Chapter II, intuitionistic fuzzy β generalized closed sets are introduced. The relationship between this newly introduced set and few of the already existing intuitionistic fuzzy sets is being discussed. Further some of the characterizations of this newly introduced set are discussed.

In Chapter III, β generalized open sets in intuitionistic fuzzy topological spaces are introduced and the relationship between this newly introduced set and some of the previously existing intuitionistic fuzzy sets are discussed.

In Chapter IV, intuitionistic fuzzy β_g $T_{1/2}$ spaces are introduced and some of the theoretical applications are discussed.

In Chapter V, intuitionistic fuzzy β generalized continuous mappings are introduced and the liaison of it with some of the previously existing continuous mappings in intuitionistic fuzzy topological spaces are investigated. Further some fascinating theorems concerning intuitionistic fuzzy β generalized continuous mappings are discussed.

In Chapter VI, intuitionistic fuzzy contra β generalized continuous mappings are introduced which is followed by the relationship of it with some of the already existing contra continuous mappings in intuitionistic fuzzy topological spaces. Further few interesting theorems concerning it are discussed.

Throughout this thesis (X, τ), (Y, σ) and (Z, δ) denote the intuitionistic fuzzy topological spaces on which no separation axioms are assumed unless otherwise explicitly mentioned.

REVIEW OF LITERATURE

Intuitionistic fuzzy topology is a challenging area of mathematics field. Atanassov [1986] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [1997] introduced the notion of intuitionistic fuzzy topological spaces. Intuitionistic fuzzy semi closed sets, intuitionistic fuzzy pre closed sets, intuitionistic fuzzy α closed sets, intuitionistic fuzzy β closed sets were introduced by Gurcay, Coker and Haydar [1997]. Joung kon Jeon, Young Bae Jun and Jin Han Park [2005], introduced intuitionistic fuzzy semi pre closed sets, intuitionistic fuzzy semi pre open sets in intuitionistic fuzzy topological spaces. Krsteska and Ekici [2007], introduced the intuitionistic fuzzy contra continuous mapping, intuitionistic fuzzy contra pre continuous mapping and intuitionistic fuzzy contra strongly pre continuous mappings. Jayanthi, D [2014] has introduced intuitionistic fuzzy generalized β continuous mappings.

1. FUZZY SETS

[Lotfi. Zadeh., 1965]

In this article, the author has introduced a new class of sets namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement, etc with respect to the fuzzy sets.

2. INTUITIONISTIC FUZZY SETS

[Krassimir T. Atanassov., 1986]

In this article, the author has provided the notion of intuitionistic fuzzy sets. This is considered to be the generalization on fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

3. FUZZY TOPOLOGICAL SPACES

[C. L. Chang., 1968]

In this article, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. In brief, the basic concepts such as fuzzy open set, fuzzy closed set, fuzzy neighbourhood, fuzzy continuity etc., are discussed in depth.

4. INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Dogan Coker., 1997]

In this article, the author has introduced intuitionistic fuzzy topological space. The notions of intuitionistic fuzzy interior and intuitionistic fuzzy closure are being provided and this is followed by the discussion of some important properties concerning them. Furthermore, the notion of intuitionistic fuzzy continuity is provided.

5. ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[H. Gurcay., D. Coker., and Haydar Es. A., 1997]

This article consists of the notions of intuitionistic fuzzy semiopen, intuitionistic fuzzy preopen, intuitionistic fuzzy α-open sets, intuitionistic fuzzy β open sets and their corresponding closed sets. Further the relationship between these sets are established.

6. INTUITIONISTIC FUZZY ALPHA – CONTUINUITY AND INTUITIONISTIC FUZZY PRECONTUINUITY

[Joung Kon Jeon., Young Bae Jun., and Jin Han Park., 2005]

In this article, the notion of intuitionistic fuzzy point is provided. This is followed by the discussion of the relationship between some of the previously defined intuitionistic fuzzy sets.

7. INTUTIONISTIC FUZZY SEMI-PRE OPEN SETS AND INTUITIONISTIC FUZZY SEMI-PRE CONTINUOUS MAPPINGS

[Young Bae Jun., and Seok – Zun Song., 2005]

This article consists of the notion of intuitionistic fuzzy semi-preopen sets and its corresponding closed sets. The properties regarding the union and intersection of theses sets are portrayed. Also, the relationship between this new class of sets and some of the previously existing sets are discussed.

8. RELATION BETWEEN SEMIPRECLOSED SETS AND BETA CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Jayanthi, D., 2013]

This article is considered to be a significant one as it clearly distinguishes intuitionistic fuzzy semipre closed sets and beta closed sets. The author has additionally provided the relationship between these two sets.

9. GENERALIZED CONTUINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Thakur, S.S., and Rekha Chaturvedi., 2006]

In this paper, the authors have discussed and studied the concept of intuitionistic fuzzy generalized continuous mappings in intuitionistic fuzzy topological spaces. They analysed some of their properties and obtained some interesting theorems.

10. INTUTIONISTIC FUZZY CONTRA STRONG PRECONTINUITY

[Biljana Krsteska., and Erdal Ekici., 2007]

This article consists of the notion of intuitionistic fuzzy contra strong pre continuity. Additionally the authors have presented the notions of intuitionistic fuzzy contra continuous mapping, intuitionistic fuzzy contra continuous mapping, intuitionistic fuzzy contra alpha continuous mapping.

CHAPTER I

PRELIMINARIES

Definition 1.1: [1] Let X be a non empty set. A ***fuzzy set*** A in X can be described in the form

$$A = \{\langle x, \mu_A(x)\rangle / x \in X\}$$

where the function $\mu_A : X \to [0,1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and $0 \le \mu_A(x) \le 1$ for each $x \in X$.

Definition 1.2: [1] An ***intuitionistic fuzzy set*** (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 1.3: [1] Let A and B be two IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 1.4: [3] An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an ***intuitionistic fuzzy closed set*** (IFCS in short) in X.

Definition 1.5: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the ***intuitionistic fuzzy interior*** and ***intuitionistic fuzzy closure*** are defined by

$$\mathrm{int}(A) = \cup \{G \,/\, G \text{ is an IFOS in X and } G \subseteq A\},$$

$$\mathrm{cl}(A) = \cap \{K \,/\, K \text{ is an IFCS in X and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $\mathrm{cl}(A^c) = (\mathrm{int}(A))^c$ and $\mathrm{int}(A^c) = (\mathrm{cl}(A))^c$.

Definition 1.6: [5] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) ***intuitionistic fuzzy semi closed set*** (IFSCS for short) if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$.

(ii) ***intuitionistic fuzzy pre closed set*** (IFPCS for short) if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$.

(iii) ***intuitionistic fuzzy α closed set*** (IFαCS for short) if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))) \subseteq A$.

(iv) ***intuitionistic fuzzy regular closed set*** (IFRCS for short) if $A = \mathrm{cl}(\mathrm{int}(A))$.

Definition 1.7: [5] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) ***intuitionistic fuzzy semi open set*** (IFSOS for short) if $A \subseteq cl(int(A))$.

(ii) ***intuitionistic fuzzy pre open set*** (IFPOS for short) if $A \subseteq int(cl(A))$.

(iii) ***intuitionistic fuzzy α open set*** (IFαOS for short) if $A \subseteq int(cl(int(A)))$.

(iv) ***intuitionistic fuzzy regular open set*** (IFROS for short) if $A = int(cl(A))$.

Definition 1.8: [5] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) ***intuitionistic fuzzy β closed set*** (IFβCS for short) if $int(cl(int(A))) \subseteq A$.

(ii) ***intuitionistic fuzzy β open set*** (IFβOS for short) if $A \subseteq cl(int(cl(A)))$.

Definition 1.9: [13] An IFS A in an IFTS (X, τ) is an ***intuitionistic fuzzy generalized closed set*** (IFGCS for short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 1.10: [15] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) ***intuitionistic fuzzy semi-pre closed set*** (IFSPCS for short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$.

(ii) ***intuitionistic fuzzy semi-pre open set*** (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) such that $B \subseteq A \subseteq cl(B)$.

Definition 1.10: [13] Two IFSs A and B are said to be ***q-coincident*** ($A_q B$ in short) if and only if there exits an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 1.11: [13] Two IFSs A and B are said to be ***not q-coincident*** ($A\, {}_q^c\, B$ in short) if and only if $A \subseteq B^c$.

Definition 1.12: [4] An ***intuitionistic fuzzy point*** (IFP for short), written as $p_{(\alpha,\beta)}$, is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 1.13: [12] An IFS A in (X, τ) is an IFQ-set if int(cl(A)) = cl(int(A))

Definition 1.14: [13] An IFTS (X, τ) is said to be an ***IFT$_{1/2}$ space*** if every IFGCS in (X, τ) is an IFCS in (X, τ).

Definition 1.15: [7] Let A be an IFS in an IFTS (X, τ). Then the β-interior and β-closure of A are defined as

$$\beta int(A) = \cup \{G / G \text{ is an IF}\beta\text{OS in X and } G \subseteq A\},$$

$$\beta cl(A) = \cap \{K / K \text{ is an IF}\beta\text{CS in X and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $\beta cl(A^c) = (\beta int(A))^c$ and $\beta int(A^c) = (\beta cl(A))^c$.

Result 1.16: Let A be an IFS in (X, τ). Then

(i) $\beta cl(A) \supseteq A \cup int(cl(int(A)))$

(ii) $\beta int(A) \subseteq A \cap cl(int(cl(A)))$

Proof: (i) Now $int(cl(int(A))) \subseteq int(cl(int(\beta cl(A))) \subseteq \beta cl(A)$, since $A \subseteq \beta cl(A)$ and $\beta cl(A)$ is an IFβCS. Therefore $A \cup int(cl(int(A))) \subseteq \beta cl(A)$.

(ii) can be proved easily by taking complement in (i).

Definition 1.17: [3] Let X and Y be two non empty sets and f: X → Y be a function. If $B = \{\langle y, (\mu_B (y), \nu_B (y) / y \in Y\rangle\}$ is an IFS in Y, then the ***preimage*** of B under f is denoted and defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X\rangle\}$$

where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$.

