Analysis of CT and MRI Image Fusion using Spatial Frequency Discrete Wavelet Transform (Haar) and Neutrosophic Set

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By

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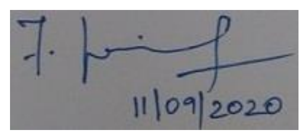
DECLARATION

DECLARATION

I declare that the dissertation entitled **Analysis of CT and MRI Image Fusion using Spatial Frequency Discrete Wavelet Transform (Haar) and Neutrosophic Set** submitted by me for the degree of Master of Philosophy (M.Phil) is the record of work carried out by me during the period from July 2019 to September 2020 under the guidance of **Dr. D. Jayanthi** Assistant Professor (SS), Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this University or any University or other similar institution of Higher Learning.



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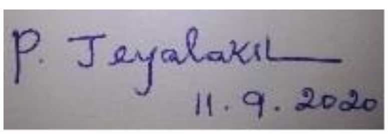
CERTIFICATE

CERTIFICATE

This is to certify that the dissertation entitled **Analysis of CT and MRI Image Fusion using Spatial Frequency Discrete Wavelet Transform (Haar) and Neutrosophic Set** submitted for the degree of Master of Philosophy (M.Phil) by **F.PRISHKA** is the record of research work carried out by her during the period from July 2019 to September 2020 under my guidance and supervision, and that this work has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this University or any other similar institution of Higher Learning.



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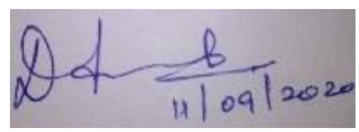

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CONTENT

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ABSTRACT

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Computer aided diagnosing and imaging techniques has been developed to solve various issues in day-to-day life. This advancement helps to improve the accuracy and efficiency in disease diagnosis and clinical care. Now a days, researches are trying to combine the CT and MRI images. But practically, it is difficult to obtain fruitful information due to the losses of data. In this regard, the medical images have to be fused without missing any information so as to avoid misconception by the medical practitioners.

There is a need for advance image fusion methods with good performance in terms of efficiency, robustness, operational, uncertainty and improved classification. Multimodal medical image fusion method is used to combine various images generated from MRI, CT, PET, SPECT scans and provide accurate information about the disease status or functions of the internal body.

Image fusion is one of the most modern, accurate and useful diagnostic techniques in medical imaging. The fusion of CT and MRI image of the same organ is to obtain a single image containing as much information as possible about the organ for diagnosis. CT images are mainly employed to visualize dense structures of bones which give the general shapes of objects. On the other hand, MRI images are used to depict the morphology of soft tissues, which is rich in details. Since these two modalities are of complementary nature, the objective is to merge both the images and obtain as much information as required. It assists physicians in extracting features that many not be normally visible in images produced by different modalities. Fused image can significantly benefit medical diagnosis and also the further image processing.

A novel approach image fusion using spatial frequency discrete wavelet transform (Haar) and neutrosophic set is proposed using the MATLAB technical computing language. The image fusion process is followed by pixel-based image fusion. In this method, the images are decomposed into low level subbands and high level subbands by

spatial frequency discrete wavelet transform (Haar). The second step for fusion process, neutrosophic set technique is applied for low level subband and average fusion method is applied for high level subbands. Finally, the two fused subbands are improved to form the final fused image by using inverse discrete wavelet transform (Haar).

In the present research, a comparative study has been made among Image fusion using spatial frequency discrete wavelet transform (Haar) and neutrosophic set and Image fusion using discrete wavelet transform (Haar) and neutrosophic set. The result shows that spatial frequency discrete wavelet transform (Haar) and neutrosophic set proves to be more advantageous based on the values of Peak Signal to Noise Ratio (PSNR), Root Mean Square Error (RMSE) and Correlation (CORR) is better compared to the discrete wavelet transform (Haar) and neutrosophic set.

CHAPTER 1

CHAPTER 1

INTRODUCTION

SECTION 1.1

DIGITAL IMAGE PROCESSING

One picture is worth more than ten thousand words. Digital images and subsequent digital image processing techniques not only has a unique place but also play a vital role in the modern world which is flooded with imaging systems, computing and multimedia applications. Without digital images and image processing, one cannot imagine this computerized global village which has been enjoying the wonders of image processing applications.

The history of image processing started in the early 1920s in the newspaper industry for Bartlane Cable picture transmission between London and New York. For effective cable transfer, images were coded at the transmitter and reconstructed at the receiver. New photographic techniques and more number of tones improved the quality of Bartlane system in mid to late 1920s.

Digital image processing research got its place in the 1960s because of fast computing techniques and the onset of space missions. Image enhancement methods came to limelight in space missions to improve the quality of space images (Rafael Gonzalez 2011). In the 1970s, digital image processing found its applications medical field after the invention of tomography and computerized axial tomography.

In the early stages of medical imaging modalities, images were very poor in resolution and quality. Hence, the medical field was in dire need of image enhancement and subsequent image processing techniques to ensure better image analysis and clinical procedures.

Applications using digital image processing have started in 1980 in various fields such as

- ❖ Medical visualization and clinical pathology
- ❖ Law enforcement
- ❖ Human computer interfaces
- ❖ Industrial inspection
- ❖ Artistic effects
- ❖ Remote sensing and so on.

Researchers have proposed in numerous algorithms for various image processing tasks such as image enhancement, image restoration, segmentation, representation, description, image compression, image fusion and so on. All the image processing concepts process images for better image analysis or image visualization or image processing based applications.

SECTION 1.2

IMAGE FUSION

The development in computing and imaging techniques, there has been enormous research happening in visual information processing and image analysis. This needs integration of images or image details or extracted features or decisions. This laborious integration task is accomplished by the concept called image fusion and hence finds wide applications in remote sensing, surveillance and navigation, change detection-target, robotics (Hall and Llinas 1997) and medical image analysis (Shihong Yue et al 2013).

Image fusion algorithms integrate multisensory images, multifocus images and images of different optic conditions of the same region. Such images exist in the remote sensing, medical imaging, object tracking, change detection, surveillance-navigation and so on.

In the literature of image fusion, various definitions or explanations have been suggested by researchers based on the field of application. A few definitions are highlighted below:

- ❖ Image fusion approach is the one which combines contents of two or more images into a composite image. Various terms such as merging, combination and integration are available in literature to express the concept of fusion (Zhijun Wang et al.2005).
- ❖ Image fusion is the process of combining two or more input images into a composite image which contains a better description of the region of interest or scene than the one provided by any of the input images, thus the fused image aids to better human visual perception or machine learning process (Mitchell 2010).

- ❖ Fusion provides wider spatial and temporal coverage, decreased uncertainty and improved reliability (Toet et al.2010).
- ❖ Sabalan Daneshvar and Hassan Ghassenisan (2010) define fusion as the process of integrating information from two or more images of an object into a single image that is more informative and appropriate for visual perception and computer analysis.
- ❖ Cosmin Ludusan and Olivier Lavialle (2012) define image fusion as the process aiming at extracting and synthesizing information from multiple sources that leads to more accurate and reliable composite output.
- ❖ Peijun Du et al. (2013) add that the objective of image fusion is to improve the accuracy of image interpretation and analysis by using complementary information.
- ❖ Image fusion in medical field combines images of same modality or multiple modalities to enhance image quality, content for better clinical analysis, diagnosis, treatment and planning. Fusion also reduces randomness and information observed from the images can be analyzed for more precise localization of abnormalities (James and Dasarathy 2014).

SECTION 1.3

CLASSIFICATION OF IMAGE FUSION ALGORITHMS

Image fusion algorithms can be classified according to fusion strategy, type of source images and fusion domain.

1.3.1 Fusion Strategy

Based on fusion strategy, fusion techniques are classified into

- ❖ Pixel level fusion
- ❖ Feature level fusion
- ❖ Decision level fusion

Pixel level fusion combines information of source images pixel by pixel or region based on certain well defined fusion criteria (Jiang and Wang 2014). This is carried out in the form of raw image representation and can preserve more original information (Jianwen Hu and Shutao Li 2012). Pixel level fusion is called low level fusion as it involves raw data from the source images.

Feature level fusion is an intermediate level fusion that extracts features to define fusion criteria. This fusion is inspired by feature based object recognition and pattern recognition. Features such as edges, textures, contrast, standard deviation and variance from the source images are observed by various image processing techniques (Lewis et al.2004). In multisensory applications, features extracted from multiple sensors are fused together to get a concatenated feature vector for object recognition or pattern recognition.

Decision level fusion or interpretation level fusion uses image description such as relational graphs, region maps (Williams et al.1999, Shapiro and Haralick 1982), voting, fuzzy logic, heuristics and prediction (Zhao et al. 2008) derived from the source images. Source images are processed individually to arrive at the decisions and subsequently the decisions are fused.

1.3.2 Type of Source Images

Based on the type of source images, fusion schemes are classified into multisensor fusion, multifocus fusion and different optic conditions or at different times. The following Table 1.1 gives different types of source images and applications of fusion.

Table 1.1 Fields of source images with their applications

Field	Source images	Applications
Medical (Multimodality images and images from same modality)	X-ray-CT, CT-MRI, PET-CT, SPECT-CT, PET-MRI, SPECT-MRI, MRI T1-MRI T2.	Radiology, oncology, molecular and brain imaging, cardiac disease diagnosis.
Remote sensing	Panchromatic-multispectral images, Range-intensity images, Multitemporal images by radar and non-radar sensors.	Lithology, soil and vegetation analysis surface representation vegetation, soil, water and forests.
Military, Security and Surveillance	Visible-infrared, Ultraviolet-visible	Weapon detection, target tracking, change detection.

1.3.3 Fusion Domain

Image fusion algorithms are carried out either in spatial domain or in transform domain. In spatial domain, pixels or regions are fused either by linear or non-linear combination of source images. In spatial domain, for pixel level fusion approaches, linear weights or non-linear weights are determined based on some salience measures and the weights are determined on individual pixels or neighbors. Popular spatial domain algorithms are min, max, average, weighted average, Principal Component Analysis (PCA), independent component analysis and Brovery transform.

In transform domain, multiscale transforms are most- frequently used and the most common steps for fusion are

- ❖ Take forward transform of source images to get multiscale transform coefficients and directions.

- Apply linear or nonlinear fusion criteria to the transform coefficients.
- Take inverse transform of fused coefficients to get the fusion output in spatial domain.
- Evaluate performance metrics for objective analysis.

