CHAPTER - VII

GENERALIZED INTUITIONISTIC FUZZY SOFT RELATIONS

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Definition : 7.1

Let $U = \{x_1, x_2, \ldots, x_n\}$ be the initial universe of elements and $E = \{e_1, e_2, \ldots, e_m\}$ be the set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, i.e., $\mu : E \to I = [0,1]$, where I^U is the collection of all fuzzy subset of U. Let F_μ be a mapping $F_\mu : E \to I^U \times I$ defined as follows:

$F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^U$. Then F_μ is called **generalized fuzzy soft set** over the soft universe (U, E).

Here for each parameter e_i, $F_\mu(e_i)$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Definition : 7.2

Let $\mathcal{F} : A \to IF^U$ and α be a fuzzy subset of A. Then $\mathcal{F}_\alpha : A \to IF^U \times [0,1]$ is a function defined as follows:

$$\mathcal{F}_\alpha(a) = (\mathcal{F}(a), \alpha(a)) = (\{(x, \mu_{\mathcal{F}(a)}(x), \nu_{\mathcal{F}(a)}(x)) : x \in U\}, \alpha(a))$$

where μ, ν denotes the degree of membership and degree of non-membership respectively. Then $\mathcal{F}_\alpha$ is called a **generalized intuitionistic fuzzy soft set** over (U, E).

Here for each parameter $e_{i,}$, $\mathcal{F}_\alpha(e_i)$ indicates not only degree of belongingness of the elements of U in $\mathcal{F}(e_i)$ but also degree of preference of such belongingness which is represented by $\alpha(e_i)$.

Example : 7.3

Let U = $\{s_1, s_2, s_3, s_4\}$ be the set of students under consideration for the best student of an academic year with respect to the given parameters $A \subseteq E$ and A = {r = "result", c = "conduct", g = "games and sports performances"}. Let $\alpha : A \rightarrow [0,1]$ be given as follows: $\alpha(r) = 0.7$, $\alpha(c) = 0.5$, $\alpha(g) = 0.6$.

We define $\mathcal{F}_\alpha$ as follows:

$\mathcal{F}_\alpha(r) = (\{(s_1, 0.8, 0.1), (s_2, 0.9, 0.05), (s_3, 0.85, 0.1), (s_4, 0.75, 0.2)\}, 0.7)$

$\mathcal{F}_\alpha(c) = (\{(s_1, 0.6, 0.3), (s_2, 0.65, 0.2), (s_3, 0.7, 0.2), (s_4, 0.65, 0.2)\}, 0.5)$

$\mathcal{F}_\alpha(g) = (\{(s_1, 0.75, 0.2), (s_2, 0.5, 0.3), (s_3, 0.5, 0.4), (s_4, 0.7, 0.2)\}, 0.6)$

Then $\mathcal{F}_\alpha$ is an generalized intuitionistic fuzzy soft set.

Definition : 7.4

Let $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ be two generalized intuitionistic fuzzy soft set over (U, E). Now $\mathcal{F}_\alpha$ is called a **generalized intuitionistic fuzzy soft subset** of $\mathcal{G}_\beta$ if

1) α is a fuzzy subset of β,
2) $A \subseteq B$,
3) $\forall$ a $\in$ A, $\mathcal{F}(a)$ is an intuitionistic fuzzy subset of $\mathcal{G}(a)$.

   i.e., $\mu_{\mathcal{F}(a)}(x) \leq \mu_{\mathcal{G}(a)}(x)$ and $\nu_{\mathcal{F}(a)}(x) \geq \nu_{\mathcal{G}(a)}(x)$, $\forall$ x $\in$ U and a $\in$ A.

   It is denoted by $\mathcal{F}_\alpha \subseteq \mathcal{G}_\beta$.

Example : 7.5

Let $\mathcal{G}_\beta$ be a generalized intuitionistic fuzzy soft set defined as follows:

$\mathcal{G}_\beta(r) = (\{(s_1, 0.85, 0.05), (s_2, 0.9, 0.025), (s_3, 0.9, 0.1), (s_4, 0.8, 0.1)\}, 0.75)$

$\mathcal{G}_\beta$ (c) = ({(s$_1$, 0.7, 0.2), (s$_2$, 0.7, 0.15), (s$_3$, 0.75, 0.2), (s$_4$, 0.65, 0.15)}, 0.6)

$\mathcal{G}_\beta$ (g) = ({(s$_1$, 0.8, 0.2), (s$_2$, 0.6, 0.3), (s$_3$, 0.7, 0.2), (s$_4$, 0.7, 0.1)}, 0.65) and

consider the generalized intuitionistic fuzzy soft set $\mathcal{F}_\alpha$ given in example 7.3.