Definition 1.18: [3] Let (X, τ) be an IFTS and A, B be intuitionistic fuzzy in X. Then the following properties hold

(i) $int(A) \subseteq A$

(ii) $A \subseteq cl(A)$

(iii) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$

(iv) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$

(v) $int(int(A)) = int(A)$

(vi) $cl(cl(A)) = cl(A)$

(vii) $int(A \cap B) = int(A) \cap int(B)$

(viii) $cl(A \cup B) = cl(A) \cup cl(B)$

(ix) $int(1_\sim) = 1_\sim$

(x) $cl(0_\sim) = 0_\sim$

Definition 1.19: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy continuous*** (IF continuous for short) ***mapping*** if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

Definition 1.20: [8] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(i) ***intuitionistic fuzzy semi continuous*** (IFS continuous for short) ***mapping*** if $f^{-1}(B) \in IFSO(X)$ for every $B \in \sigma$

(ii) ***intuitionistic fuzzy α- continuous*** (IFα continuous for short) ***mapping*** if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$

(iii) ***intuitionistic fuzzy pre continuous*** (IFP continuous for short) ***mapping*** if $f^{-1}(B) \in IFPO(X)$ for every $B \in \sigma$

Definition 1.21: [15] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy semi-pre continuous*** (IFSP continuous for short) mapping if $f^{-1}(B) \in IFSPO(X)$ for every $B \in \sigma$

Every IFS continuous mapping and IFP continuous mappings are IFSP continuous mapping but the converses may not be true in general [13].

Definition 1.22: [9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(i) ***intuitionistic fuzzy contra continuous*** (IFC continuous for short) ***mapping*** if $f^{-1}(B) \in IFO(X)$ for each IFCS B in Y

(ii) ***intuitionistic fuzzy contra α- continuous*** (IFCα continuous for short) ***mapping*** if $f^{-1}(B) \in IF\alpha O(X)$ for each IFCS B in Y

(iii) ***intuitionistic fuzzy contra pre continuous*** (IFCP continuous for short) ***mapping*** if $f^{-1}(B) \in IFPO(X)$ for each IFCS B in Y

Corollary 1.23: [3] Let A, $A_i (i \in J)$ be intuitionistic fuzzy sets in X and B, $B_j (j \in K)$ be intuitionistic fuzzy sets in Y and f: X → Y be a function. Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]

d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]

e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

g) $f^{-1}(0\sim) = 0\sim$

h) $f^{-1}(1\sim) = 1\sim$

i) $f^{-1}(B^c) = (f^{-1}(B))^c$

Definition 1. 24 : [3] Let A , B and C be intuitionistic fuzzy sets in X. Then

(i) $(A \subseteq B)$ and $(C \subseteq D) \Rightarrow (A \cup C) \subseteq (B \cup D)$ and $(A \cap C) \subseteq (B \cap D)$

(ii) $A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq (B \cap C)$

(iii) $A \subseteq C$ and $B \subseteq C \Rightarrow (A \cup B) \subseteq C$

(iv) $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$

(v) $(A \cup B)^c = A^c \cap B^c$

(vi) $(A \cap B)^c = A^c \cup B^c$

(vii) $A \subseteq B \Rightarrow B^c \subseteq A^c$

(viii) $(A^c)^c = A$

(ix) $(0\sim)^c = 1\sim$

(x) $(1\sim)^c = 0\sim$

CHAPTER II

INTUITIONISTIC FUZZY β GENERALIZED CLOSED SETS

In this section we have introduced intuitionistic fuzzy β generalized closed sets and studied some of their properties. Some interesting and important theorems are also obtained.

Definition 2.1: An IFS A in an IFTS (X, τ) is said to be an ***intuitionistic fuzzy β generalized closed set*** (IFβGCS for short) if βcl(A) ⊆ U whenever A ⊆ U and U is an IFβOS in (X, τ).

The complement A^c of an IFβGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy β generalized open set (IFβGOS in short) in X.

The family of all IFβGCSs of an IFTS (X, τ) is denoted by IFβGC(X).

Example 2.2: Let X = {a, b} and G = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then τ = {0~, G, 1~} is an IFT on X. Let A = $\langle x, (0.4_a, 0.3_b)\ (0.6_a, 0.7_b)\rangle$ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

We have A ⊆ G. As βcl(A) = A, βcl(A) ⊆ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X.

Theorem 2.3: Every IFCS in (X, τ) is an IFβGCS in (X, τ) but not conversely.

Proof: Let A be an IFCS. Therefore cl(A) = A. Let A ⊆ U and U be an IFβOS. Since βcl(A) ⊆ cl(A) = A ⊆ U, we have βcl(A) ⊆ U. Hence A is an IFβGCS in (X, τ).

Example 2.4: Let X = {a, b} and G = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then τ = {0~, G, 1~} is an IFT on X. Let A = $\langle x, (0.4_a, 0.3_b)\ (0.6_a, 0.7_b)\rangle$ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

We have $A \subseteq G$. As $\beta cl(A) = A$, $\beta cl(A) \subseteq G$, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFCS, since $cl(A) = G^c \neq A$.

Theorem 2.5: Every IFRCS in (X, τ) is an IFβGCS in (X, τ) but not conversely.

Proof: Let A be an IFRCS, by Definition 1.6. Since every IFRCS is an IFCS [13], by Theorem 2.3, A is an IFβGCS.

Example 2.6: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)\rangle$ be an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

We have $A \subseteq G$. As $\beta cl(A) = A$, $\beta cl(A) \subseteq G$, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFRCS, since $cl(int(A)) = cl(0\sim) = 0\sim \neq A$.

Theorem 2.7: Every IFSCS in (X, τ) is an IFβGCS in (X, τ) but not conversely.

Proof: Assume A is an IFSCS, by Definition 1.6. Let $A \subseteq U$ and U be an IFβOS. Since $\beta cl(A) \subseteq scl(A) = A$ and $A \subseteq U$, by hypothesis, we have $\beta cl(A) \subseteq U$. Hence A is an IFβGCS.

Example 2.8: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)\rangle$ be an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

We have $A \subseteq G$. As $\beta cl(A) = A$, $\beta cl(A) \subseteq G$, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFSCS, since $int(cl(A)) = int(G^c) = G \nsubseteq A$.

Theorem 2.9: Every IFαCS in (X, τ) is an IFβGCS in (X, τ) but not conversely.

Proof: Assume A is an IFαCS, by Definition 1.6. Let $A \subseteq U$ and U be an IFβOS. Since $\beta cl(A) \subseteq \alpha cl(A) = A$ and $A \subseteq U$, by hypothesis, we have $\beta cl(A) \subseteq U$. Hence A is an IFβGCS.

Example 2.10: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0_\sim, G, 1_\sim\}$ is an IFT on X. Let $A = \langle x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)\rangle$ be an IFS in X.

Then, $IF\beta C(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

We have $A \subseteq G$. As $\beta cl(A) = A$, $\beta cl(A) \subseteq G$, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFαCS, since $cl(int(cl(A))) = cl(int(G^c)) = cl(G) = G^c \nsubseteq A$.

Theorem 2.11: Every IFPCS in (X, τ) is an IFβGCS in (X, τ) but not conversely.

Proof: Assume A is an IFPCS, by Definition 1.6. Let $A \subseteq U$ and U be an IFβOS. Since $\beta cl(A) \subseteq pcl(A) = A$ and $A \subseteq U$, by hypothesis, we have $\beta cl(A) \subseteq U$. Hence A is an IFβGCS.

Example 2.12: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, Then $\tau = \{0_\sim, G, 1_\sim\}$ is an IFT on X. Let $A = \langle x, (0.5_a, 0.7_b) (0.5_a, 0.3_b)\rangle$ be an IFS in X.

Then, $IF\beta C(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, \mu_b < 0.6 \text{ whenever } \mu_a \geq 0.5, \mu_a < 0.5 \text{ whenever } \mu_b \geq 0.6, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Now $A \subseteq 1_\sim$. As $\beta cl(A) = 1_\sim \subseteq 1_\sim$, we have A is an IFβGCS in X, but not an IFPCS since $cl(int(A)) = cl(G) = 1_\sim \nsubseteq A$.

Remark 2.13: Every IFGCS and every IFβGCS are independent to each other.

Example 2.14: Let $X = \{a, b\}$ and $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle x,(0.3_a, 0.1_b),(0.7_a, 0.8_b)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on X. Let $A = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ be an IFS in X. Then $A \subseteq G_1$ and $cl(A) = G_1^c \subseteq G_1$. Therefore A is an IFGCS in X.

Now $IF\beta C(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, \text{either } \mu_a \geq 0.5 \text{ and } \mu_b \geq 0.5 \text{ or } \mu_a < 0.3 \text{ and } \mu_b < 0.1, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1 \}$.

Since $A \subseteq G_1$ where G_1 is an IFβOS in X, but $\beta cl(A) = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)\rangle \nsubseteq A$, A is not an IFβGCS.

Example 2.15: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

We have A ⊆ G. As βcl(A) = A, βcl(A) ⊆ G, where G is an IFβOS in X. This implies that A is an IFβGCS in X, but not an IFGCS in X, since cl(A) = $G^c \nsubseteq G$.

Theorem 2.16: Every IFβCS in (X, τ) is an IFβGCS in (X, τ) but not conversely.

Proof: Assume A is an IFβCS, by Definition 1.8, then βcl(A) = A. Let A ⊆ U and U be an IFβOS. Then βcl(A) ⊆ U, by hypothesis. Therefore A is an IFβGCS.

Example 2.17: Let X = {a, b} and G = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩, then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨x, (0.5_a, 0.8_b) (0.5_a, 0.2_b)⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / provided $\mu_b < 0.7$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$ }.

Now A ⊆ 1 ~ and βcl(A) = 1~ ⊆ 1~. This implies that A is an IFβGCS in X, but not an IFβCS, since int(cl(int(A))) = int(cl(G)) = int(1 ~) = 1 ~ ⊈ A.

Theorem 2.18: Every IFSPCS in (X, τ) is an IFβGCS in (X, τ) but not conversely.

Proof: Assume A is an IFSPCS, by Definition 1.10. Since every IFSPCS is an IFβCS [6], by Theorem 2.16, A is an IFβGCS.

Example 2.19: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0 ~, G, 1 ~} is an IFT on X. Let A = ⟨x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Here A is an IFβCS in X. As int(cl(int(A))) = 0~ ⊆ A. Therefore A is an IFβGCS in X.