Some of the transform domain methods available in literature are pyramid transform methods, stationary wavelet transform (SWT), discrete wavelet transform (DWT), dual tree complex wavelet transform (DTCWT), Contourlet transform (CTT), Curvelet (CVT), nonsubsampled contourlet (NSCT) and ridgelet transform.

The following image fusion methods are widely used in the medical image fusion domain:

- Average Method
- Max-Min Intensity method
- Wavelet transform
- Curvelet transform
- Brovery transform
- Principal Component Analysis Method

(i) Average Method

Average Method is one of the easiest methods for fusing the input images. The average method is used to compute the average A(i,j) of all the input images. This type of method is easy to interpret and implement. The average method removes the irrelevant features and unwanted noises in the resultant image. In other words, average method collects all the pixel values of the input images and computes the average.

$$A(i,j) = \frac{\{x(i,j)+y(i,j)\}}{2}$$

Where,

x(i,j) = Input image ‘x’

y(i,j) = Input image ‘y’

(ii) Max-Min Intensity Method

Max-Min Intensity method is divided into methods such as select maximum method and select minimum method. The select maximum method selects the highest region of the input image generated from various modalities. In addition, the select maximum method produces the output image with high frequency regions with greater pixel values. The select minimum method is identical to the select maximum method. Hence, the select minimum method produces the resultant fused image with low frequency regions and low pixel values.

(iii) Wavelet Transform

Wavelet transform based image fusion methods, functions with varying frequency and limited duration. Wavelet transform method mine the high frequency information from one image in both time and space domains and inject it. Wavelet methods decompose the input images into one frequency subband and three high frequency subbands. Wavelet transform methods are widely used in signal processing communication applications. Wavelet transform method represents the useful information in a convenient tree structure in a timely manner. Various numerical and mathematical models are applied in the wavelet methods to execute injection starting from simple addition and aggregate operation to more compound mathematical models.

(iv) Curvelet Transform

Curvelet transform based image fusion method is developed based on segmentation concept. Curvelet transform methods divide the input image into various small overlapping tiles. Finally, ridgelet transform is applied to each of the tiles to carry out the edge detection task. The fused image generated from the Curvelet transform based image fusion method provides more information by preventing image denoising.

(v) Brovery Transform

Brovery transform based image fusion method is also called color normalized fusion method which is used to fuse the input images generated from the various sensor devices. The resultant image generated from Brovery transform method is used for the visual interpretation of various meaningful results. Brovery transform based image fusion method does not require any transform method to transfer luminance data into panchromatic images.

(vi) Principal Component Analysis Method

Principal Component Analysis based image fusion method converts the correlated variables into uncorrelated variables. The converted variables from the Principal Component Analysis (PCA) based image fusion methods are called as principal components. The primary function of PCA based image fusion method is to compute a solid and most favorable report of the input data set. The first principal component is computed based on the variance in the data where as the second principal component is controlled to lie in the sub-space vertical of the first. Finally, the third principal component is identified based on the almost variance path in the sub-space vertical to the first and second principal components. PCA based image fusion method does not follow any predetermined set of vectors from the input images generated from various image modalities.

SECTION 1.4

MEDICAL IMAGE FUSION

In recent past, many fusion algorithms have been proposed for medical images, because of complementary image details provided by various medical imaging modalities. Hence, image fusion algorithms have started evolving to combine two or more medical images into a composite image for additional clinical information, better diagnosis and treatment planning. Medical image fusion often leads to more image details not apparent in separate images. It also reduces storage cost by storing a fused image instead of many source images (Yong yang et al. 2010).

Most of the medical imaging modalities are divided into structural and functional systems. Some of the medical imaging modalities which gained popularity for fusion over the years are X-ray-CT, CT-MRI, PET-CT, SPECT-CT, PET-MRI and SPECT-MRI (Sabalan Daneshwaran and Hassan Ghassemian 2010).

Low resolution bone structures are exposed by X-ray images. High resolution structural and anatomical information are provided by CT and MRI systems, where as PET and SPECT provide low resolution functional information.

Image fusion has its applications in medical field in radiology, oncology, molecular imaging, brain imaging, cardiac disease diagnosis and ultrasound (Kavitha et al. 2012).

SECTION 1.5

IMAGE MODALITIES

Nowadays, image processing methods are used in various applications to make the decisions and solve various issues in day-to-day life. Generally, image processing methods are utilized in medical fields of disease diagnosis, clinical care and other healthcare services. The advancement in medical image processing is increased noticeably. Hence, various types of advance image generation sources are increased. They produce huge sizes of medical images continuously. The following image generation sources play a vital role in medical applications:

- Computed Tomography (CT)
- Magnetic Resonance Image (MRI)
- Positron Emission Tomography (PET)
- Positron Emission Tomography and Computed Tomography (PET-CT)
- Single photon Emission Computed Tomography (SPECT)
- Ultrasound
- X-ray

1.5.1 Computed Tomography (CT)

Computed Tomography (CT) is used to take X-ray with various angles to produce cross sectional images. In other words, CT scan uses various types of digital geometry processes to take X-ray images from different directions. Images generated from the CT scans are used to image the various types of heart diseases, broken bones, prostate cancers, liver cancer, breast cancer, internal bleeding and blood clots and so on. In addition, Computed Tomography (CT) images are also used to observe the function of the internal organs, identify fracture of bones, monitor the soft tissue, identify the location of tumors, monitor the vascular condition and blood flow, observe the blood vessel conditions, identify the traumatic injuries and diagnose the cardiovascular disease.

In general, Computed Tomography (CT) images are present in the form of three dimensional and printed on the film and then stored in external storage devices. Computed Tomography (CT) images are often used to identify various cancers and their location, size and present condition. During a CT scan, patients do not feel any pain or risk. CT scans are very simple and accurate method for disease diagnosis. In many cases CT scans are used to save the patient's life by efficiently monitoring the emerging diseases, internal bleeding and injuries.

1.5.2 Magnetic Resonance Image (MRI)

Magnetic Resonance Imaging (MRI) is used in medical image processing to observe the conditions of internal tissues and organs. MRI scan uses magnetic field and radio waves to obtain the internal scans. MRI scans are widely used to identify the difference between the normal individual's body and patient's body. In other words, MRI scan uses magnetic field and radio waves to image the anatomy and physiological processes of the patients who are affected by various diseases and health issues.

The individuals who are affected by various diseases and individuals who are facing health issues are placed inside an MRI machine to observe the MRI scan. During the MRI scan patients must remain idle for a period of time. In general, MRI scans are used to monitor the blood vessels flow, identify the abnormal tissue, monitor the tumors in the breast, identify the tears in the ligament, function of bones and monitor the internal organs in the heart, kidney, liver and spleen.

1.5.3 Positron Emission Tomography (PET)

Positron Emission Tomography (PET) scans uses advance camera and radioactive chemical to observe the internal organs of the patients. A radioactive chemical is in the form of glucose and it is often used during the PET scans to collect the cancer cells from the patient's body. This type of chemical is also called a tracer. In general, a radioactive chemical is injected into the human body via intravenous and its chemical, moves into the internal body where the cancer cells or abnormal cells are present.

In other words, PET scans use radioactive drug to image the metabolic processes of the patients. The essential role of PET scan is to observe the tissues and organs of the patients who are affected by various health problems such as heart disease, prostate cancer, liver cancer, breast cancer and tuberculosis. During the PET scans advance cameras are used to take the photos of positrons and then they are stored into an external storage device. Compared to Computed Tomography (CT) scans or Magnetic Resonance Imaging (MRI), PET scan does not provide more details about the internal organs because the PET scan shows only about the location of the radioactive chemical. Nowadays, health care organization and image processing experts combine the PET scans and CT scans to monitor the internal organs and the location of the radioactive chemical efficiently.

1.5.4 Positron Emission Tomography and Computed Tomography (PET-CT)

Nowadays, PET and CT scans are combined to take a complete scan that observes the internal organs and monitor the abnormal anatomic location. This type of scan produces results more effectively than the two scans (PET and CT scans) taken separately. PET-CT scans are widely used to identify cancer in the early stage. Results from this type of scans are used by the doctor to take necessary treatment. Efficient use of PET-CT scan may help to recover people from various cancers and other diseases.

PET-CT scans are also used to determine the levels and areas of cancer spread in the human body, continuous monitoring of effectiveness of cancer therapy, observe the blood flow between heart muscle, determine the heart attack effects, monitor the abnormal conditions of the brain and identify the brain disorders, brain tumors and bleedings in the brain.

1.5.5 Single Photon Emission Computed Tomography (SPECT)

A single Photon Emission Computed Tomography (SPECT) scan is used to monitor the internal blood flow and functions of internal organs. This type of scans integrates two different technologies, namely Computed Tomography (CT) and a radioactive material also called a tracer. In general, a radioactive chemical is in the form of glucose and it is often used during the PET scans to collect the cancer cells from the patient's body. Radioactive chemical or tracer is injected into the human body via intravenous. Tracer or radioactive chemical moves to the internal body where the cancer cells or abnormal cells are present. Moreover, radioactive chemical injects the gamma rays into the body which are identified by the scanner.

Advance computers and cameras are used to collect the gamma rays from the human body and then it is stored as two-dimensional cross-sections in the external storage devices. Finally, 2 dimensional cross-sections are converted into 3 dimensional images for efficient monitoring of the human brain and its abnormal conditions. In other words, SPECT scan uses various types of gamma cameras to image the functions of organ, internal blood flow and brain. In general, gamma cameras rotate over a 360 degree around the patient to take a single SPECT scan in three dimensional formats.

1.5.6 Ultrasound

Ultrasound imaging is one of the medical imaging technologies to observe the swelling, Pain and infection in the internal body. Ultrasound technology uses high frequency sound waves to collect the internal pictures of the human body. The high frequency sound waves created from the ultrasound imaging technology converts the returning sound echoes into a digital image. The resultant image generated from the ultrasound technology is used to identify the heart diseases, monitor the human body parts affected after a major heart attack, determine the baby health conditions, monitor the pregnancy status and identify the abnormal situations in the blood vessels.

1.5.7 X-ray

X-ray technology is one of the familiar imaging technologies used to take pictures of the internal body. Ionizing radiation is injected from the X-ray machines to the human body that shows the body parts by black and white shades. In general, bones have more calcium than other parts of the body. Hence, the resultant image shows the bones as white color and other parts as black color. X-ray images are used to observe the fractures in the bone, identify the swallowed objects, monitor the blood vessels and observe the condition of the lungs.