Then $\mathcal{F}_\alpha$ is a generalized intuitionistic fuzzy soft subset of $\mathcal{G}_\beta$.

Definition : 7.6

The **intersection** of two generalized intuitionistic fuzzy soft sets $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ is denoted by $\mathcal{F}_\alpha \cap \mathcal{G}_\beta$ and defined by a generalized intuitionistic fuzzy soft set $\mathcal{H}_\delta : A \cap B \rightarrow IF^U \times [0,1]$ such that for each $e \in A \cap B$

$$\mathcal{H}_\delta(e) = (\{(x, \mu_{\mathcal{H}(e)}(x), \nu_{\mathcal{H}(e)}(x)) : x \in U\}, \delta(e))$$

where $\mu_{\mathcal{H}(e)}(x) = \mu_{\mathcal{F}(e)}(x) * \mu_{\mathcal{G}(e)}(x)$, $\nu_{\mathcal{H}(e)}(x) = \nu_{\mathcal{F}(e)}(x) \lozenge \nu_{\mathcal{G}(e)}(x)$, $\delta(e) = \alpha(e) * \beta(e)$.

Definition : 7.7

The **union** of two generalized intuitionistic fuzzy soft sets $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ is denoted by $\mathcal{F}_\alpha \cup \mathcal{G}_\beta$ and defined by a generalized intuitionistic fuzzy soft set $\mathcal{H}_\delta : A \cup B \rightarrow IF^U \times [0,1]$ such that for each $e \in A \cup B$

$$\mathcal{H}_\delta(e) = (\{(x, \mu_{\mathcal{F}(e)}(x), \nu_{\mathcal{F}(e)}(x)) : x \in U\}, \alpha(e)) \text{ if } e \in A - B$$

$$= (\{(x, \mu_{\mathcal{G}(e)}(x), \nu_{\mathcal{G}(e)}(x)) : x \in U\}, \beta(e)) \text{ if } e \in B - A$$

$$= (\{(x, \mu_{\mathcal{H}(e)}(x), \nu_{\mathcal{H}(e)}(x)) : x \in U\}, \delta(e)) \text{ if } e \in A \cap B$$

where $\mu_{\mathcal{H}(e)}(x) = \mu_{\mathcal{F}(e)}(x) \lozenge \mu_{\mathcal{G}(e)}(x)$, $\nu_{\mathcal{H}(e)}(x) = \nu_{\mathcal{F}(e)}(x) * \nu_{\mathcal{G}(e)}(x)$, $\delta(e) = \alpha(e) \lozenge \beta(e)$.

Example : 7.8

Let us consider the generalized intuitionistic fuzzy soft sets $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ defined in example 7.3 and 7.5 respectively. Let us consider the t-norm $*$ and the t-conorm $\Diamond$ as follows: $a * b = ab$ and $a \Diamond b = a + b - ab$. Then

$(\mathcal{F}_\alpha \cup \mathcal{G}_\beta)(r)$ = ({(s_1, 0.97, 0.005), (s_2, 0.99, 0.00125), (s_3, 0.985, 0.01), (s_4, 0.95, 0.02)}, 0.68)

$(\mathcal{F}_\alpha \cup \mathcal{G}_\beta)(c)$ = ({(s_1, 0.88, 0.06), (s_2, 0.895, 0.03), (s_3, 0.925, 0.04), (s_4, 0.8775, 0.1625)}, 0.1625)

$(\mathcal{F}_\alpha \cup \mathcal{G}_\beta)(g)$ = ({(s_1, 0.95, 0.04), (s_2, 0.8, 0.09), (s_3, 0.85, 0.08), (s_4, 0.91, 0.02)}, 0.86).

Since {r, c, g} $\in A \cap B$,

$(\mathcal{F}_\alpha \cap \mathcal{G}_\beta)(r)$ = ({(s_1, 0.68, 0.145), (s_2, 0.81, 0.07375), (s_3, 0.765, 0.19), (s_4, 0.6, 0.28)}, 0.12)

$(\mathcal{F}_\alpha \cap \mathcal{G}_\beta)(c)$ = ({(s_1, 0.42, 0.44), (s_2, 0.455, 0.32), (s_3, 0.525, 0.36), (s_4, 0.4225, 0.7375)}, 0.375)

$(\mathcal{F}_\alpha \cap \mathcal{G}_\beta)(g)$ = ({(s_1, 0.6, 0.36), (s_2, 0.3, 0.5), (s_3, 0.35, 0.52), (s_4, 0.49, 0.28)}, 0.39).