Since IFPC(X) = {0 ~, 1 ~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

But A is not an IFSPCS in X, as we cannot find any IFPCS B such that int(B) $\subseteq$ A $\subseteq$ B in X.

In the following diagram, we have provided relations between various types of intuitionistic fuzzy closedness.

$$\begin{tikzcd}
 & \text{IFCS} \arrow[r] \arrow[d] \arrow[dr] & \text{IFGCS} \arrow[dr, leftrightarrow, "/" marking] & \\
\text{IFRCS} \arrow[ur] \arrow[r] \arrow[dr] & \text{IF}\alpha\text{CS} \arrow[r] \arrow[d] & \text{IFPCS} \arrow[r] \arrow[d] & \text{IF}\beta\text{GCS} \\
 & \text{IFSCS} \arrow[r] & \text{IFSPCS} \arrow[r] & \text{IF}\beta\text{CS} \arrow[u]
\end{tikzcd}$$

The reverse implications are not true in general in the above diagram.

Remark 2.20: The union of any two IFβGCS is not an IFβGCS in general as seen from the following example.

Example 2.21: Let X = {a, b} and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ where $G_1 = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$ and $G_2 = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$. Then the IFSs $A = \langle x, (0.6_a, 0.5_b)\ (0.4_a, 0.3_b)\rangle$ and $B = \langle x, (0.4_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ are IFβGCSs in (X, τ) but $A \cup B$ is not an IFβGCS in (X, τ).

Then $\text{IF}\beta\text{C}(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / provided $\mu_b < 0.7$ whenever $\mu_a \geq 0.6$, $\mu_a < 0.6$ whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

As $\beta cl(A) = A$, we have A is an IFβGCS in X and $\beta cl(B) = B$, we have B is an IFβGCS in X. Now $A \cup B = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)\rangle \subseteq G_1$, where G_1 is an IFβOS, but $\beta cl(A \cup B) = 1_\sim \nsubseteq G_1$.

Theorem 2.22: Let (X, τ) be an IFTS. Then for every $A \in$ IFβGC(X) and for every $B \in$ IFS(X), $A \subseteq B \subseteq \beta cl(A) \Rightarrow B \in$ IFβGC(X).

Proof: Let $B \subseteq U$ and U be an IFβOS. Then since, $A \subseteq B$, $A \subseteq U$. By hypothesis, $B \subseteq \beta cl(A)$. Therefore $\beta cl(B) \subseteq \beta cl(\beta cl(A)) = \beta cl(A) \subseteq U$, since A is an IFβGCS. Hence $B \in$ IFβGC(X).

Theorem 2.23: An IFS A of an IFTS (X, τ) is an IFβGCS if and only if $A\,{}_q{}^c\,F \Rightarrow \beta cl(A)\,{}_q{}^c\,F$ for every IFβCS F of X.

Proof: (Necessity): Let F be an IFβCS and $A\,{}_q{}^c\,F$, then $A \subseteq F^c$ [9], where F^c is an IFβOS. Then $\beta cl(A) \subseteq F^c$, by hypothesis. Hence again [13] $\beta cl(A)\,{}_q{}^c\,F$.

Sufficiency: Let U be an IFβOS such that $A \subseteq U$. Then U^c is an IFβCS and $A \subseteq (U^c)^c$. By hypothesis, $A\,{}_q{}^c\,U^c \Rightarrow \beta cl(A)\,{}_q{}^c\,U^c$. Hence by [13], $\beta cl(A) \subseteq (U^c)^c = U$. Therefore $\beta cl(A) \subseteq U$. Hence A is an IFβGCS.

Theorem 2.24: Let (X, τ) be an IFTS. Then every IFS in (X, τ) is an IFβGCS if and only if IFβO(X) = IFβC(X).

Proof : (Necessity): Suppose that every IFS in (X, τ) is an IFβGCS. Let $U \in$ IFβO(X), and by hypothesis, $\beta cl(U) \subseteq U \subseteq \beta cl(U)$. This implies $\beta cl(U) = U$. Therefore $U \in$ IFβC(X). Hence IFβO(X) $\subseteq$ IFβC(X). Let $A \in$ IFβC(X), then $A^c \in$ IFβO(X) $\subseteq$ IFβC(X). That is, $A^c \in$ IFβC(X). Therefore $A \in$ IFβO(X). Hence IFβC(X) $\subseteq$ IFβO(X). Thus IFβO(X) = IFβC(X).

Sufficiency: Suppose that IFβO(X) = IFβC(X). Let $A \subseteq U$ and U be an IFβOS. By hypothesis $\beta cl(A) \subseteq \beta cl(U) = U$, since $U \in$ IFβC(X). Therefore A is an IFβGCS in X.

Theorem 2.25: If A is an IFβOS and an IFβGCS in (X, τ) then A is an IFβCS in (X, τ).

Proof: Since $A \subseteq A$ and A is an IFβOS, by hypothesis, $\beta cl(A) \subseteq A$. But $A \subseteq \beta cl(A)$. Therefore $\beta cl(A) = A$. Hence A is an IFβCS.

Theorem 2.26: Let A be an IFβGCS in (X, τ) and $p_{(\alpha,\beta)}$ be an IFP in X such that $int(p_{(\alpha,\beta)})\,{}_q\,\beta cl(A)$, then $int(cl(int(p_{(\alpha,\beta)})))\,{}_q\,A$.

Proof: Let A be an IFβGCS and let $(int(p_{(\alpha,\beta)}))\,{}_q\,\beta cl(A)$.

Suppose $int(cl(int(p_{(\alpha,\beta)})))\ {}_{q}^{c}\ A$, since by [13] $A \subseteq [int(cl(int(p_{(\alpha,\beta)})))]^c$. This implies $[int(cl(int(p_{(\alpha,\beta)})))]^c$ is an IFβOS. Then by hypothesis,

$\beta cl(A) \subseteq [int(cl(int(p_{(\alpha,\beta)})))]^c$

$= cl(int(cl[(p_{(\alpha,\beta)})]^c.$

$\subseteq cl(cl[(p_{(\alpha,\beta)})]^c.$

$= cl[(p_{(\alpha,\beta)})]^c.$

$= (int(p_{(\alpha,\beta)}))^c$. This implies $int(p_{(\alpha,\beta)})\ {}_{q}^{c}\ \beta cl(A)$, which is a contradiction to the hypothesis. Hence $int(cl(int(p_{(\alpha,\beta)})))\ {}_{q}\ A$.

Theorem 2.27: Let F ⊆ A ⊆ X where A is an IFβOS and an IFβGCS in X. Then F is an IFβGCS in A if and only if F is an IFβGCS in X.

Proof: Necessity: Let U be an IFβOS in X and F ⊆ U. Also let F be an IFβGCS in A. Then clearly F ⊆ A ∩ U and A ∩ U is an IFβOS in A. Hence the β closure of F in A, $\beta cl_A(F) \subseteq A \cap U$. By Theorem 2.25, A is an IFβCS. Therefore βcl(A) = A and the β closure of F in X, $\beta cl(F) \subseteq \beta cl(F) \cap \beta cl(A) = \beta cl(F) \cap A = \beta cl_A(F) \subseteq A \cap U \subseteq U$. That is, βcl(F) ⊆ U whenever F ⊆ U. Hence F is an IFβGCS in X.

Sufficiency: Let V be an IFβOS in A such that F ⊆ V. Since A is an IFβOS in X, V is an IFβOS in X. Therefore βcl(F) ⊆ V, since F is an IFβGCS in X. Thus $\beta cl_A(F) = \beta cl(F) \cap A \subseteq V \cap A \subseteq V$. Hence F is an IFβGCS in A.

Theorem 2.28: For an IFS A, the following conditions are equivalent:

(i) A is an IFOS and an IFβGCS

(ii) A is an IFROS

Proof: (i) ⇒ (ii) Let A be an IFOS and an IFβGCS. Then βcl(A) ⊆ A and A ⊆ βcl(A) this implies that βcl(A) = A. Therefore A is an IFβCS, since int(cl(int(A))) ⊆ A. Since A is an IFOS, int(A) = A. Therefore int(cl(A)) ⊆ A. Since A is an IFOS, it is an IFPOS. Hence A ⊆ int(cl(A)). Therefore A = int(cl(A)). Hence A is an IFROS.

(ii) ⇒ (i) Let A be an IFROS. Therefore A = int(cl(A)). Since every IFROS in an IFOS and A ⊆ A. This implies int(cl(A)) ⊆ A. That is int(cl(int(A))) ⊆ A. Therefore A is an IFβCS. Hence A is an IFβGCS.

Theorem 2.29: For an IFOS A in (X, τ), the following conditions are equivalent.

(i) A is an IFCS

(ii) A is an IFβGCS and an IFQ-set

Proof: (i) ⇒ (ii) Since A is an IFCS, it is an IFβGCS. Now int(cl(A)) = int(A) = A = cl(A) = cl(int(A)), by hypothesis. Hence A is an IFQ-set, by Definition 1.14.

(ii) ⇒ (i) Since A is an IFOS and an IFβGCS, by Theorem 2.28, A is an IFROS. Therefore A = int(cl(A)) = cl(int(A)) = cl(A), by hypothesis. A is an IFCS.

Theorem 2.30: Let (X, τ) be an IFTS, then for every A ∈ IFSPC(X) and for every B in X, int(A) ⊆ B ⊆ A ⇒ B ∈ IFβGC(X).

Proof: Let A be an IFSPCS in X. Then there exists an IFPCS, (say) C such that int(C) ⊆ A ⊆ C. By hypothesis, B ⊆ A. Therefore B ⊆ C. Since int(C) ⊆ A, int(C) ⊆ int(A) and int(C) ⊆ B, by hypothesis. Thus int(C) ⊆ B ⊆ C and by [5], B ∈ IFSPC(X). Hence by Theorem 2.18, B ∈ IFβGC(X).

CHAPTER III

INTUITIONISTIC FUZZY β GENERALIZED OPEN SETS

In this section we have investigated the basic properties of an intuitionistic fuzzy β generalized open sets and obtained some interesting results.

Example 3.1: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then τ = {0 ~, G, 1 ~} is an IFT on X. Let A = ⟨ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS.

Theorem 3.2: Every IFOS, IFROS [14], IFαOS [5], IFSOS [5], IFPOS [5], IFβOS [5] and IFSPOS [15] and is an IFβGOS but the converses are not true in general.