SECTION 1.6

MOTIVATION OF RESEARCH

Motivation for image fusion is the result of recent advancements in the field of image scanners or sensors. As the new image scanners are of high resolution and are available at low cost, multiple sensors are used in a wide range of imaging applications. These sensors are of high spatial and spectral resolution and also faster scan rates. The images taken by these sensors are more reliable, informative and contain complete picture of the scanned environment. Thus, they help in improved performance of dedicated imaging systems. As the number of scenes increase in a medical application, the more proportionate amount of image data is collected. To improve imaging system's performance, development of additional sensors is permitted by a corresponding increase in die processing power of the system. A sensor grabs multiple images of a location and one of them will be considered for analysis. However, the considered image may not have good spatial and spectral resolution. To overcome this and to generate a fused image with high spatial and spectral resolution, this research work identifies the need for image fusion by developing new techniques to improve the performance of existing fusion techniques. There is need to provide the technique of image fusion more effectively through analytical study of medical images using wavelet transforms.

Image fusion technique within short time is able to overcome many obstacles that face the subject of image enhancement compared to traditional techniques used in various field like remote sensing, military surveillance and medical field. Therefore, more research is needed in this field to develop medical technology and so this is what this study is going to try to do combining most of the images features in one image using image fusion technique. This technique has an important role in providing information required by medical doctors in better medical diagnostics services.

SECTION 1.7

RESEARCH OBJECTIVES

The basic objectives of proposed research work are:

- ❖ Study of image fusion techniques and medical imaging modalities.
- ❖ To develop a spatial frequency discrete wavelet transform (Haar) and neutrosophic set.
- ❖ To evaluate the performance of the fused images using parameters such as Peak Signal to Noise Ratio, Root Mean Square Error and Correlation.

SECTION 1.8

CONTRIBUTION OF RESEARCH

This thesis contributes in the area of image fusion. Specifically it introduces a novel image fusion technique based on spatial frequency discrete wavelet (Haar) and neutrosophic set for improving the details of medical images for clinical diagnosis. The proposed technique gives improved medical images with higher PSNR. The importance of this work is improving the medical services such as disease diagnosis, monitoring and analysis.

Thesis Outline

Outline of the thesis is as follows:

Chapter 1 includes the theory related to the types of fusion techniques and medical imaging.

Chapter 2 covers a literature review which provides a summary of the work done on various image fusion techniques using medical images.

Chapter 3 deals with the preliminary definitions that are needed for the present study.

Chapter 4 analyzes the CT and MRI image fusion using spatial frequency discrete wavelet transform (Haar) and neutrosophic set.

Chapter 5 deals with the comparison of the performance of medical image fusion in terms of Peak Signal to Noise Ratio, Root Mean Square Error and Correlation.

CHAPTER 2

CHAPTER 2

REVIEW OF LITERATURE

Image fusion algorithms have started involving on the onset of fast techniques in development of imaging modalities and effective multimedia applications. In the literature, lots of image fusion algorithms have been suggested for various applications such as surveillance, navigation, security, remote sensing, medical imaging, object recognition, change detection, tracking, etc. In all these fields of applications, algorithms have been proposed in spatial and transform domain for multifocus images, multisensor images and images from different optic conditions. Medical imaging modalities are classified under multisensor category and vary among themselves in contrast and resolution to demonstrate the details of region of interest.

This chapter gives a brief review on the literature for various image fusion schemes in spatial domain and transform domain. Section 2.1 gives brief review on multimodal image fusion. Section 2.2 elaborates various wavelet based image fusion methods. Medical image fusion algorithms are given in section 2.3. Section 2.4 discusses review on morphological fusion methods. Section 2.5 gives brief review on fuzzy logic based fusion methods. Intuitionistic fuzzy based fusion methods are given in Section 2.6. Neutrosophic techniques are discussed in Section 2.7. Section 2.8 discusses review on other fusion methods.

SECTION 2.1

MULTIMODAL IMAGE FUSION

Rottensteiner et al. (2005) have developed a novel image fusion approach based on the hierarchical rule-based technique. The proposed approach is tested with remote sensing images of the urban areas to detect various buildings, trees and soil.

Huang and Jing (2007) have developed a novel pulse coupled neural network based multifocus image fusion approach. The proposed approach decomposes the input image into 8×8 pixel blocks. Image Laplacian features are computed for each block of the input image and the results generated by the Laplacian algorithm. The experimental results prove the efficiency of the visual effect and objective evaluation criteria of the final fused images generated from the novel pulse coupled neural network based approach.

Zhang (2010) has reviewed various multimodal image fusion approaches such as remote sensing images, computer vision, medical image processing and defense security, etc. This paper also reviews various hierarchical classification methods such as pixel level, feature level and decision level.

Wang et al. (2010) have proposed a novel multimodal image fusion approach based on the neural networks with dual channel pulse coupled method. In general discrete wavelet transform methods are widely used in multimodal image fusion. This type of existing methods initially decomposes the original images, where as the proposed dual channel pulse coupled neural network method does not decompose the original image. Hence the proposed method not employs more dual channel pulse coupled neural networks.

Bhateja et al. (2015) have developed two stage multimodal image fusion approach based on the Non-Subsampled Contourlet Transform and Stationary Wavelet Transforms. This hybrid approach is primarily developed to process the images generated from two distinct medical imaging sensor modalities.

Xu et al. (2016) have proposed multimodal image fusion algorithm based on particle swarm optimization with quantum mechanics. The optimized output of this algorithm is given to the pulse-coupled neural networks to get desired output in disease diagnosis.

SECTION 2.2

WAVELET BASED IMAGE FUSION METHODS

Wavelet based fusion methods work on the basis of extracting detail information from the source images. This detail information are injected into a fused image by a simple addition, averaging, maximum, minimum and more complex mathematical strategies. Wavelet based methods require more decomposition levels to fuse high resolution images. These methods can be combined with other methods such as singular value decomposition and entropy.

Yu et al (2001) fused the medical images by the wavelet based method with a maximum selection fusion rule.

Discrete wavelet transform of image signals produces a nonredundant image representation, it can provide better spatial and spectral localization of image information as compared to other multiresolution representations. The research results discrete wavelet transform schemes have some advantages over pyramid schemes such as increased directional information, no blocking artifacts that often occur in pyramid fused images, better signal to noise ratio and so on (Li et al 2002).

Myungjin choi et al (2005) proposed a remote sensing fusion scheme based on Curvelet transform. Curvelet are able to preserve edge details than wavelets, there by resulting in better fusion of multispectral and panchromatic images. This method uses fusion rules similar to that of wavelet fusion.

Jianwen Hu and Shutao Li (2012) presented a fusion scheme using multiscale directional bilateral bank. Source images are decomposed into the directional detail subbands and approximation subbands are fused according to the fusion criteria adopted for the subbands. Details subbands are fused using choose maximum fusion scheme and approximation subband are fused by averaging method.

A wavelet based fusion scheme that combines pixel and region based algorithms using fuzzy logic was proposed by Jamal Saeedi and Karim Faez (2012). Weighted averaging, selection by pixel based decision map and selection by region based decision map are used to fuse high frequency coefficients. To fuse low frequency coefficients, optimization based rule is implemented.

Yong Li (2013) proposed a multifocus image fusion method based on Curvelet fusion. This method uses adaptive weights to fuse high frequency coefficients and regional variance to fuse low frequency coefficients.

Zhiping Xu (2013) proposed a fusion scheme decomposing the input images into coarse and detailed layers in the multilevel scheme. This method utilizes local energy and contrast for coefficient selection in different layers. It preserves more details in the source images and improves the quality of the fused images. Superposition of selected coefficients in coarse and detailed layers gives the final fusion output.

Nonsubsampled shearlet transform is a fusion method in transform proposed by Ming Yin et al. (2014). This method uses singular value decomposition to estimate local structure details for low subband coefficients. Extracted features in low frequency subband reflect edge information effectively. By comparing the clarity indices of source images, band pass subband coefficient with higher clarity index is selected as fused value.

SECTION 2.3

MEDICAL IMAGE FUSION METHODS

In general, functional images provide low spatial resolution. On the other side, anatomical images give spatial resolution and lead to detection of lesions with millimeter accuracy. Functional images are able to detect lesions before the anatomy is damaged. Thus the fusion of anatomical and functional images could avoid undesired effects (Gonzalo Pajares and Jesus manuel de la Cruz 2004).

Weibei Dou et al (2007) proposed a four stage fuzzy fusion frame of brain tumour tissues on MRI images. The stages are (i) Registration of multispectral MRI images (ii) Defining fuzzy models to describe the characteristics of tumour (iii) Carrying out fusion based on fuzzy fusion operators (iv) Making adjustments by fuzzy region growing using fuzzy connecting concept.

James and Dasarathy (2014) classify medical image fusion research based on popular image fusion schemes, imaging systems and region of interest. The trivial technical challenges in medical image fusion are the limitation of imaging devices, cost of imaging and nature of clinical diagnosis and reliability of fused images.

SECTION 2.4

MORPHOLOGICAL FUSION METHODS

Morphological methods are applied in the spatially relevant details present in the medical images using structuring elements that define opening and closing operations. Fusion of CT and MRI images in brain diagnosis involves the detection of scale specific features. Morphological fusion strategies are highly sensitive to noise, outliers, shape and size of the features.

An edge preserved fusion scheme based on multiscale toggle contrast operator is proposed by Xiangzhi Bai (2011) which uses multiscale structuring elements with same shape and varying sizes, multiscale dilation and erosion features. Fused image is constructed by combining dilation and erosion features with base image.

Yong Jiang and Minghui Wang (2013) proposed a fusion scheme with morphological component analysis. This method combines cartoons and texture component of source images instead of combining a single component. Cartoon image describes the smooth changes with respect to the illumination and edges. Texture map gives texture information of regions covered by edges.

Xiangzhi Bai (2013) presented image fusion scheme based on multi-scale top-hat transform which uses multiscale structuring elements with the same shape and varying sizes. This is used to extract the bright and dim features for image fusion.

SECTION 2.5

FUZZY LOGIC BASED FUSION METHODS

Harpreer singh (2004) proposed Fuzzy and neuro fuzzy algorithm. The work supplemented by algorithms which helps us to analyze the output qualitatively on attributes like entropy, statistical moments and uniformity.