Theorem : 7.9

Let $\mathcal{F}_\alpha$, $\mathcal{G}_\beta$ and $\mathcal{H}_\delta$ be any three generalized intuitionistic fuzzy soft sets over (U, E). Then the following holds:

1) $\mathcal{F}_\alpha \cup \mathcal{G}_\beta = \mathcal{G}_\beta \cup \mathcal{F}_\alpha$
2) $\mathcal{F}_\alpha \cap \mathcal{G}_\beta = \mathcal{G}_\beta \cap \mathcal{F}_\alpha$

3) $\mathcal{F}_\alpha \cup (\mathcal{G}_\beta \cup \mathcal{H}_\delta) = (\mathcal{F}_\alpha \cup \mathcal{G}_\beta) \cup \mathcal{H}_\delta$

4) $\mathcal{F}_\alpha \cap (\mathcal{G}_\beta \cap \mathcal{H}_\delta) = (\mathcal{F}_\alpha \cap \mathcal{G}_\beta) \cap \mathcal{H}_\delta$

Proof :

Since the t-norm function and t-conorm functions are commutative and associative, therefore the theorem follows.

Remark : 7.10

Let $\mathcal{F}_\alpha$, $\mathcal{G}_\beta$ and $\mathcal{H}_\delta$ be any three generalized intuitionistic fuzzy soft sets over (U, E). If we consider $a * b = \min\{a, b\}$ and $a \lozenge b = \max\{a, b\}$ then the following holds:

1) $\mathcal{F}_\alpha \cap (\mathcal{G}_\beta \cup \mathcal{H}_\delta) = (\mathcal{F}_\alpha \cap \mathcal{G}_\beta) \cup (\mathcal{F}_\alpha \cap \mathcal{H}_\delta)$

2) $\mathcal{F}_\alpha \cup (\mathcal{G}_\beta \cap \mathcal{H}_\delta) = (\mathcal{F}_\alpha \cup \mathcal{G}_\beta) \cap (\mathcal{F}_\alpha \cup \mathcal{H}_\delta)$

But in general above relations do not hold.

Definition : 7.11

Let $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ be two generalized intuitionistic fuzzy soft set over (U, E). Then **generalized intuitionistic fuzzy soft relation** (in short GIFSR) R from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ is a function $R : A \times B \to IF^U \times [0; 1]$ satisfying

$R(a, b) \subseteq \mathcal{F}_\alpha(a) \cap \mathcal{G}_\beta(b), \ \forall \ (a, b) \in A \times B$.

Definition : 7.12

Let R be a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$. Then $\mathbf{R}^{-1}$ is defined as follows:

$R^{-1}(b, a) = R(a, b), \ \forall \ (a, b) \in A \times B$.

Note : 7.13

If R is a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ then R^{-1} is a GIFSR from $\mathcal{G}_\beta$ to $\mathcal{F}_\alpha$.

Theorem : 7.14

If R_1 and R_2 are GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$,

1) $(R_1^{-1})^{-1} = R_1$.
2) $R_1 \subseteq R_2 \Rightarrow R_1^{-1} \subseteq R_2^{-1}$.

Proof :

Let $(a, b) \in A \times B$.

1) $(R_1^{-1})^{-1}(a, b) = R_1^{-1}(b, a) = R_1(a, b)$

   Hence $(R_1^{-1})^{-1} = R_1$.
2) $R_1(a, b) \subseteq R_2(a, b)$

   $\Rightarrow (R_1^{-1})^{-1}(a, b) \subseteq (R_2^{-1})^{-1}(a, b)$

   $\Rightarrow R_1^{-1}(b, a) \subseteq R_2^{-1}(b, a)$

   Hence $R_1^{-1} \subseteq R_2^{-1}$.

Definition : 7.15

The **composition** $\circ$ of two GIFSR R_1 and R_2 is defined by $(R_1 \circ R_2)(a, c) = \bigcup_{b \in B} (R_1(a, b) \cap R_2(b, c))$, $\forall$ $(a, c) \in A \times C$, where R_1 is a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ and R_2 is a GIFSR from $\mathcal{G}_\beta$ to $\mathcal{H}_\delta$.

Theorem : 7.16

Let R_1 be a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ and R_2 be a relation $\mathcal{G}_\beta$ to $\mathcal{H}_\delta$. Then $R_1 \circ R_2$ is a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{H}_\delta$.