Proof: Straightforward.

Example 3.3: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then τ = {0 ~, G, 1 ~} is an IFT on X. Let A = ⟨ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS. But it is not an IFOS in X, since int(A) = G ≠ A.

Example 3.4: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then τ = {0 ~, G, 1 ~} is an IFT on X. Let A = ⟨ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS. But it is not an IFROS in X, since $int(cl(A)) = int(1\sim) = 1\sim \neq A$.

Example 3.5: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ be an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS. But it is not an IFαOS in X, since $int(cl(int(A))) = int(cl(G)) = int(G^c) = G \not\supseteq A$.

Example 3.6: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ be an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS. But it is not an IFSOS in X, since $cl(int(A)) = cl(G) = G^c \not\supseteq A$.

Example 3.7: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ be an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, \mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Now $A^c \subseteq 1\sim$. As $\beta cl(A^c) = 1\sim \subseteq 1\sim$. We have A^c is an IFβGCS in X and hence A is an IFβGOS in X. But it is not an IFPOS, since $A \not\subseteq int(cl(A)) = int(G^c) = 0\sim$.

Example 3.8: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.5_a, 0.2_b), (0.5_a, 0.8_b)\rangle$ be an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / provided $\mu_b < 0.7$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Now $A^c \subseteq 1\sim$ and $\beta cl(A^c) = 1\sim \subseteq 1\sim$. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS in X. But it is not an IFβOS, since $cl(int(cl(A))) = cl(int(G^c)) = cl(0\sim) = 0\sim \not\supseteq A$.

Example 3.9: Let X = {a, b} and G = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let A = $\langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ be an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Here A^c is an IFβCS in X, as $int(cl(int(A^c))) = 0\sim \subseteq A$. Therefore A^c is an IFβGCS in X and hence A is an IFβGOS.

Since IFPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

But A is not an IFSPOS in X, since there exists no IFPOS B such that $A \subseteq B \subseteq cl(A)$ in X.

Remark 3.10: Every IFGOS and every IFβGOS are independent to each other.

Example 3.11: Let X = {a, b} and $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle x, (0.3_a, 0.1_b), (0.7_a, 0.8_b)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ is an IFT on X. Let A = $\langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ be an IFS in X. Then $A^c \subseteq G_1$ and $cl(A^c) = G_1^c \subseteq G_1$.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_a \geq 0.5$ and $\mu_b \geq 0.5$ or $\mu_a < 0.3$ and $\mu_b < 0.1$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Therefore A^c is an IFGCS in X but it is not an IFβGCS in X and hence A is an IFGOS in X. But it is not an IFβGOS, since $\beta int(A) \not\supseteq U$ whenever $A \supseteq U$ and U is an IFβCS in (X, τ).

Example 3.12: Let X = {a, b} and G = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let A = $\langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ be an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

We have $A^c \subseteq G$. As βcl(A^c) = A^c, βcl(A^c) ⊆ G, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS in X. But it is not an IFGOS in X, since $G^c \not\subseteq$ int(A) whenever $G^c \subseteq A$, where G^c is an IFβCS in X.

Remark 3.13: The intersection of any two IFβGOS is not an IFβGOS in general as seen from the following example.

Example 3.14: Let X = {a, b} and τ = $\{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$ and $G_2 = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$. Then the IFSs $A = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $B = \langle x, (0.4_a, 0.2_b), (0.4_a, 0.8_b)\rangle$ are IFβGOSs in (X, τ) but A ∩ B is not an IFβGOS in (X, τ). Let us prove A^c and B^c are IFβGCS.

Then IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/$ provided $\mu_b < 0.7$ whenever $\mu_a \geq 0.6$, $\mu_a < 0.6$ whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

As βcl(A^c) = A^c, A^c is an IFβGCS in X and hence A is an IFβGOS in X, and βcl(B^c) = B^c, we have B^c is an IFβGCS in X and hence B is an IFβGOS in X.

Now to prove $A \cap B = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$ is an IFβGOS. Let us prove $(A \cap B)^c$ is an IFβGCS. Now since $(A \cap B)^c = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)\rangle \subseteq G_1$ but βcl $(A \cap B)^c = 1\sim \not\subseteq G_1$.

Therefore $(A \cap B)^c$ is not an IFβGCS in X and hence A ∩ B is not an IFβGOS in X.

Theorem 3.15: Let (X, τ) be an IFTS. Then for every A ∈ IFβGO(X) and for every B ∈ IFS(X), βint(A) ⊆ B ⊆ A ⇒ B ∈ IFβGO(X).

Proof: Let A be any IFβGOS of X and B be any IFS of X. Let βint(A) ⊆ B ⊆ A. Then A^c is an IFβGCS and $A^c \subseteq B^c \subseteq$ βcl(A^c). Therefore B^c is an IFβGCS [15] which implies B is an IFβGOS in X. Hence B ∈ IFβGO(X).

Theorem 3.16: An IFS A of an IFTS (X, τ) is an IFβGOS if and only if $F \subseteq \beta int(A)$ whenever F is an IFβCS and $F \subseteq A$.

Proof: Necessity: Suppose A is an IFβGOS. Let F be an IFβCS such that $F \subseteq A$. Then F^c is an IFβOS and $A^c \subseteq F^c$. By hypothesis A^c is an IFβGCS, we have $\beta cl(A^c) \subseteq F^c$. Therefore $F \subseteq \beta int(A)$.

Sufficiency: Let F be an IFβCS such that $F \subseteq A$ and $F \subseteq \beta int(A)$. Then $(\beta int(A))^c \subseteq F^c$ and $A^c \subseteq F^c$. This implies that $\beta cl(A^c) \subseteq F^c$, where F^c is an IFβOS. Therefore A^c is an IFβGCS. Hence A is an IFβGOS.

Theorem 3.17: Let (X, τ) be an IFTS. Then for every $A \in IFS(X)$ and for every $B \in IF\beta O(X)$, $B \subseteq A \subseteq int(cl(int(B))) \Rightarrow A \in IF\beta GO(X)$.

Proof: Let B be an IFβOS. Then $B \subseteq cl(int(cl(B)))$. By hypothesis, $A \subseteq int(cl(int(B))) \subseteq int(cl(int(cl(int(cl(B)))))) \subseteq int(cl(cl(int(cl(B))))) = int(cl(int(cl(B)))) \subseteq int(cl(cl(A))) \subseteq int(cl(A))$ as $B \subseteq A$. Therefore A is an IFPOS and by Theorem 3.2, A is an IFβGOS. Hence $A \in IF\beta GO(X)$.

Theorem 3.18: If A is an IFRCS and B is an IFβOS, then $A \cup B$ is an IFβGOS.

Proof: Let B be an IFβOS and A be an IFRCS. Then $B \subseteq cl(int(cl(B)))$ and $cl(int(A)) = A$. Therefore $A \cup B \subseteq A \cup (cl(int(cl(B))) = cl(int(A)) \cup cl(int(cl(B))) \subseteq cl(int(cl(A))) \cup cl(int(cl(B))) = cl(int(cl(A)) \cup int(cl(B))) \subseteq cl(int(cl(A) \cup cl(B))) \subseteq cl(int(cl(A \cup B)))$. Therefore $A \cup B$ is an IFβOS and by Theorem 3.2, $A \cup B$ is an IFβGOS.

Theorem 3.19: Let (X, τ) be an IFTS then for every $A \in IFSPO(X)$ and for every IFS B in X, $A \subseteq B \subseteq cl(A) \Rightarrow B$ IFβGO(X).

Proof: Let A be an IFSPOS in X. Then there exists an IFPOS, (say) C such that $C \subseteq A \subseteq cl(C)$. By hypothesis, $A \subseteq B$. Therefore $C \subseteq B$. Since $A \subseteq cl(C)$, $cl(A) \subseteq cl(C)$ and $B \subseteq cl(C)$, by hypothesis. Hence by [15], B is an IFSPOS. As every IFSPOS is an IFβGOS by Theorem 3.2, $B \in IF\beta GO(X)$.

Theorem 3.20: Let (X, τ) be an IFTS and $A, B \subset X$, If B is IFβGO(X) and $\beta int(B) \subset A$ then $A \cap B$ is IFβGO(X).

Proof: Since B is IFβGO(X) and $\beta int(B) \subset A$, $\beta int(B) \subset A \cap B \subset B$, by Theorem 3.15, $A \cap B$ is IFβGO(X).

CHAPTER IV

Applications of intuitionistic fuzzy β generalized closed sets

The concept of intuitionistic fuzzy β $T_{1/2}$ space was introduced by Jayanthi, D [7] in 2014. In this section we have discussed some applications of intuitionistic fuzzy β generalized closed sets.

Definition 4.1: If every IFβGCS in (X, τ) is an IFβCS in (X, τ), then the space can be called as an intuitionistic fuzzy β generalized $T_{1/2}$ space ($IF\beta_g T_{1/2}$ in short).

Example 4.2: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0 ~, G, 1 ~} is an IFT on X. Let A = ⟨x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Therefore the space (X, τ) is an intuitionistic fuzzy β generalized $T_{1/2}$ space, as every IFβGCS is an IFβCS in this (X, τ).

Definition 4.3: An IFTS (X, τ) is an intuitionistic fuzzy β generalized pre $T_{1/2}$ ($IF\beta_{gp}T_{1/2}$ in short) space if every IFβGCS is an IFPCS in X.

Example 4.4: Let X = {a, b} and G = ⟨x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)⟩. Then τ = {0 ~, G, 1 ~} is an IFT on X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Now, IFPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

The space (X, τ) is an intuitionistic fuzzy β generalized pre $T_{1/2}$ space, as every IFβGCS is an IFPCS in this (X, τ).

Definition 4.5: An IFTS (X, τ) is an intuitionistic fuzzy β generalized a $T_{1/2}$ ($IF\beta_{ga}T_{1/2}$ in short) space if every IFβGCS is an IFCS in X.

Definition 4.6: An IFTS (X, τ) is an intuitionistic fuzzy β generalized semi $T_{1/2}$ ($IF\beta_{gs}T_{1/2}$ in short) space if every IFβGCS is an IFSCS in X.

Definition 4.7: An IFTS (X, τ) is an intuitionistic fuzzy β generalized α $T_{1/2}$ ($IF\beta_{g\alpha}T_{1/2}$ in short) space if every IFβGCS is an IFαCS in X.