MRI and PET image fusion using fuzzy logic and image local features is proposed by Umer Javed and Muhammad Mohsin Riaz (2014). This paper illustrates to maximally combine useful information present in MRI and PET images. Image local features are extracted and combined with fuzzy logic to compute weights for each pixel.

Myna and Prakash (2015) presented CT and MRI image fusion using fuzzy logic and discrete wavelet transform. Experiments are conducted for both Madami type fuzzy logic system and Sugeno type with varying number of membership functions. The results are analyzed using various performance metrices and Sugeno fuzzy logic system has produced better results compared to Madami fuzzy logic system.

Sasilakshmi and Ramya (2015) proposed fuzzy based multimodal medical image fusion. The real medical images are registered and a fusion using fuzzy logic is presented. The quality measures of the fused image is measured and the proposed method gives better results in terms of image quality metrices compared to other existing methods.

Kumaraswamy (2016) Presented common fusion methods like principal component analysis and wavelet transform along with quality assessment metrices. Exploratory outputs demonstrated in order that fuzzy based image fusion technique can actively retains more information compared to principal component analysis and wavelet transform approaches enhancing the spatial and spectral resolution of the satellite image.

Giriraj Prasad Rathor and Sanjeev Gupta (2017) proposed a new image fusion technique based on discrete wavelet transform to source images. It computes the weight of each source images and fuses the coefficients through weighted averaging method to acquire a combined picture.

Agarwal Ruchi Sanjay (2017) proposed type-2 fuzzy technique is applied on low level subbands and average fusion method is applied on the high level subbands in order to enhance the most prominent features present in CT and MRI image.

Srinivasa Reddy (2017) proposed neuro fuzzy based iterative image fusion of medical images. The method fuses images based on fuzzy inference system and experimental results are presented.

Gowsalya and Priyadharshini (2017) presented multimodal medical image fusion using nonsubsampled counterlet transform with type-2 fuzzy logic.

Mamata and Ramya (2018) presented image fusion using novel discrete wavelet transform and type-2 fuzzy logic systems. Experiments are conducted for already existing methods with varying number of membership functions.

Kullayama and Byna Niranjan (2019) proposed the image fusion using spatial frequency discrete wavelet transform and type-2 fuzzy logic. The images are instinctively decomposed into low level subbands and high level subbands by spatial frequency discrete wavelet transform. The next step for fusion process, type-2 fuzzy techniques is applied for low level and average fusion method is applied for high level subbands in order to intensify the most conspicuous features present in images.

SECTION 2.6

INTUITIONISTIC FUZZY FUSION METHODS

Balasubramaniam and Ananthi (2014) proposed a new way to fuse two or more images by using maximum and minimum operators in intuitionistic fuzzy sets. The paper evaluates the performance measures and the comparison results shows better results when compared to the other existing methods.

Ananthi and Balasubramaniam (2015) proposed a novel way to fuse several images using interval valued intuitionistic fuzzy sets. Spatial and textural information of the images are extracted and blended using maximum and minimum operations in interval valued intuitionistic fuzzy sets.

Rajkumar Soundrapandiyan (2017) presented intuitionistic fuzzy based multimodality medical image fusion. For fusion initially all source images are fused using discrete wavelet transform with average, maximum and entropy fusion rules.

Rajkumar Soundrapandiyan and Rishin Haldar (2018) discussed multimodality medical image fusion using block based intuitionistic fuzzy sets. The intuitionistic fuzzy sets are partitioned into image blocks and then recombined by the generated membership function. The experimental results shows better image visualization generated through the proposed method when compared to the other existing methods.

Asmit Kumar Dutta (2018) presented an image fusion technique using intuitionistic fuzzy sets. Registration is the major issue of intuitionistic fuzzy and the research work found solution for the problem. Image features were filtered and integrated with intuitionistic fuzzy logic and compute pixels. The proposed method produced better results compared to the existing methods.

SECTION 2.7

NEUTROSOPHIC TECHNIQUES

Salama and Florentin Smarandache (2014) presented a neutrosophic technique in image processing and discuss distance between the neutrosophic sets, the hamming distance, the normalized hamming distance and Euclidean distance.

Mangalraj and Anupam Agarwal (2017) proposed a novel regional based fusion rule. The indeterminacy on the input data is handled to provide a comprehensive fusion result, where the compared method fails. The proposed approach is tested against the multi-resolution analysis and multi-geometric analysis based techniques quantitatively and qualitatively based on the metrices.

Salama and Florentin Smarandache (2018) proposed a new technique for enchancing images. It will work on removing noise contained in the image as well as improving its contrast based on three different enhancing transforms by embedding the image into neutrosophic domain.

SECTION 2.8

OTHER FUSION METHODS

Bruzzone and Melgani (2003) using pixel and context based fusion strategies for multisensory data.

Dempter-shafer evidence theory is used by Le Hegarat-Mascle and seltz (2004) to provide decision level fusion for change detection on multispectral images.

Zheng (2007) proposed a fusion scheme based on discrete wavelet transform and PCA. Source images are decomposed into approximate images and detail images. PCA is applied to approximate images to evaluate weights for the fusion rule. Fused output for approximate images is obtained by weighted sum of the same. Detail images are obtained by consistency checking using select maximum principle.

Bovolo (2010) analyzed the impacts of pan sharpening techniques on change detection using an unsupervised similarity measure. Li (2010) analyzed the fusion of geographic information system data with images to detect changes.

Changtao He (2010) presented a fusion scheme integrating the advantages of PCA fusion methods. The low resolution intensity component of PET image in HIS space is replaced by a gray level MRI image with high spatial resolution and also minimizes redundancy present in the fusion result.

Senthil Kumar (2011) discussed maximum principle rule for low frequency elements are separated from the source images using Gaussian low pass filter. High frequency components are fused by weighted normalized sum of the difference images.

Cosmin Ludusan and Olivier lavialle (2012) proposed image geometry driven, antistrophic fusion model which is expressed using antistrophy reinforcing discretization scheme based on error estimation theory and partial differential equation.

Hengjun Zhao (2013) presented a neighbor distance based multifocus image fusion. Pixel’s sharpness is measured using neighbor distance which is deduced from the oriented distance in differential geometry.

CHAPTER 3

CHAPTER 3

PRELIMINARIES

SECTION 3.1

WAVELET

3.1.1 Wavelet Definition

The term ‘wavelet’ refers to an oscillatory vanishing wave with time-limited extend, which has the ability to describe the time-frequency plane, with atoms of different time supports. Generally, wavelets are purposefully crafted to have specific properties that make them useful for image processing.

Wavelet is a waveform which exists for limited time interval whose average value is zero. But the Fourier transform of a sine wave extends from $-\infty$ to $+\infty$. They are smooth and can be predicted. A mother wavelet is shifted and scaled to break a signal in wavelet transform. Wavelet is the basis function of wavelet transform. It is a short oscillation beginning at zero, then increasing to a peak value and then back to zero as shown in Fig.3.1. Wavelets can be used to extract information from an unknown signal by convolving a wavelet with portions of known signal.

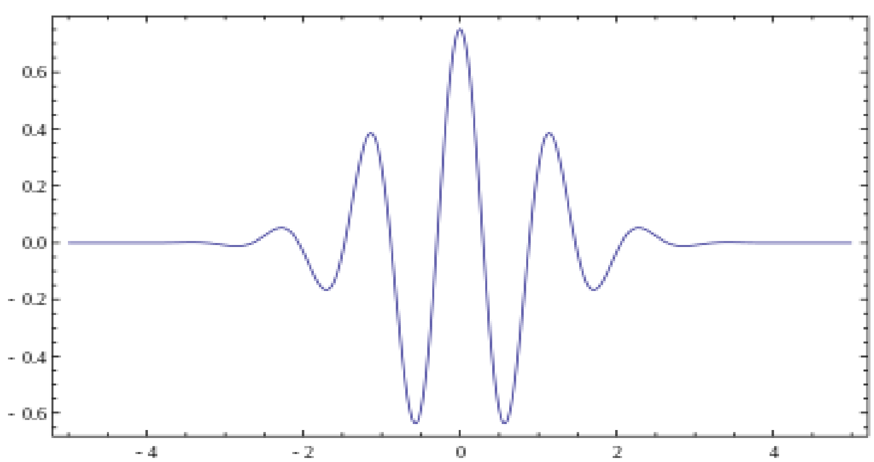


Fig.3.1 Wavelet

3.1.2 Wavelet Characteristics

Wavelet is a mathematical tool that can be used to extract information from many kinds of data, including audio signals and images. Mathematically, the wavelet ψ is a function of zero average, having the energy concentrated in time:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

In order to be more flexible in extracting time and frequency informations, a family of wavelets can be constructed from a function $\psi(t)$, also known as the 'Mother Wavelet', which is confined in a finite interval. 'Daughter Wavelet', $\psi_{u,s}$(t) are then formed by translation with a factor u and dilation with a scale parameter s:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi \left(\frac{t-u}{s}\right)$$

3.1.3 Wavelet Analysis

The wavelet analysis is performed by projecting the signal to be analyzed on the wavelet function. It implies a multiplication and integration:

$$\langle x(t), \psi_{u,s}(t) \rangle = \int x(t)\, \psi_{u,s}(t)dt$$

Depending on the signal characteristics that we want to analyze, we can use different scales and transitions of the mother wavelet. The particularity of the wavelet analysis is that it allows us to change freely the size of the analysis function, to make it suitable for the needed resolution, time or frequency domain. For high resolution in time-domain analysis we want to capture all the sudden changes that appear in the signal, we do that by using a contracted version of the mother wavelet. Conversely, for high resolution in the frequency domain we will be using a dilated version of the same function.

3.1.4 History of wavelets

The history of the development of the wavelets can be related to various works which are carried out in different fields. Haar introduced the first wavelet in 1909 and it was known as Haar wavelet. Dennis Gabor built a family of functions in 1946, by translating and modulating a generator function and it was named as Gabor atoms or Gabor functions.

Continuous wavelet transform which has initially addressed as cochlear transform was found by George Zweig in 1975. Continuous wavelet transform was formulated by Grossmann and Morlet in 1982. Meyer developed this theory in association with Daubechies who developed orthogonal wavelets.

Mallat who developed the implementation of filter-bank scheme for discrete wavelet transform. The filter-bank theory and wavelet theory are closely related and also known that the concept of Multiresolution analysis is very much related to the wavelet theory.

Though the continuous wavelet transform and discrete wavelet transform are considered to the basic wavelet transforms, there are many more that are related to wavelet theory, namely the wavelet packets transform and the complex wavelet transform.