Proof :

By definition

$R_1(a, b) \subseteq \mathcal{F}_\alpha(a) \cap \mathcal{G}_\beta(b)$

$= \{(\{x, \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x), \nu_{\mathcal{F}(a)}(x) \lozenge \nu_{\mathcal{G}(b)}(x)\}, \alpha(a) * \beta(b)) : x \in U\},$

$\forall\ (a, b) \in A \times B.$

$R_2(b, c) \subseteq \mathcal{G}_\beta(b) \cap \mathcal{H}_\delta(c)$

$= \{(\{x, \mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x), \nu_{\mathcal{G}(b)}(x) \lozenge \nu_{\mathcal{H}(c)}(x)\}, \beta(b) * \delta(c)) : x \in U\},$

$\forall\ (b, c) \in B \times C.$

Therefore

$(R_1 \circ R_2)(a, c) = \bigcup_{b \in B} (R_1(a, b) \cap R_2(b, c))$

$= \bigcup_{b \in B} \{(\{x, (\mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x)) * (\mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x)), (\nu_{\mathcal{F}(a)}(x) \lozenge \nu_{\mathcal{G}(b)}(x))$

$\lozenge (\nu_{\mathcal{G}(b)}(x) \lozenge \nu_{\mathcal{H}(c)}(x))\}, (\alpha(a) * \beta(b)) * (\beta(b) * \delta(c))) : x \in U\},\ \forall\ (a, c) \in A \times C.$

Now

$(\mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x)) * (\mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x)) = \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x)$

$\leq \mu_{\mathcal{F}(a)}(x) * 1 * \mu_{\mathcal{H}(c)}(x)$

$= \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{H}(c)}(x)$

$(\nu_{\mathcal{F}(a)}(x) \lozenge \nu_{\mathcal{G}(b)}(x)) \lozenge (\nu_{\mathcal{G}(b)}(x) \lozenge \nu_{\mathcal{H}(c)}(x)) = \nu_{\mathcal{F}(a)}(x) \lozenge \nu_{\mathcal{G}(b)}(x) \lozenge \nu_{\mathcal{H}(c)}(x)$

$\geq \nu_{\mathcal{F}(a)}(x) \lozenge 0 \lozenge \nu_{\mathcal{H}(c)}(x)$

$$= \nu_{\mathcal{F}(a)}(x) \Diamond \nu_{\mathcal{H}(c)}(x).$$

Also, $(\alpha(a) * \beta(b)) * (\beta(b) * \delta(c)) = \alpha(a) * \beta(b) * \delta(c)$

$$\leq \alpha(a) * 1 * \delta(c)$$

$$= \alpha(a) * \delta(c).$$

Hence, $\bigcup_{b \in B} (R_1(a, b) \cap R_2(b, c)) \subseteq \mathcal{F}_\alpha \cap \mathcal{H}_\delta$.

Thus $R_1 \circ R_2$ is a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{H}_\delta$.

Theorem : 7.17

$(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$ where R_1 is a IFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ and R_2 are GIFSR from $\mathcal{G}_\beta$ to $\mathcal{H}_\delta$.

Proof :

Let $a \in A$, $b \in B$, $c \in C$.

$(R_1 \circ R_2)^{-1}(c, a) = (R_1 \circ R_2)(a, c)$

$$= \bigcup_{b \in B} (R_1(a, b) \cap R_2(b, c))$$

$$= \bigcup_{b \in B} (R_2(b, c) \cap R_1(a, b))$$

$$= \bigcup_{b \in B} (R_2^{-1}(c, b) \cap R_1^{-1}(b, a))$$

$$= (R_2^{-1} \circ R_1^{-1})(c, a).$$

Hence $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$.

An application of generalized intuitionistic fuzzy soft relation in decision making.

There are several application of generalized intuitionistic fuzzy soft set theory to deal with uncertainties from our different kinds of daily life problems. Here we present such an application for solving a socialistic decision making problem.

Suppose there are six men in the universe U and $U = \{b_1, b_2, b_3, b_4, b_5, b_6\}$ and the parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$ where each e_i, $1 \leq i \leq 9$ indicates an attributes of b_j, $1 \leq j \leq 6$.

e_1 stands for "education qualification", e_2 stands for "hard working", e_3 stands for "responsible", e_4 stands for "government employee", e_5 stands for "non-government employee", e_6 stands for "businessman", e_7 stands for "family status", e_8 stands for "spiritual and ideal". e_9 stands for "handsome".

Suppose an woman Miss.Y wishes to marry a man on the basis of some criteria listed above. Our aim is to find out the most appropriate partner for Miss. Y.