Theorem 4.8: Every $IF\beta_{gp}T_{1/2}$ space is an $IF\beta_{g}T_{1/2}$ space but not conversely.

Proof: Let (X, τ) be an $IF\beta_{gp}T_{1/2}$ space and let A be an IFβGCS in X. By hypothesis A is an IFPCS in X. Since every IFPCS is an IFβCS, A is an IFβCS in X. Hence (X, τ) is an $IF\beta_{g}T_{1/2}$ space.

Example 4.9: Let X = {a, b} and G = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩. Then τ = {0 ~, G, 1 ~} is an IFT on X.

Now, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$} and

IFβO(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a > 0.5$ whenever $\mu_b \leq 0.4$, $\mu_a \leq 0.5$ whenever $\mu_b > 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

The space (X, τ) is an intuitionistic fuzzy β generalized $T_{1/2}$ space, as every IFβGCS is an IFβCS in this (X, τ), but (X, τ) is not an $IF\beta_{gp}T_{1/2}$ space. Since A = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩ is an IFβGCS in (X, τ), but as cl(int(A)) = cl(G) = 1~ $\not\subseteq$ A, A is not an IFPCS.

Theorem 4.10: Every $IF\beta_{gs}T_{1/2}$ space is an $IF\beta_{g}T_{1/2}$ space but not conversely.

Proof: Let (X, τ) be an $IF\beta_{gs}T_{1/2}$ space and let A be an IFβGCS in X. By hypothesis A is an IFSCS in X. Since every IFSCS is an IFβCS, A is an IFβCS in X. Hence (X, τ) is an $IF\beta_{g}T_{1/2}$ space.

Example 4.11: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0~, G, 1~} is an IFT on X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

The space (X, τ) is an intuitionistic fuzzy β generalized $T_{1/2}$ space, as every IFβGCS is an IFβCS in this (X, τ), but (X, τ) is not an IF$\beta_{gs}T_{1/2}$ space. Since A = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFβGCS in (X, τ), but as int(cl(A)) = int(G^c) = G ⊈ A, A is not an IFSCS.

Theorem 4.12: Every IF$\beta_{g\alpha}T_{1/2}$ space is an IF$\beta_gT_{1/2}$ space but not conversely.

Proof: Let (X, τ) be an IF$\beta_{g\alpha}T_{1/2}$ space and let A be an IFβGCS in X. By hypothesis A is an IFαCS in X. Since every IFαCS is an IFβCS, A is an IFβCS in X. Hence (X, τ) is an IF$\beta_gT_{1/2}$ space.

Example 4.13: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0~, G, 1~} is an IFT on X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

The space (X, τ) is an intuitionistic fuzzy β generalized $T_{1/2}$ space, as every IFβGCS is an IFβCS in this (X, τ), but (X, τ) is not an IF$\beta_{g\alpha}T_{1/2}$ space. Since A = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFβGCS in (X, τ), but as cl(int(cl(A))) = cl(int(G^c)) = cl(G) = G^c ⊈ A, A is not an IFαCS.

Theorem 4.14: Every IF$\beta_{gsp}T_{1/2}$ space is an IF$\beta_gT_{1/2}$ space but not conversely.

Proof: Let (X, τ) be an IF$\beta_{gsp}T_{1/2}$ space and let A be an IFβGCS in X. By hypothesis A is an IFSPCS in X. Since every IFSPCS is an IFβCS, A is an IFβCS in X. Hence (X, τ) is an IF$\beta_gT_{1/2}$ space.

Example 4.15: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0~, G, 1~} is an IFT on X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

The space (X, τ) is an intuitionistic fuzzy β generalized $T_{1/2}$ space, as every IFβGCS is an IFβCS in this (X, τ).

Now IFPC(X) = {0 ~, 1 ~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Let A = ⟨x, (0.4_a, 0.3_b) (0.6_a, 0.7_b)⟩ be an IFS in X and A is not an IFSPCS in X, as we cannot find any IFPCS B such that int(B) ⊆ A ⊆ B in X. Therefore (X, τ) is not an IF$\beta_{gsp}T_{1/2}$ space.

Theorem 4.16: An IFTS (X, τ) is an IF$\beta_g T_{1/2}$ space if and only if IFβO(X) = IFβGO(X).

Proof: Necessity: Let A be an IFβGOS in (X, τ), then A^c is an IFβGCS in (X, τ). By hypothesis, A^c is an IFβCS in (X, τ) and therefore A is an IFβOS in (X, τ). Hence IFβO(X) = IFβGO(X).

Sufficiency: Let A be an IFβGCS in (X, τ). Then A^c is an IFβGOS in (X, τ). By hypothesis A^c is an IFβOS in (X, τ) and therefore A is an IFβCS in (X, τ). Hence (X, τ) is an IF$\beta_g T_{1/2}$ space.

Theorem 4.17: An IFTS (X, τ) is an IF$\beta_{ga}T_{1/2}$ space if and only if IFβGO(X) = IFO(X).

Proof: Necessity: Let A be an IFβGOS in (X, τ), then A^c is an IFβGCS in (X, τ). By hypothesis A^c is an IFCS in (X, τ). Hence A is an IFOS in (X, τ). Thus IFβGO(X) = IFO(X).

Sufficiency: Let A be an IFβGCS in (X, τ). Then A^c is an IFβGOS in (X, τ). By hypothesis A^c is an IFOS in (X, τ). Therefore A is an IFCS in (X, τ). Hence (X, τ) is an IF$\beta_{ga}T_{1/2}$ space.

Theorem 4.18: Let (X, τ) is an IF$\beta_g T_{1/2}$ space. Then

(i) Any union of IFβGCS is an IFβGCS,

(ii) Any intersection of IFβGOS is an IFβGOS.

Proof: (i) Let $\{A_i\}_{i \in J}$ be a collection of IFβGCS. Since (X, τ) is an IF$\beta_g T_{1/2}$ space, every IFβGCS is an IFβCS and hence each A_i, $i \in J$ is an IFβCS in (X, τ). But any union of IFβCS is an IFβCS [3], $\cup A_i$ for every $i \in J$ is an IFβCS. Since every IFβCS is an IFβGCS [7], $\bigcup_{i\in j} A_i$ is also an IFβGCS in X.

(ii) can be proved by taking complement in (i).

Remark 4.19: Not every IF$\beta_g T_{1/2}$ space is an IF$T_{1/2}$ space.

Example 4.20: Let $X = \{a, b\}$ and $G = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Since all IFβGCS in X are IFβCS in (X, τ) is an IF$\beta_g T_{1/2}$ space. But it is not an IF$T_{1/2}$ space since if $A = \langle x, (0.6_a, 0.7_b)\ (0.4_a, 0.3_b)\rangle$,then $cl(A) = 1\sim \subseteq 1\sim$ whenever $A \subseteq 1\sim$ and therefore A is an IFGCS in X but as $cl(A) = 1\sim \neq A$, A is not an IFCS in X. Therefore (X, τ) is not an IF$T_{1/2}$ space.

Theorem 4.21: For any IFS A in (X, τ) where X is an IF$\beta_g T_{1/2}$ space, $A \in IF\beta GO(X)$ if and only if for every IFP $p_{(\alpha,\beta)} \in A$, there exists an IFβGOS in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Proof: Necessity: If $A \in IF\beta GO(X)$, then we can take $B = A$ so that $p_{(\alpha,\beta)} \in B \subseteq A$ for every IFP $p_{(\alpha,\beta)} \in A$.

Sufficiency: Let A be an IFS in (X, τ) and assume that there exists $B \in IF\beta GO(X)$ such that $p_{(\alpha,\beta)} \in B \subseteq A$. Since X is an IF$\beta_g T_{1/2}$ space, B is an IFβOS. Then $A = \cup_{p_{(\alpha,\beta)} \in A} \{ p_{(\alpha,\beta)} \} \subseteq \cup_{p_{(\alpha,\beta)} \in A} B \subseteq A$. Therefore $A = \cup_{p_{(\alpha,\beta)} \in A} B$, which is an IFβOS. Hence A is an IFβGOS.

Theorem 4.22: Let (X, τ) be an IF$\beta_g T_{1/2}$ space, then the following conditions are equivalent:

(i) $A \in IF\beta GO(X)$

(ii) $A \subseteq cl(int(cl(A)))$

(iii) $cl(A) \in IFRC(X)$.

Proof: (i) ⇒ (ii) Let A be an IFβGOS. Then since X is an $IF\beta_gT_{1/2}$ space, A is an IFβOS. Therefore A ⊆ cl(int(cl(A))).

(ii) ⇒ (iii) Let A ⊆ cl(int(cl(A))). Then cl(A) ⊆ cl(cl(int(cl(A)))) = cl(int(cl(A))) ⊆ cl(cl(A)) = cl(A). Therefore cl(A) = cl(int(cl(A))). Hence cl(A) ∈ IFRC(X).

(iii) ⇒ (i) Since cl(A) is an IFRCS, cl(A) = cl(int(cl(A))) and since A ⊆ cl(A), A ⊆ cl(int(cl(A))). Therefore A is an IFβOS. Hence by Theorem 3.2, A ∈ IFβGO(X).

Theorem 4.23: Let (X, τ) be an IFTS and let X be an $IF\beta_gT_{1/2}$ space, then the following conditions are equivalent:

(i) A ∈ IFβGC(X)

(ii) int(cl(int(A))) ⊆ A

(iii) int(A) ∈ IFRO(X).

Proof: (i) ⇒ (ii) Let A be an IFβGCS. Then since X is an $IF\beta_gT_{1/2}$ space, A is an IFβCS. Therefore int(cl(int(A))) ⊆ A.

(ii) ⇒ (iii) Let int(cl(int(A))) ⊆ A. Then int(A) ⊇ int(int(cl(int(A)))) = int(cl(int(A))) ⊇ int(int(A) = int(A). Therefore int(cl(int(A))) = int(A). Hence int(A) ∈ IFRO(X).

(iii) ⇒ (i) Since int(A) is an IFROS, int(A) = int(cl(int(A))) and since int(A) ⊆ (A), int(cl(int(A))) ⊆ A. Therefore A is an IFβCS which implies A^c is an IFβOS. Hence by Theorem 3.2, A^c is an IFβGOS. Therefore A ∈ IFβGC(X).