3.1.5 Wavelet Properties

With the introduction of wavelet concept there is more number of wavelet functions developed by different persons. To be more precise, development of several different methods to create or compute wavelet functions have been carried out. Thus these developments resulted in each of them coming up with a set of wavelet functions. In each of these methods it was noted that some properties usually specified a parameter has been followed by a set of wavelet transform. These sets are being defined as that of wavelet families.

The following section will list out some basic wavelet properties and also will introduce the classification of some particular wavelet families based on those properties without considering the type of wavelet analysis. The usage of a particular wavelet family will vary with that of the properties of its wavelets and also in accordance to its application. The main qualitative parameters of wavelets are the following:

(i) Wavelet Filter Support:

Finite support can be said as the one of the most important and required wavelet properties because of the following reasons:

- Compact support will result in finite multiplication in the discrete wavelet transform, which will finally yield to simple practical implementation of that of a wavelet analysis.
- All computations are exact, since there is no need to approximate the wavelet function.
- The next main reason is about the good time localization in the wavelet.

(ii) Orthogonality:

The orthogonality property of the wavelet guarantees the independence of wavelets in time, making them very much excusive for the signal analysis. Since it is not possible to impose both the orthogonality and symmetry of wavelets simultaneously at the same time there is always a trade-off between them. Design of orthogonal wavelets which are symmetric to the degree possible was one of the major issues in research. It is also possible to exploit wavelets which are non-orthogonal.

(iii) Symmetry and Antisymmetry:

Phase characteristic of the scaling function and the mother wavelet plays a crucial role. The negative derivative of the phase is a group delay. It is very desirable in filters that have a linear phase and constant group delay. The linear phase of wavelet filter requires its scaling function to be symmetric or antisymmetric.

(iv) Existence of Scaling Function:

The simple rule about this property is the analysis is not orthogonal, when the scaling function does not exist. Since that the rules is more theoretical in nature and thus has no impact on the choice made on wavelet family to be applied in practice, since the non-orthogonality property of that of the particular wavelet family is always known.

(v) Number of Vanishing Moments:

Polynomial functions of certain order are suppressed along with the moment functions by some wavelets. Thus resulting in sparse representation of the wavelet analysis, which can be significantly helpful in saving the memory space while implementing. Again, as the number of vanishing moment increases the length of the wavelet also increases.

(vi) Time-Frequency Localization:

A good localization of wavelet in either time or frequency determines the possibility of a wavelet to detect particular phenomena in that domain. However, both the compact support and the band-limitations property cannot be achieved simultaneously.

If more frequency resolution is attained, the worse will be time resolution. Analysis in time proceeds by shifting the wavelet along the time axis and analysis in frequency proceeds by scaling the wavelet. Widening the wavelet, shifts its frequency content towards low frequencies and narrowing the wavelet shifts its frequency content towards high frequencies.

(vii) Expression:

An explicit representation of the wavelet will generally result in faster computation of their elements as well as coefficients of wavelet analysis.

3.1.6 Evolution of Wavelet Transform

Wavelet transform is the basic and simplest transform among numerous multiscale transform. The wavelet transform provides a time-frequency representation of a signal, resulting in the time-frequency localization of the signal. For many years, classical image processing was concentrated on the characterization of signals and on the designing of time-invariant and space-invariant operators that modify stationary signal properties. But the biggest amount of information is concentrated in the transients rather than its stationary signals. In the following, the evolution of the wavelet transform will be described, having as departure point of the Fourier transform.

(i) Fourier Transform

In 19th century, Joseph Fourier, a French mathematician and physicist showed that any periodic function can be decomposed in a series of simple oscillating functions, namely sines and cosines.

The generalization to the non-periodic signals has come only a century later and took the name of Fourier transform. The Fourier transform decomposes a signal in complex exponential functions at different frequencies. The equations used in the decomposition and reconstruction part are the following:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega$$

In the above equations, t stands for time, $\omega = 2\pi f$ for frequency, x denotes the signal in the time domain and X denotes the signal in the frequency domain. The computation of the Fourier transform is done over all times, making no distinction between signals stationary parts and transient ones.

The scaling property of the Fourier transform states that if we have scaled version of the version of the original x_s(t):

$$x_s(t) = x(st)$$

Then, its corresponding Fourier transform will be $X_s(\omega)$:

$$X_s(\omega) = \frac{1}{|s|} X\left(\frac{\omega}{s}\right)$$

We can observe from the last two equations that if we reduce the time spread of x by s (if s >1) than the Fourier transform is dilated by s, meaning that if what we have gained in time localization, we have lost in frequency localization. Projecting the signal on complex exponentials leads to good frequency analysis, but no time localization. The poor time localization is the main disadvantage of the Fourier transform, making it not suitable for all kind of applications.

(ii) Short Time Fourier Transform

The base concept of the Short Time Fourier Transform describes, the frequency content of a signal changes over time, we can cut the signal into blocks and compute the spectrum of each block.

For computing Short Time Fourier Transform we simply multiply the original signal by a window function, which is non-zero for only a short period of time, and then we compute the Fourier transform of the obtained signal.

The result is a two-dimensional representation of the signal, which can be written mathematically as:

$$X(\mathrm{T}, \omega) = \int_{-\infty}^{\infty} x(t)\omega(t - \mathrm{T}).e^{-j\omega t}\mathrm{dt}$$

Where $\omega(t)$ is the window function, commonly a Hann window or a Gaussian centered around zero, and x(t) is the signal to be analyzed. This equation can be interpreted as an analysis of the signal by a sliding window in time or by sliding bandpass filter in frequency. A particularity of this transform is the fact that the window is of constant length throughout the whole analysis process, meaning that the transform has a fixed resolution in time and frequency.

Time and frequency energy concentrations are restricted by the Heisenberg uncertainty principle. If we consider a finite energy function, $f \in L^2(R)$ and we consider it centered around zero in time and its Fourier transform, F(ω) centered around zero in frequency, then the temporal variance, ${\sigma^2}_t$ and the frequency variance, ${\sigma^2}_\omega$ of the wave function satisfy the condition, ${\sigma^2}_t{\sigma^2}_\omega \geq \frac{\pi}{2}$

$${\sigma^2}_t = \frac{1}{\|f\|^2}\int_{-\infty}^{\infty} t^2\,|f(t)|^2 dt$$

$$\sigma^2{}_{\omega} = \frac{1}{8\pi\|f\|^2}\int_{-\infty}^{\infty}\omega^2\,|f(\omega)|^2 d\omega$$

By $\|f\|$ we have denoted the norm of the function f, computed as $\sqrt{\int_{-\infty}^{\infty}|f(t)|^2 dt}$

Depending on the time localization we can choose width of the analysis window, namely a short window for a good time but poor frequency localization or a wide window for good frequency localization with the price of poor time localization.

(iii) Comparison of the Wavelet with Fourier Transform and Short Time Fourier Transform

Wavelet analysis is actually used to compute several magnifications of signals with distinct resolutions. The Fourier analysis can be done using basic building blocks also known as time frequency atoms namely sine and cosine waves.

In wavelets there are two different wavelets namely mother wavelet and child wavelets. Mother wavelet is oscillating and it is translated and dilated by some transactions and dilations so as to generate child wavelets. These two are used as building blocks of the wavelet analysis.

(iv) Why Prefer Wavelet?

An image can be decomposed at different levels of resolution and be sequentially processed from low resolution to high resolution using wavelet decomposition because wavelets are localized in both space and frequency domains.

It is easy to capture local features in a signal. Another advantage of wavelet basis is that it supports multi-resolution. With wavelet based decomposition, the window sizes vary and allow analyzing the signal at different resolution levels.

Wavelet transform decomposes an image into various sub-images based on local frequency content. It represents an image as a sum of wavelet functions with different location and scales. Any decomposition of an image into wavelets involves a pair of waveforms. These represent the high frequencies corresponding to the detailed parts of an image called as wavelet function. The other represent low frequencies or smooth parts of an image called scaling function.

The principle of the wavelet decomposition is to transform the original raw image into several components with single low-resolution component called ‘approximation’ and the other components called ‘details’. The approximation component is obtained after applying a bi-orthogonal low-pass wavelet in each direction that is horizontal and vertical followed by a sub-sampling of each image factor of two for each dimension. The details are obtained with the application of low-pass filter in one direction and a high-pass in the other or a high-pass in both the directions.

The noise is mainly present in the detail components. A higher level of decomposition is obtained by repeating the same operations on the approximation. For small details it is not obvious in diagnosis of images to know what is needed to eliminate or to preserve.

The basic approach of wavelet based image processing is

- Compute the two-dimensional wavelet transform of an image.
- Alter the transform coefficients.
- Compute the inverse transform.

The images are considered to be matrices with N rows and M columns. At every level of decomposition the horizontal data is filtered, then the approximation and details produced form this are filtered on columns. At every level, four sub images are obtained, the approximation, the vertical detail, the horizontal detail and the diagonal detail. The next level of decomposition can be obtained by the decomposition of approximation sub-image.

The horizontal edges of the original image are present in the horizontal detail coefficients of the upper-right quadrant. The vertical edges of the image can be similarly identified in the vertical detail coefficients of the lower-left quadrant. To combine this information into a single edge image, we simply zero the approximation coefficients of the generated transform. Compute the inverse of it and obtain the absolute value.

SECTION 3.2

WAVELET TRANSFORM

The aim of the image transform is to pack as much information as possible into smallest number of coefficients. Need of image transform is to convert the data into a form where the compression is easier which facilitates reduction of redundant irrelevant information. Fast data computation is also possible in transform domain.

Wavelet has finite energy and limited duration signal which is referred as a basic function. Each basis function represents small wave. Representing an image in the form of basis function is called wavelet transform. It gives both time resolution and frequency resolution. Wavelet transform provides multi-resolution analysis for a given signal or image.

If we consider a function $x \in L^2(R)$ and for analysis we use mother wavelet with its scaled and translated versions, we can write the wavelet transform of $x(t)$ at time u and scale s as:

$$W_x(\mathrm{u,s}) = \langle x(t), \psi_{u,s} \rangle = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-u}{s} \right) dt$$

In this wavelet transform can be seen as a convolution between the signal to be analyzed and the reverse function, $\frac{1}{\sqrt{s}} \psi^* \left(\frac{-t}{s} \right)$ derived from the mother wavelet.