Suppose the wishing parameters for Miss. Y is $A \subseteq E$ where $A = \{ e_3, e_4, e_7, e_9\}$. Let the preference of the criterions for Miss. Y be described by the fuzzy subset $\alpha : A \rightarrow [0,1]$ of A, as follows:

$\alpha(e_3) = 0.4$, $\alpha(e_4) = 0.6$, $\alpha(e_7) = 0.6$, $\alpha(e_9) = 0.8$.

Consider the generalized intuitionistic fuzzy soft sets $\mathcal{F}_\alpha$ as a collection of intuitionistic fuzzy approximation as below:

$\mathcal{F}_\alpha(e_3) = (\{(b_1,0.3,0.3), (b_2,0.5,0.3), (b_3,0.3,0.2), (b_4,0.6,0.3), (b_5,0.4,0.3), (b_6,0.3, 0.4)\}, 0.4)$

$\mathcal{F}_\alpha(e_4)$ = ({(b_1,0,0.8), (b_2,1,0), (b_3,0.8,0.02), (b_4,0,0.24), (b_5,0,0.2), (b_6,0,0.06)}, 0.6)

$\mathcal{F}_\alpha(e_7)$ = ({(b_1,0.6,0.3), (b_2,0.5,0.4), (b_3,0.6,0.3), (b_4,0.7,0.2), (b_5,0.7,0.28), (b_6,0.8, 0.02)}, 0.5)

$\mathcal{F}_\alpha(e_9)$ = ({(b_1,0.5,0.3), (b_2,0.4,0.3), (b_3,0.6,0.4), (b_4,0.5,0.3), (b_5,0.6,0.3), (b_6,0.7, 0.2)}, 0.8)

Algorithm:

1) Input a weighted intuitionistic fuzzy soft set in tabular form.
2) Choose mid-level soft set (or, input a threshold intuitionistic fuzzy set or, give a threshold value pair or, choose top-bottom-level decision rule or, choose top-top-level decision rule or, choose bottom-bottom-level decision rule) for decision making.
3) Compute the level soft set with respect to the threshold intuitionistic fuzzy set (or, the threshold-level soft set or, the mid-level soft set or, the top-bottom-level soft set or, the top-top-level soft set or, the bottom-bottom-level soft set) in tabular form.
4) Compute the weighted choice value α_i' of b_i $\forall$ i.
5) If the maximum score occurs in k-th row then Miss. Y will marry to b_k.
6) If k has more than one value then one of b_k may be chosen.

Tabular representation of the generalized intuitionistic fuzzy soft set $\mathcal{F}_\alpha$

	e_3, $\alpha(e_3)$ = 0.4	e_4, $\alpha(e_4)$ = 0.6	e_7, $\alpha(e_7)$ = 0.6	e_9, $\alpha(e_9)$ = 0.8
b_1	(0.3, 0.3)	(0, 0.8)	(0.6, 0.3)	(0.5, 0.3)
b_2	(0.5, 0.3)	(1, 0)	(0.5, 0.4)	(0.4, 0.3)
b_3	(0.3, 0.2)	(0.8, 0.02)	(0.6, 0.3)	(0.6, 0.4)
b_4	(0.6, 0.3)	(0, 0.24)	(0.7, 0.2)	(0.5, 0.3)
b_5	(0.4, 0.3)	(0, 0.2)	(0.7, 0.28)	(0.6, 0.3)
b_6	(0.3, 0.4)	(0, 0.06)	(0.8, 0.02)	(0.7, 0.2)

Here we deal with the problem by mid-level decision rule. It is clear that the mid-threshold of ($\mathcal{F}_\alpha$, A) is {(e_3, 0.4, 0.3), (e_4, 0.3,0.22), (e_7, 0.65, 0.25), (e_9, 0.55, 0.3)}.

Tabular representation of mid-level set with degree of preferences.

	e_3, $\alpha(e_3) = 0.4$	e_4, $\alpha(e_4) = 0.6$	e_7, $\alpha(e_7) = 0.6$	e_9, $\alpha(e_9) = 0.8$	α_i
b_1	0	0	0	0	0.0
b_2	1	1	0	0	1.0
b_3	0	1	0	0	0.6
b_4	1	0	1	0	0.9
b_5	1	0	0	1	1.2
b_6	0	0	1	1	1.3

Clearly the maximum score is scored by the man b_6.

Decision: Miss. Y will marry to b_6. In case, if she does not want to marry b_6 due to certain reasons, her second choice will be b_5.