CHAPTER V

Intuitionistic Fuzzy β Generalized Continuous Mappings

In this section we have introduced intuitionistic fuzzy β generalized continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced mapping and the already existing mappings.

Definition 5.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy β generalized continuous*** (IFβG continuous for short) **mapping** if $f^{-1}(V)$ is an IFβGCS in (X, τ) for every IFCS V of (Y, σ).

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in the following examples.

Similarly we shall use the notation $B = \langle y, (\mu_u, \mu_v), (\nu_u, \nu_v)\rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

Example 5.2: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $\text{IF}\beta\text{C}(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping.

Theorem 5.3: Every IF continuous mapping is an IFβG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFβGCS, by Theorem 2.3, $f^{-1}(V)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Example 5.4: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping but since $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is not an IFCS in X, as $cl(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$, f is not an IF continuous mapping.

Theorem 5.5: Every IFS continuous mapping is an IFβG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFβGCS, by Theorem 2.7, $f^{-1}(V)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Example 5.6: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping. We have $int(cl(f^{-1}(G_2^c))) = int(G_1^c) = G_1 \not\subseteq f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$. Hence $f^{-1}(G_2^c)$ is not an IFSCS in X. Hence f is not an IFS continuous mapping.

Theorem 5.7: Every IFP continuous mapping is an IFβG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFβGCS, by Theorem 2.11, $f^{-1}(V)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Example 5.8: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.3_v), (0.5_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping. We have $cl(int(f^{-1}(G_2^c))) = cl(G_1) = 1\sim \not\subseteq f^{-1}(G_2^c) = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$. Hence $f^{-1}(G_2^c)$ is not an IFPCS in X. Hence f is not an IFP continuous mapping.

Theorem 5.9: Every IFα continuous mapping is an IFβG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFβGCS, by Theorem 2.9, $f^{-1}(V)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Example 5.10: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping. We have $cl(int(cl(f^{-1}(G_2^c)))) = cl(int(G_1^c)) = cl(G_1) = G_1^c \not\subseteq f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$. Hence $f^{-1}(G_2^c)$ is not an IFαCS in X. Hence f is not an IFα continuous mapping.

Theorem 5.11: Let f: (X, τ) → (Y, σ) be a mapping and $f^{-1}(A)$ be an IFRCS in X for every IFCS A in Y. Then f is an IFβG continuous mapping but not conversely.

Proof: Let A be an IFCS in Y and $f^{-1}(A)$ be an IFRCS in X. Since every IFRCS is an IFβGCS, by Theorem 2.5, $f^{-1}(A)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Example 5.12: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping. We have $cl(int(f^{-1}(G_2^c))) = cl(0_\sim) = 0_\sim \neq f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$. Hence $f^{-1}(G_2^c)$ is not an IFRCS in X. Hence f is not continuous mapping as in Theorem 5.11.

Theorem 5.13: Every IFβ continuous mapping is an IFβG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFβ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFβCS in X. Since every IFβCS is an IFβGCS, by Theorem 2.16, $f^{-1}(V)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Example 5.14: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.2_v), (0.5_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.5_u, 0.8_v), (0.5_u, 0.2_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / provided $\mu_b < 0.7$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping. We have $int(cl(int(f^{-1}(G_2^c)))) = int(cl(G_1)) = int(1\sim) = 1\sim \nsubseteq f^{-1}(G_2^c) = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b)\rangle$. Hence $f^{-1}(G_2^c)$ is not an IFβCS in X. Hence f is not an IFβ continuous mapping.

Theorem 5.15: Every IFSP continuous mapping is an IFβG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFSP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. Since every IFSPCS is an IFβGCS, by Theorem 2.18, $f^{-1}(V)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Example 5.16: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping.

Since $IFPC(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Since there exists no IFPCS B in X such that $int(B) \subseteq f^{-1}(G_2^c) \subseteq B$, $f^{-1}(G_2^c)$ is not an IFSPCS in X. Hence f is not an IFSP continuous mapping.

Remark 5.17: IFG continuous mappings and IFβG continuous mappings are independent to each other.

Example 5.18: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2^c)$ is an IFβGCS in (X, τ). Therefore f is an IFβG continuous mapping. We have $cl(f^{-1}(G_2^c)) = G_1^c \nsubseteq G_1$. Hence $f^{-1}(G_2^c)$ is not an IFGCS in X. Hence f is not an IFG continuous mapping.

Example 5.19: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)\rangle$, $G_2 = \langle x, (0.3_a, 0.1_b), (0.7_a, 0.8_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is IFG continuous mapping, as $f^{-1}(G_3^c) \subseteq G_1$ and $cl(f^{-1}(G_3^c)) = G_1^c \subseteq G_1$.

Now $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/$ either $\mu_a \geq 0.5$ and $\mu_b \geq 0.5$ or $\mu_a < 0.3$ and $\mu_b < 0.1, 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

We have $f^{-1}(G_3^c) \subseteq G_1$ but $\beta cl(f^{-1}(G_3^c)) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle = 1\sim \nsubseteq G_1$. Hence $f^{-1}(G_3^c)$ is not an IFβGCS in X. Hence f is not an IFβG continuous mapping.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram ‘cts.’ means continuous.

$$\begin{tikzcd}
 & & \text{IFG cts.} \arrow[drr, "/" marking] & & \\
\text{IF cts.} \arrow[urr] \arrow[r] \arrow[dr] & \text{IF}\alpha\text{ cts.} \arrow[r] \arrow[d] & \text{IFP cts.} \arrow[rr] \arrow[d] & & \text{IF}\beta\text{G cts.} \\
 & \text{IFS cts.} \arrow[r] & \text{IFSP cts.} \arrow[r] & \text{IF}\beta\text{ cts.} \arrow[ur] &
\end{tikzcd}$$

Theorem 5.20: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFβG continuous mapping if and only if the inverse image of each IFOS in Y is an IFβGOS in X.

Proof: (Necessity): Let A be an IFOS in Y. This implies A^c is IFCS in Y. Then $f^{-1}(A^c)$ is an IFβGCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβGOS in X.

(Sufficiency): Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis $f^{-1}(A^c)$ is IFβGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFβGOS in X. Therefore $f^{-1}(A)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Theorem 5.21: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFβG continuous mapping then for each IFP $p_{(\alpha,\beta)}$ of X and each $A \in \sigma$ such that $f(p_{(\alpha,\beta)}) \in A$, there exists an IFβGOS B of X such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and $A \in \sigma$ such that $f(p_{(\alpha,\beta)}) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFβGOS in X such that $p_{(\alpha,\beta)} \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 5.22: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFβG continuous mapping then for each IFP $p_{(\alpha,\beta)}$ of X and each $A \in \sigma$ such that $f(p_{(\alpha,\beta)})\ {}_q \in A$, there exists an IFβGOS B of X such that $p_{(\alpha,\beta)\,q}\ B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and $A \in \sigma$ such that $f(p_{(\alpha,\beta)})\ {}_q\ A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is an IFβGOS in X such that $p_{(\alpha,\beta)\,q}\ B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 5.23: Let f: (X, τ) → (Y, σ) be an IFβG continuous mapping, then f is an IFβ continuous mapping if X is an IFβg$T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then f^{-1} (V) is an IFβGCS in X, by hypothesis. Since X is an IFβg$T_{1/2}$ space, f^{-1} (V) is an IFβCS in X. Hence f is an IFβ continuous mapping.

Theorem 5.24: Let f: (X, τ) → (Y, σ) be an IFβG continuous mapping, then f is an IF continuous mapping if X is an IF$\beta_{ga}T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then f^{-1} (V) is an IFβGCS in X, by hypothesis. Since X is an IF$\beta_{ga}T_{1/2}$ space, f^{-1} (V) is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 5.25: Let f: (X, τ) → (Y, σ) be an IFβG continuous mapping and g: (Y, σ) → (Z, δ) is an IF continuous mapping then g∘ f : (X, τ) → (Z, δ) is an IFβG continuous mapping.

Proof: Let V be an IFCS in Z. Then g^{-1} (V) is an IFCS in Y, by hypothesis. Since f is an IFβG continuous mapping, f^{-1} (g^{-1} (V)) is an IFβGCS in X. Hence g∘f is an IFβG continuous mapping.

Theorem 5.26: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IF$\beta_gT_{1/2}$ spaces.

(i) f is an IFβG continuous mapping,

(ii) f^{-1}(B) is an IFβGOS in X for each IFOS B in Y,

(iii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$, there exists an IFβGOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

Proof: (i) ⇒ (ii) is obvious from the Theorem 5.20.

(ii) ⇒ (iii) Let B be any IFOS in Y and let $p_{(\alpha,\beta)} \in X$. Given $f(p_{(\alpha,\beta)}) \in B$. By hypothesis f^{-1}(B) is an IFβGOS in X. Take $A = f^{-1}(B)$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) = A$. This implies $p_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let A be an IFCS in Y. Then its complement, say B is an IFOS in Y. Let $p_{(\alpha,\beta)} \in X$ and $f(p_{(\alpha,\beta)}) \in B$. Then there exists an IFβGOS, say C in X such that $p_{(\alpha,\beta)} \in C$

and $f(C) \subseteq B$. Therefore $p_{(\alpha,\beta)} \in C \subseteq f^{-1}(B)$ and hence $f^{-1}(B)$ is an IFβGOS in X, by Theorem 2.9. That is $f^{-1}(A^c)$ is an IFβGOS in X and hence $f^{-1}(A)$ is an IFβGCS in X. Thus f is an IFβG continuous mapping.

Theorem 5.27: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFT Y that satisfies $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for every IFS B in Y. Then f is an IFβG continuous mapping.

Proof: Let B be an IFOS in Y. Then int(B) = B. By hypothesis $f^{-1}(B) \subseteq cl(int(cl(f^{-1}(B))))$. This implies $f^{-1}(B)$ is an IFβOS in X. Therefore it is an IFβGOS in X and hence f is an IFβG continuous mapping, by Theorem 5.20.

Theorem 5.28: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an $IF\beta_gT_{1/2}$ space:

(i) f is an IFβG continuous mapping,

(ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IFβGOS in X,

(iii) $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for every IFS B in Y.

Proof: (i) ⇒ (ii) is obviously true by Theorem 5.20.