3.2.1 Multi-resolution Analysis

Representation of signals or images at different resolutions is called multi-resolution. For better interpretation, the large objects or high contrast images require low resolution, where as the small objects or low contrast images require high resolution. This can be achieved by multi-resolution approach. Different resolutions of signals or images are achieved by filtering and sub sampling operations. Sub sampling is nothing but reducing sampling rate or removing of some samples. This multi-resolution concept is very useful in wavelet analysis in image processing.

SECTION 3.3

CLASSIFICATION OF WAVELET TRANSFORM

Wavelet transform is defined as the sum over all time of the signal multiplied by scaled, shifted version of mother wavelet. A set of wavelets are used to analyze the data and complementary wavelet sets are used to decompose the data without overlapping. Decomposition process is reversible. Wavelet transform can be broadly classified as:

- Continuous wavelet transform
- Discrete wavelet transform

3.3.1 Continuous Wavelet Transform

Continuous wavelet transform decomposes a continuous time signal over dilated and translated functions called wavelets into a highly redundant function.

Wavelet transform is a mathematical tool that has emerged as a latest tool for multi-resolution analysis of continuous time signal. It could handle the signals at different times with different frequencies or scales in a different way. This enables wavelet transforms to provide the time-frequency representation of a signal. In wavelet transform, higher frequencies have good time localization and lower frequencies have good frequency localization. This can be achieved by multiplying the signal with an orthogonal wavelet function.

Given an input signal$x(t)$, the continuous wavelet transform of $x(t)$ is defined as

$$X_{\omega}(a,b) = \frac{1}{\sqrt{b}} x(t)\, \psi\left(\frac{t-a}{b}\right) dt$$

Where the location factors a can be a real number and the scaling factor b can be a positive real number. The mother wavelet $\psi(t)$ is a well-defined function so that the continuous wavelet transform has low computational complexity, and it is reversible. As b is large, $\psi\left(\frac{t-a}{b}\right)$ is more like a high frequency signal, and thus the output $X_{\omega}(a,b)$

would represent the high frequency component of $x(t)$ after performing the inner product with $\psi\left(\frac{t-a}{b}\right)$ is small.

3.3.2 Discrete Wavelet Transform

The main drawback of continuous wavelet transform is its redundancy. To overcome this scale and translation parameters were discretized. This version of wavelet transforms was called as wavelet families. Frames that are used in wavelet transform derive a group of coefficients in the transformation domain. The number of coefficients derived is finite in number. The coefficients thus yielded, corresponds to the points on a two-dimensional lattice or grid. These coefficients are discrete points in the scale-translation domain. The mother wavelet can be indexed by two integers j and n. It corresponds to the points in the lattice. 'j' represents the discrete scale steps and 'n' represents the discrete translation steps. Let ${s_0}^j$ be the dilation parameter and $\mathrm{n}u_0{s_0}^j$ be the translation parameter. Here s_0 is the discrete scale and u_0 is a translation step.

Wavelet frame of a continuous time signal x(t) with finite energy is given by,

$$W_x(j,n) = \langle x(t), \psi_{j,n} \rangle = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{{s_0}^j}} \psi^*\left(\frac{\mathrm{t}-\mathrm{n}u_0{s_0}^j}{{s_0}^j}\right) dt$$

Original signal is represented by the wavelet coefficients. The representation is based on the mother wavelet chosen. Wavelet frames are defined for the positive values of s_0. Since the mother wavelet can be used, the frames can also have negative scales. They also provide good time and frequency localization. One and two are the most commonly used values for s_0 and u_0 respectively. Here discretization of scale takes place and this result in Dyadic wavelet in time, which has a constant time interval, u = 2^jT.

Discrete Wavelet Transform of a signal x(t) can be expressed as,

$$\mathrm{DWT}_x(2^j, u) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{2^j}} \psi^*\left(\frac{\mathrm{t}-\mathrm{u}}{2^j}\right) dt$$

Continuous wavelet transform is performed by scaling the analysis window, applying a time shift to the window and then the signal for which continuous wavelet transform has to be found is multiplied with it and integrated for all the time. In discrete wavelet transform the filters with different frequencies are used to analyze the signal at different scales. This can be implemented by a bank of low pass and high pass filters. High frequency component of the signal are analyzed by high pass filters and low frequency components by low pass filter.

A measure which quantifies the detail information of a signal is called a resolution of a signal. Filtering helps to change the resolution. Up sampling and down sampling helps in scaling. Removal of some samples from a signal by reducing the sample rate is called subsampling or down sampling.

For example, when a signal is subsampled by k, it reduces in the signal by k times. When new samples are added to a signal the sampling rate increases. This is called up sampling. For example, up sampling by k means adding k new samples. This may be usually a value obtained by interpolation of two samples.

The procedure for decomposition of a signal using discrete wavelet transform begins with sending the signal through a bank of digital half band low pass filter whose impulse response is h[n]. This is similar to convolution of a signal with the filter's impulse response. This procedure can be defined mathematically as,

$$\mathrm{x[n]} * \mathrm{h[n]} = \sum_{k=-\infty}^{\infty} \mathrm{x[k]} * \mathrm{h[n-k]}$$

The operation involved in above equation will remove the frequencies which are half of the highest frequency in the signal. This is followed by subsampling of signal by 2. Hence the signal will have half the number of points. Now the signal scale is doubled. Amount of information depends on resolution and this is influenced by filtering operations. Thus, information content is reduced by half because of this filtering.

Filtering is followed by subsampling. Subsampling will not affect resolution. The signal subsampled by 2 can be expressed as,

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] * h[2n - k]$$

Discrete wavelet transform is computed by decomposing a signal into coarse approximation by approximation coefficients of the wavelet filter or by the sampling function and detail information by the detailed coefficients of the wavelet filter or by the wavelet function corresponds to high pass filtering.

Successive filtering with low pass and high pass filters result in decomposition of a signal. After First level of decomposition of a signal x[n], the output at a high pass filter is expressed as,

$$y_H[k] = \sum_n x[n]g[2k - n]$$

And the output at the low pass filter is expressed as,

$$y_L[k] = \sum_n x[n]h[2k - n]$$

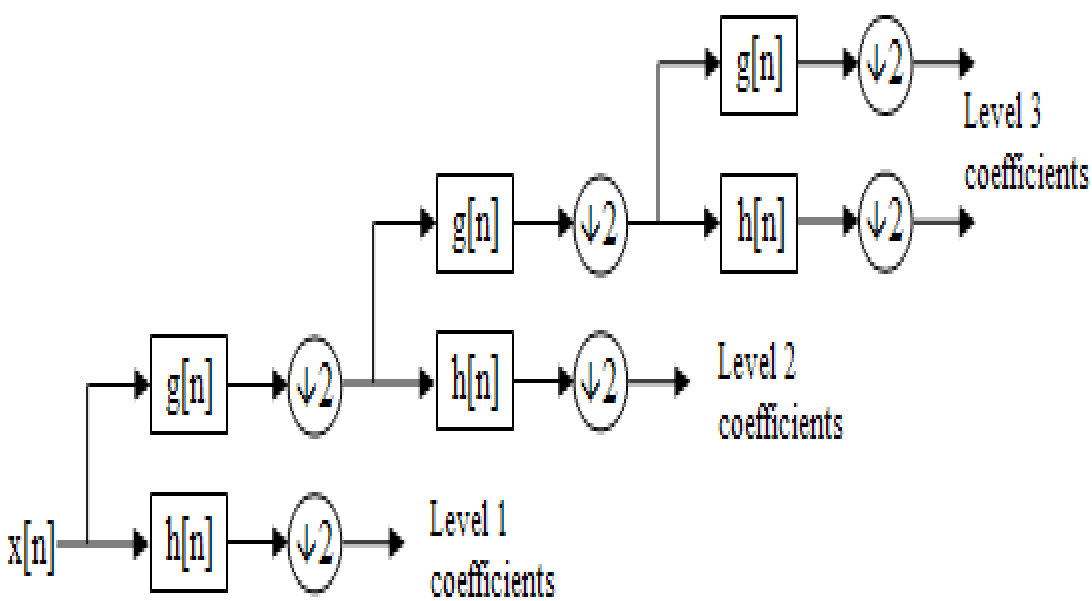


Fig.3.2 Third level wavelet decomposition

The procedure described above is called subband coding. This process can be repeated for more levels of decomposition. Filtering and subsampling at each level will produce half the number of samples and time resolution is also reduced by half.
Third level wavelet decomposition is shown in Fig.3.2. In this figure, h[n] and g[n] are low pass filters and high pass filters respectively.

Computation in discrete wavelet transform is fast. To perform discrete wavelet transform the length of the data vector must be an integer power of 2. Discrete wavelet transforms the data into a different vector, whose length is same as the data vector. For this purpose, discrete wavelet transforms which is made up of wavelet filter coefficients. The number of coefficients depends on the type of wavelet used. The transformation matrix is invertible and orthogonal.

To perform the reconstruction of the original data vector an inverse matrix is used. The transpose of the transformation matrix is the inverse matrix. Hence discrete wavelet transform of an input signal is rotating a function space, from input space domain to another domain.

SECTION 3.4

WAVELET FAMILIES

There are many members in the wavelet family. A few of them found to be more general are given below:

Haar wavelet is one of the oldest and simplest wavelet. Therefore, any discussion starts with the haar wavelet. Daubechies wavelet is popular and they are used in numerous applications. These are also called Maxflat wavelets as their frequency responses have maximum flatness at frequencies 0 and R. The Haar, Daubechies, Symlets and Coiflets are compactly supported orthogonal wavelets. These wavelets along with Meyer wavelets are capable of perfect reconstruction.

(i) Haar Wavelet

In wavelet transform haar wavelet is one of the simplest wavelet transforms. The Haar wavelet is used in computer engineering applications, such as signal and image compression. Haar wavelet is a sequence of rescaled square-shaped functions which together form a wavelet family or basis. Wavelet analysis is identical to Fourier analysis in target function over an interval to be represented in terms of orthonormal function basis. The Haar basis function is shown in Figure 3.3. The Haar sequence is recognized as the first known wavelet basis.