(ii) ⇒ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFβGOS in X. Since X is an $IF\beta_gT_{1/2}$ space, $f^{-1}(int(B))$ is an IFβOS in X. Therefore $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(int(B))))) \subseteq cl(int(cl(f^{-1}(B))))$.

(iii) ⇒ (i) Let B be an IFCS in Y. Then its complement, say A is an IFOS in Y, then int(A) = A. Now by hypothesis $f^{-1}(int(A)) \subseteq cl(int(cl(f^{-1}(A))))$. This implies $f^{-1}(A) \subseteq cl(int(cl(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFβOS in X. Since every IFβOS is an IFβGOS, $f^{-1}(A)$ is an IFβGOS in X. Thus $f^{-1}(B)$ is an IFβGCS in X, since $f^{-1}(A) = f^{-1}(B^c)$. Hence f is an IFβG continuous mapping.

Theorem 5.29: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IF\beta_gT_{1/2}$ spaces:

(i) f is an IFβG continuous mapping,

(ii) $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(\beta cl(B))$ for each IFCS B in Y,

(iii) $f^{-1}(\beta int(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for each IFOS B of Y,

(iv) $f(int(cl(int(A)))) \subseteq cl(f(A))$ for each IFS A of X.

Proof: (i) ⇒ (ii) Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFβGCS in X. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(B)$ is an IFβCS. Therefore $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) = f^{-1}(\beta cl(B))$.

(ii) ⇒ (iii) can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let $A \in X$. Then $B = f(A)$ in Y and therefore $A \subseteq f^{-1}(B)$. Here $int(f(A)) = int(B)$ is an IFOS in Y. Then (iii) implies that $f^{-1}(\beta int(int(B))) \subseteq cl(int(cl(f^{-1}(int(B))))) \subseteq cl(int(cl(f^{-1}(B))))$. Now $(cl(int(cl(A^c))))^c \subseteq (cl(int(cl(f^{-1}(B)^c))))^c \subseteq (f^{-1}(\beta int(int(B^c))))^c$. Therefore $int(cl(int(A))) \subseteq f^{-1}(\beta cl(cl(B)))$. Now $f(int(cl(int(A)))) \subseteq f(f^{-1}(\beta cl(cl(B)))) \subseteq cl(B) = cl(f(A))$.

(iv) ⇒ (i) Let B be any IFCS in Y, then $f^{-1}(B)$ is an IFS in X. By hypothesis $f(int(cl(int(f^{-1}(B))))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B) = B$. Now $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(f(int(cl(int(f^{-1}(B)))))) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFβCS and hence it is an IFβGCS in X. Thus f is an IFβG continuous mapping.

Theorem 5.30: A mapping $f: (X, \tau) \to (Y, \sigma)$ is an IFβG continuous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: Let A be an IFOS in Y then A^c is an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(cl(A^c)) = f^{-1}(A^c)$, since A^c is an IFCS. Now $(int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(A^c) = (f^{-1}(A))^c$. This implies $f^{-1}(A) \subseteq int(cl(int(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFαOS and hence it is an IFβGOS. Therefore f is an IFβG continuous mapping, by Theorem 5.20.

Theorem 5.31: Let $f: (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IF$\beta_g T_{1/2}$ space.

(i) f is an IFβG continuous mapping,

(ii) $f^{-1}(B)$ is an IFβGCS in X for every IFCS B in Y,

(iii) $int(cl(int(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: (i) ⇒ (ii) is obvious from Definition 5.1.

(ii) ⇒ (iii) Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, f^{-1} (cl(A)) is an IFβGCS in X. Since X is an IF$\beta_g T_{1/2}$ space, f^{-1} (cl(A)) is an IFβCS. Therefore int(cl(int(f^{-1} cl(A))))) ⊆ f^{-1} (cl(A)). Now int(cl(int(f^{-1}(A)))) ⊆ int(cl(int(f^{-1} (cl(A))))) ⊆ f^{-1} (cl(A)).

(iii) ⇒ (i) Let A be an IFCS in Y. By hypothesis int(cl(int(f^{-1} (A)))) ⊆ f^{-1} (cl(A)) = f^{-1} (A). This implies f^{-1} (A) is an IFβCS in X and hence it is an IFβGCS. Thus f is an IFβG continuous mapping.

CHAPTER VI

Intuitionistic fuzzy contra β generalized continuous mappings

In this chapter, we have introduced intuitionistic fuzzy contra β generalized continuous mappings and investigated some of their properties.

Definition 6.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy contra β generalized continuous*** (IFCβG continuous for short) mapping if $f^{-1}(V)$ is an IFβGCS in (X, τ) for every IFOS V of (Y, σ).

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in the following examples.

Similarly we shall use the notation $B = \langle y, (\mu_u, \mu_v), (\nu_u, \nu_v)\rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

Example 6.2: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping.

Remark 6.3: Every IFC continuous mapping, IFCα continuous mapping, IFCP continuous mapping, IFCS continuous mapping, IFCβ continuous mapping and IFCSP continuous mapping are IFCβG continuous mapping but the converses are not true in general. This can be seen from the following examples.

Example 6.4: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_{a}, 0.4_{b}), (0.5_{a}, 0.6_{b})\rangle$, $G_2 = \langle y, (0.4_{u}, 0.3_{v}), (0.6_{u}, 0.7_{v})\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_{u}, 0.3_{v}), (0.6_{u}, 0.7_{v})\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_{a}, 0.3_{b}), (0.6_{a}, 0.7_{b})\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping but since $f^{-1}(G_2) = \langle x, (0.4_{a}, 0.3_{b}), (0.6_{a}, 0.7_{b})\rangle$ is not an IFCS in X, as $cl(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$, f is not an IFC continuous mapping.

Example 6.5: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_{a}, 0.4_{b}), (0.5_{a}, 0.6_{b})\rangle$, $G_2 = \langle y, (0.4_{u}, 0.3_{v}), (0.6_{u}, 0.7_{v})\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_{u}, 0.3_{v}), (0.6_{u}, 0.7_{v})\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_{a}, 0.3_{b}), (0.6_{a}, 0.7_{b})\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $cl(int(cl(f^{-1}(G_2)))) = cl(int(G_1^c)) = cl(G_1) = G_1^c \not\subseteq f^{-1}(G_2)$. Hence $f^{-1}(G_2)$ is not an IFαCS in X. Hence f is not an IFCα continuous mapping.

Example 6.6: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_{a}, 0.6_{b}), (0.5_{a}, 0.4_{b})\rangle$, $G_2 = \langle y, (0.5_{u}, 0.7_{v}), (0.5_{u}, 0.3_{v})\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.5_{u}, 0.7_{v}), (0.5_{u}, 0.3_{v})\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.5_{a}, 0.7_{b}), (0.5_{a}, 0.3_{b})\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, \mu_b < 0.6 \text{ whenever } \mu_a \geq 0.5, \mu_a < 0.5 \text{ whenever } \mu_b \geq 0.6, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $cl(int(f^{-1}(G_2))) = cl(G_1) = 1\sim \not\subseteq f^{-1}(G_2)$. Hence $f^{-1}(G_2)$ is not an IFPCS in X. Hence f is not an IFCP continuous mapping.

Example 6.7: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \ / \ 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $int(cl(f^{-1}(G_2))) = int(G_1^c) = G_1 \not\subseteq f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$. Hence $f^{-1}(G_2)$ is not an IFSCS in X. Hence f is not an IFCS continuous mapping.

Example 6.8: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.8_v), (0.5_u, 0.2_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_2 = \langle y, (0.5_u, 0.8_v), (0.5_u, 0.2_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \ /$ provided $\mu_b < 0.7$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $int(cl(int(f^{-1}(G_2)))) = int(cl(G_1)) = int(1\sim) = 1\sim \not\subseteq f^{-1}(G_2) = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b)\rangle$. Hence $f^{-1}(G_2)$ is not an IFβCS in X. Hence f is not an IFCβ continuous mapping.

Example 6.9: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and

$f(b) = v$. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping.

Since $IFPC(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ /either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$ and

There exists no IFPCS B in X such that $int(B) \subseteq f^{-1}(G_2) \subseteq B$, $f^{-1}(G_2)$ is not an IFSPCS in X. Hence f is not an IFCSP continuous mapping.

Remark 6.10: IFCG continuous mappings and IFCβG continuous mappings are independent to each other.

Example 6.11: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $cl(f^{-1}(G_2)) = G_1^c \nsubseteq G_1$. Hence $f^{-1}(G_2)$ is not an IFGCS in X. Hence f is not an IFCG continuous mapping.

Example 6.12: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)\rangle$, $G_2 = \langle x, (0.3_a, 0.1_b), (0.7_a, 0.8_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFCG continuous mapping, where G_3 is an IFOS in (Y, σ).

Now $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_a \geq 0.5$ and $\mu_b \geq 0.5$ or $\mu_a < 0.3$ and $\mu_b < 0.1$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

We have $f^{-1}(G_3) \subseteq G_1$ but $\beta cl(f^{-1}(G_3)) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle = 1_\sim \not\subseteq G_1$. Hence $f^{-1}(G_3)$ is not an IFβGCS in X. Hence f is not an IFCβG continuous mapping.

Theorem 6.13: A mapping f: (X, τ) → (Y, σ) is an IFCβG continuous mapping if and only if the inverse image of each IFCS in Y is an IFβGOS in X.

Proof: (Necessity): Let A be an IFCS in Y. This implies A^c is an IFOS in Y. Then $f^{-1}(A^c)$ is an IFβGCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβGOS in X.

(Sufficiency): Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis $f^{-1}(A^c)$ is IFβGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFβGOS in X. Therefore $f^{-1}(A)$ is an IFβGCS in X. Hence f is an IFCβG continuous mapping.

Theorem 6.14: Let f: (X, τ) → (Y, σ) be a mapping and let $f^{-1}(A)$ be an IFβOS in X for every IFCS A in Y. Then f is an IFCβG continuous mapping.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFβOS in X, by hypothesis. Since every IFβOS is an IFβGOS by Theorem 3.2, $f^{-1}(A)$ is an IFβGOS in X. Hence f is an IFCβG continuous mapping, by Theorem 6.13.