The Haar wavelet's mother wavelet function ψ(t) can be described as

$$\psi(t) = \begin{cases} 1 & 0 \leq t \leq \frac{1}{2}, \\ -1 & \frac{1}{2} \leq t \leq 1, \\ 0 & otherwise \end{cases}$$

Its scaling function φ(t) can be described as

$$\varphi(t) = \begin{cases} 1 & 0 \leq t \leq 1, \\ 0 & otherwise \end{cases}$$

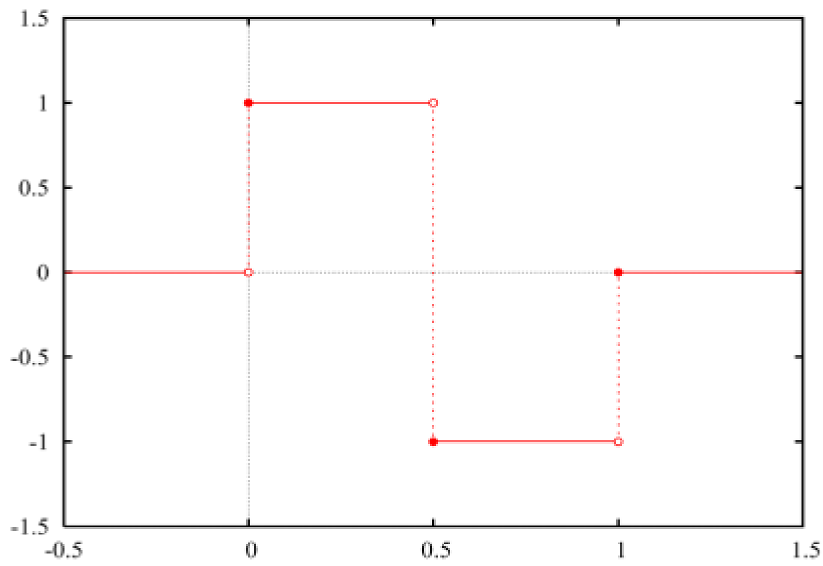


Fig.3.3 Haar wavelet basis function

(ii) Daubechies

Ingrid Daubechies developed new type of wavelets called Daubechies. Daubechies are compactly supported orthogonal wavelets and found application in discrete wavelet transform. It represents as (dbN) where db indicates family and N represent the either filter coefficients or vanishing moment order of wavelet function. Daubechies wavelet basis functions are shown in Figure 3.4. Analysis of Daubechies wavelet is shown in Figure 3.5.

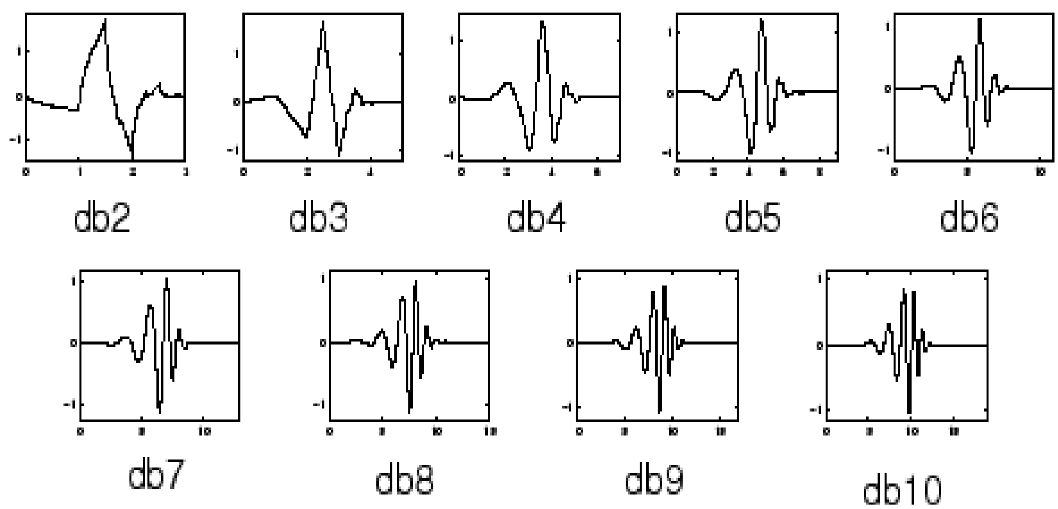


Fig.3.4 Daubechies wavelet basis function

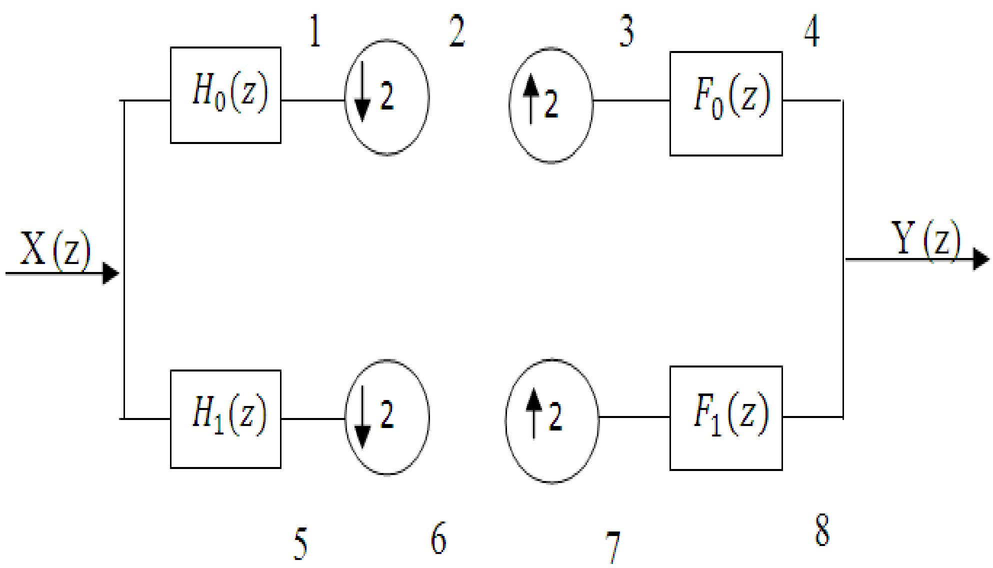


Fig.3.5 Wavelet analysis for decomposition and reconstruction

(iii) Coiflets

The wavelet function has 2N moments equal to 0 and the scaling function has 2N-1 moments equal to 0. The two functions have a support of length 6N-1. Cofilet wavelet basis function is shown in Figure 3.6.

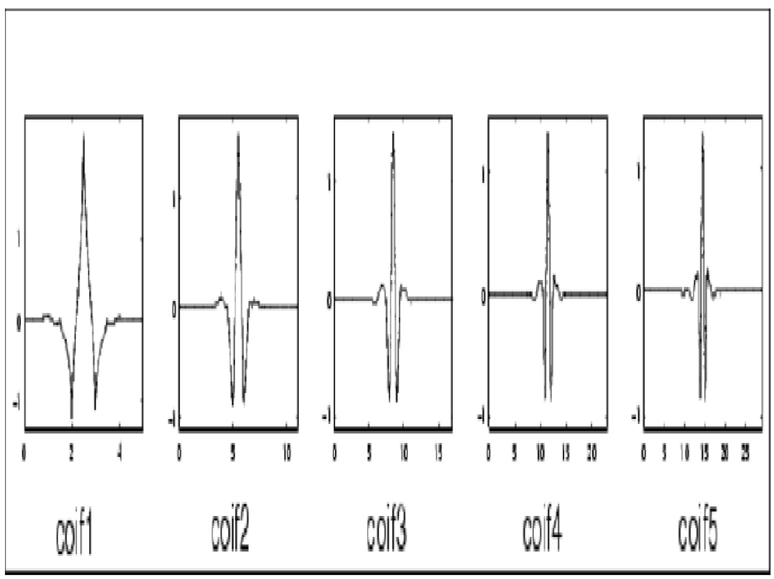


Fig.3.6 Coiflet wavelet basis function

(iv) Biorthogonal wavelet

Biorthogonal wavelet exhibits the property of linear phase, which is needed for signal and image reconstruction. Interesting properties are derived by employing two wavelets, one for decomposition and the other for reconstruction instead of the same single one. Biorthogonal wavelet is shown in Figure 3.7

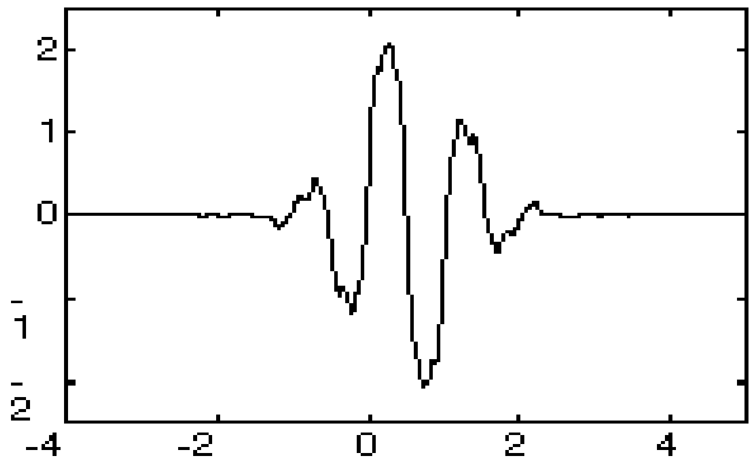


Fig. 3.7 Biorthogonal wavelet

(v) Symlets

The symlets are nearly symmetrical wavelet. The properties of the two wavelet families are similar. Symlet wavelet basis function is shown in Figure 3.8

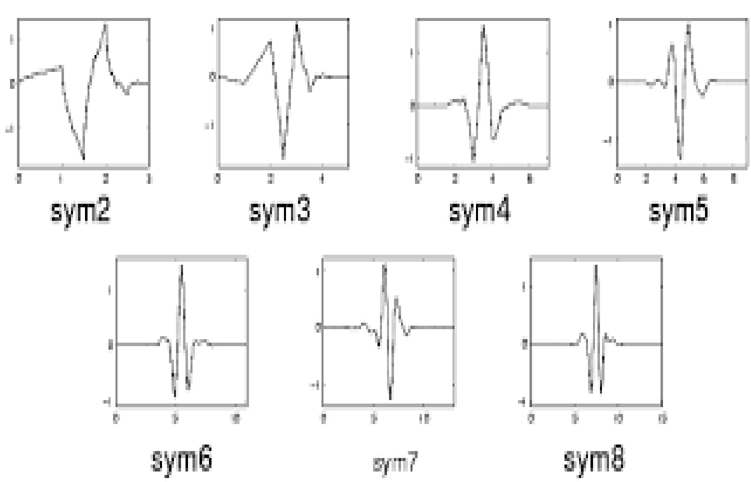


Fig. 3.8 Symlet wavelet

(vi) Mexican Hat

This wavelet has no scaling function and is derived from a function that is proportional to the second derivative function of the Gaussian probability density function. Mexican Hat is shown in Figure 3.9