Theorem 6.15: Let f: (X, τ) → (Y, σ) be a bijective mapping. Suppose that one of the following properties hold:

(i) $f^{-1}(cl(B)) \subseteq int(\beta cl(f^{-1}(B)))$ for each IFS B in Y

(ii) $cl(\beta int(f^{-1}(B))) \subseteq f^{-1}(int(B))$ for each IFS B in Y

(iii) $f(cl(\beta int(A))) \subseteq int(f(A))$ for each IFS A in X

(iv) $f(cl(A)) \subseteq int(f(A))$ for each IFβOS A in X

Then f is an IFCβG continuous mapping.

Proof: (i) ⇒ (ii) is obvious by taking complement of (i).

(ii) ⇒ (iii) Let $A \subseteq X$. Put $B = f(A)$ in Y. This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in X. Now $cl(\beta int(A)) = cl(\beta int(f^{-1}(B))) \subseteq f^{-1}(int(B))$ by (ii). Therefore $f(cl(\beta int(A))) \subseteq f(f^{-1}(int(B))) = int(B) = int(f(A))$.

(iii) ⇒ (iv) Let $A \subseteq X$ be an IFβOS. Then $\beta int(A) = A$. By hypothesis, $f(cl(\beta int(A))) \subseteq int(f(A))$. Therefore $f(cl(A)) = f(cl(\beta int(A))) \subseteq int(f(A))$.

Suppose (iv) holds. Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFS in X and $\beta int(f^{-1}(A))$ is an IFβOS in X. Hence by hypothesis, $f(cl(\beta int(f^{-1}(A)))) \subseteq int(f(\beta int(f^{-1}(A)))) \subseteq int(f(f^{-1}(A))) = int(A) \subseteq A$. Therefore $cl(\beta int(f^{-1}(A))) = f^{-1}(f(cl(\beta int(f^{-1}(A))))) \subseteq f^{-1}(A)$. Now $cl(int(f^{-1}(A))) \subseteq cl(\beta int(f^{-1}(A))) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFPCS in X and hence an IFβGCS in X, by Theorem 6.11. Thus f is an IFCβG continuous mapping.

Theorem 6.16: Let $f: (X, \tau) \to (Y, \sigma)$ be a mapping. Suppose that one of the following properties hold:

(i) $f(\beta cl(A)) \subseteq int(f(A))$ for each IFS A in X

(ii) $\beta cl(f^{-1}(B)) \subseteq f^{-1}(int(B))$ for each IFS B in Y

(iii) $f^{-1}(cl(B)) \subseteq \beta int(f^{-1}(B))$ for each IFS B in Y

Then f is an IFCβG continuous mapping.

Proof: (i) ⇒ (ii) Let $B \subseteq Y$. Then $f^{-1}(B)$ is an IFS in X. By hypothesis, $f(\beta cl(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. Now $\beta cl(f^{-1}(B)) \subseteq f^{-1}(f(\beta cl(f^{-1}(B)))) \subseteq f^{-1}(int(B))$.

(ii) ⇒ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. Now $f^{-1}(A) = f^{-1}(cl(A)) \subseteq \beta int(f^{-1}(A)) \subseteq f^{-1}(A)$, by hypothesis. This implies $f^{-1}(A)$ is an IFβOS in X and hence an IFβGOS in X, by Theorem 3.2. Therefore f is an IFCβG continuous mapping.

Theorem 6.17: Let $f: (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then f is an IFCβG continuous mapping if $cl(f(A)) \subseteq f(\beta int(A))$ for every IFS A in X.

Proof: Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. By hypothesis $cl(f(f^{-1}(A))) \subseteq f(\beta int(f^{-1}(A)))$. Since f is an onto, $f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl(f(f^{-1}(A))) \subseteq f(\beta int(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(\beta int(f^{-1}(A)))) = \beta int(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFβOS in X and hence an IFβGOS in X, by Theorem 3.2. Thus f is an IFCβG continuous mapping.

Theorem 6.18: If f: (X, τ) → (Y, σ) is an IFCβG continuous mapping, where X is an IF$\beta_g T_{1/2}$ space, then the following conditions hold:

(i) $\beta cl(f^{-1}(B)) \subseteq f^{-1}(int(\beta cl(B)))$ for every IFOS in Y,

(ii) $f^{-1}(cl(\beta int(B))) \subseteq \beta int(f^{-1}(B))$ for every IFCS B in Y.

Proof: (i) Let $B \subseteq Y$ be an IFOS. By hypothesis $f^{-1}(B)$ is an IFβGCS in X. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(B)$ is an IFβCS in X. This implies $\beta cl(f^{-1}(B)) = f^{-1}(B) \subseteq f^{-1}(int(B)) \subseteq f^{-1}(int(\beta cl(B)))$.

(ii) can be proved easily by taking the complement of (i).

Theorem 6.19: If f: (X, τ) → (Y, σ) is an IFCβG continuous mapping and g: (Y, σ) → (Z, δ) is an IF continuous mapping then $g \circ f$: (X, τ) → (Z, δ) is an IFCβG continuous mapping.

Proof: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFOS in Y, since g is an IF continuous mapping. Since f is an IFCβG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFβGCS in X. Therefore $g \circ f$ is an IFCβG continuous mapping.

Theorem 6.20: If f: (X, τ) → (Y, σ) is an IFCβG continuous mapping and g: (Y, σ) → (Z, δ) is an IFC continuous mapping then $g \circ f$: (X, τ) → (Z, δ) is an IFβG continuous mapping.

Proof: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IFC continuous mapping. Since f is an IFCβG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFβGOS in X. Therefore $g \circ f$ is an IFβG continuous mapping.

Theorem 6.21: For a mapping f: (X, τ) → (Y, σ), where X is an IF$\beta_g T_{1/2}$ space, the following are equivalent:

(i) f is an IFCβG continuous mapping

(ii) For every IFCS A in Y and for every IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$ then $p_{(\alpha,\beta)}\ _q\ \beta int(f^{-1}(A))$

(iii) For every IFCS in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$ then there exists an IFβGOS B such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$.

Proof: (i) ⇒ (ii) Let f be an IFCβG continuous mapping. Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})_q\, A$ then $p_{(\alpha,\beta)\,q}\, f^{-1}(A)$. By hypothesis $f^{-1}(A)$ is an IFβGOS in X. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(A)$ is an IFβOS in X. Hence $\beta int(f^{-1}(A)) = f^{-1}(A)$. This implies $p_{(\alpha,\beta)\,q}\, \beta int(f^{-1}(A))$.

(ii) ⇒ (i) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$ and let $f(p_{(\alpha,\beta)})_q\, A$ then $p_{(\alpha,\beta)\,q}\, f^{-1}(A)$. By hypothesis this implies $p_{(\alpha,\beta)\,q}\, \beta int(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \beta int(f^{-1}(A))$. But $\beta int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\beta int(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFβOS in X and hence an IFβGOS in X, by Theorem 3.2. This implies f is an IFCβG continuous mapping.

(ii) ⇒ (iii) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})_q\, A$ then $p_{(\alpha,\beta)\,q}\, f^{-1}(A)$. By hypothesis this implies $p_{(\alpha,\beta)\,q}\, \beta int(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \beta int(f^{-1}(A))$. But $\beta int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\beta int(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFβOS in X and hence an IFβGOS in X, by Theorem 3.2. Let $f^{-1}(A) = B$. Therefore $p_{(\alpha,\beta)\,q}\, B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

(iii) ⇒ (ii) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})_q\, A$ then $p_{(\alpha,\beta)\,q}\, f^{-1}(A)$. By hypothesis there exists an IFβGOS B in X such that $p_{(\alpha,\beta)\,q}\, B$ and $f(B) \subseteq A$. Let $B = f^{-1}(A)$. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(A)$ is an IFβOS in X and $\beta int(f^{-1}(A)) = f^{-1}(A)$. Therefore $p_{(\alpha,\beta)\,q}\, \beta int(f^{-1}(A))$.

Theorem 6.22: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFCβG continuous mapping if $f^{-1}(\beta cl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.

Proof: Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. Since every IFCS is an IFβCS, $\beta cl(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\beta cl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = int(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X. Hence f is an an IFC continuous mapping. Then by Remark 3.3, f is an IFCβG continuous mapping.

Theorem 6.23: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFCβG continuous mapping, where X is an IF$\beta_g T_{1/2}$ space if and only if $f^{-1}(\beta cl(B)) \subseteq \beta int(f^{-1}(cl(B)))$ for every IFS B in Y.

Proof: Necessity: Let $B \subseteq Y$ be an IFS. Then cl(B) is an IFCS in Y. By hypothesis, $f^{-1}(cl(B))$ is an IFβGOS in X. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(cl(B))$ is an IFβOS in X. Therefore $f^{-1}(\beta cl(B)) \subseteq f^{-1}(cl(B)) = \beta int(f^{-1}(cl(B)))$.

Sufficiency: Let B ⊆ Y be an IFCS. Then cl(B) = B. By hypothesis, f^{-1}(βcl(B)) ⊆ βint(f^{-1}(cl(B))) = βint(f^{-1}(B)). But βcl(B) = B. Therefore f^{-1}(B) = f^{-1}(βcl(B)) ⊆ βint(f^{-1}(B)) ⊆ f^{-1}(B). This implies f^{-1}(B) is an IFβOS in X and hence an IFβGOS in X, by Theorem 3.2. Hence f is an IFCβG continuous mapping.

SUMMARY AND CONCLUSION

In this thesis the notions of intuitionistic fuzzy β generalized closed set, intuitionistic fuzzy β generalized open set, intuitionistic fuzzy β_g $T_{1/2}$ space, intuitionistic fuzzy β generalized continuous mapping, intuitionistic fuzzy contra β generalized continuous mapping are discussed. In each of the chapters, the notions of the previously mentioned concepts are given. This is followed by their respective examples and corresponding comparisons. Furthermore, some interesting and unique theorems concerning each of the above mentioned concepts are presented.

We conclude that this particular research work made us enjoy the interesting outcomes of adding new concepts to the already existing concepts of intuitionistic fuzzy sets. The comparison of these new concepts with the already existing ones made us realize the fascinating nature of intuitionistic fuzzy sets. Additionally the pleasing theorems under each of the topics under consideration created a passion to work with the enthralling aspects of intuitionistic fuzzy sets.

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2. **Saranya, M.,** and **Jayanthi, D.,** On Intuitionistic Fuzzy β Generalized Open Sets, (to appear).

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