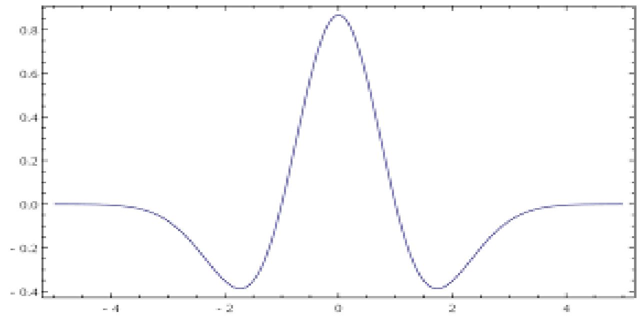


Fig. 3.9 Mexican Hat wavelet

(vii) Morlet

This wavelet has no scaling function. Morlet wavelet is shown in Figure 3.10

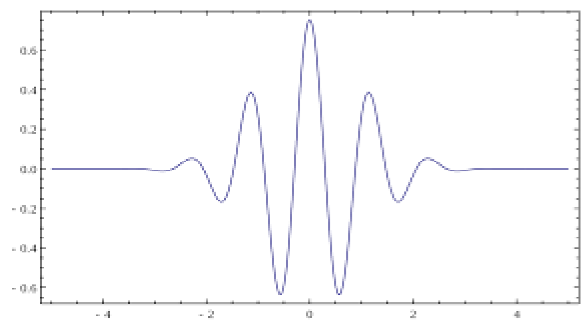


Fig.3.10 Morlet wavelet

(viii) Meyer wavelet

The Meyer wavelet and scaling function are defined in the frequency domain. Meyer wavelet is shown in Figure 3.11

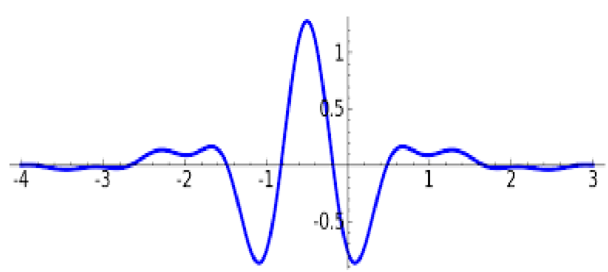


Fig.3.11 Meyer wavelet

SECTION 3.5
FUSION WITH WAVELET TRANSFORM

The idea of image fusion based on the wavelet transform is to apply the multiresolution decomposition on each source image. The coefficients of both the low and high frequency bands are then subjected to fusion rules as displayed in the centre block of Figure 3.12. The inverse discrete wavelet transform is applied for the combined wavelet coefficients and fused image is obtained. The fusion steps based on the wavelet transform are followed as:

Step 1: The input images must be registered to assure the corresponding pixels are aligned.

Step 2: The input images are decomposed using the wavelet transform.

Step 3: The transform coefficients are then fused using different fusion in the low and high frequency bands.

Step 4: An inverse wavelet transform is applied on the fused coefficients, and the fused image is constructed.

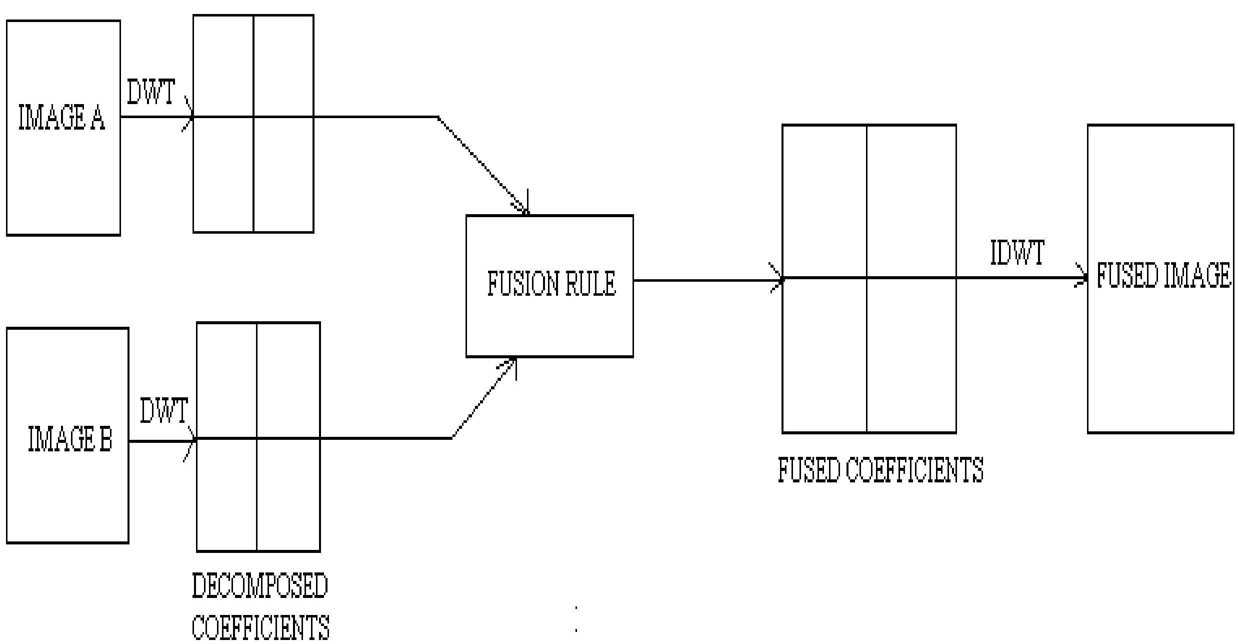
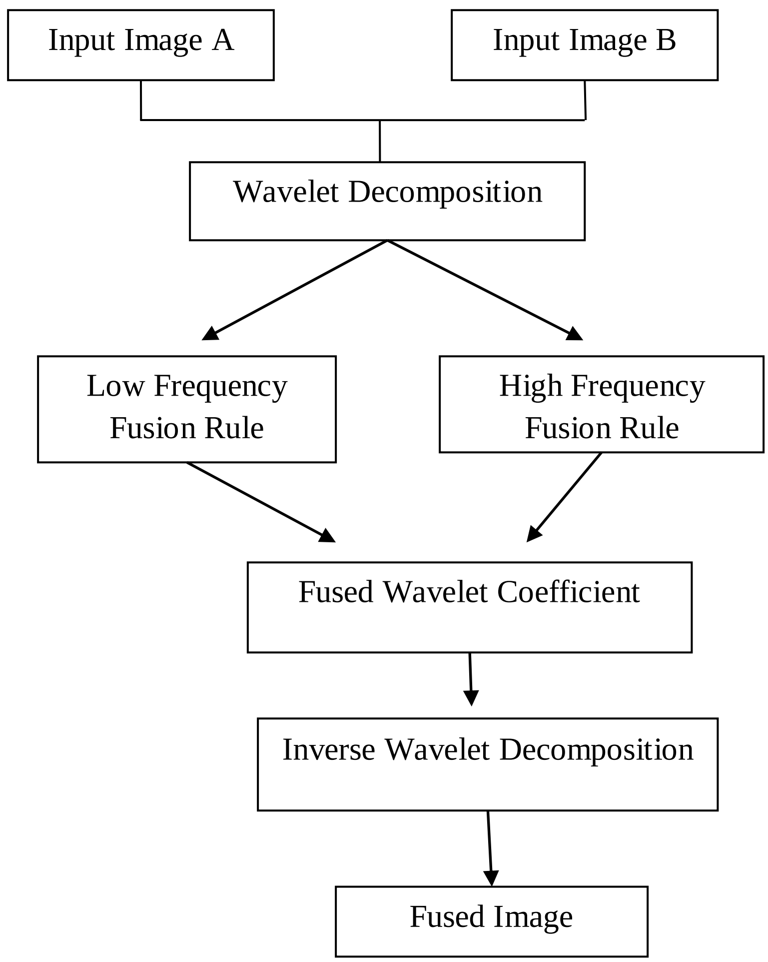


Fig. 3.12 The image fusion scheme using the wavelet transforms

3.5.1 Fusion Rule

According to the theoretical models of the human visual system, the human eyes have different sensitivity to the wavelet coefficients of low and high resolution bands. Based on this information, different fusion rules are applied to the low and high frequency bands separately. The flowchart of the fusion rule is followed by:



SECTION 3.6

NEUTROSOPHIC SETS AND SYSTEMS

Neutrosophic sets are generalized sets of fuzzy, intuitionistic fuzzy and interval valued fuzzy sets. Neutrosophic sets define indeterminate information as well as determinate information. The following discusses the detailed evolution of neutrosophic sets:

3.6.1 Evolution of Neutrosophic sets and systems

(i) Fuzzy sets

Lotfi zadeh defined fuzzy logic as practical complexity increases precision statements lose meaning. Fuzzy logic trades between significance and precision encountered by humans in day to day difficulties. Fuzzy logic is convenient way to map the input to output among various existing systems.

Fuzzy logic is easy to understand and flexible to use. In terms of time fuzzy logic is efficient and quickly dealing imprecise and nonlinearity. Fuzzy logic starts with fuzzy sets. Fuzzy set consists of without crisp or definite boundaries. Membership function is a curve which maps input space to the membership values between 0 and 1. The shape of the curve is defined based on simplicity, speed, convenience and efficiency.

Fuzzy set is extension of classical set where X is universe of disclosure, x denotes elements in X. Then the fuzzy set A in X is defined as

$$\mathrm{A} = \{x, \mu_A(x)/x \in X\}$$

Where $\mu_A(x)$ is mapping function of A in X. The membership function maps each element in X to membership value between 0 and 1.

In evaluating the real world problem decision making conflicting, inconsistent, indeterminate information is not expresses in terms of crisp values. Crisp values are induce imprecision and confusion to the inaccurate results. To reduce the fuzziness and vagueness of subjective information Zadeh proposed fuzzy set theory in 1965 and the decision making methods have developed by Bellman and zadeh in 1970 using fuzzy theory. Fuzzy set theory gives truth function which describes acceptance value categorized by an attribute but the constraint lies; it doesn't represent false function developed by Jian Guo in 2013.

(ii) Intuitionistic Fuzzy sets

However, the fuzzy set theory is not define false function which is rejection mapping value of corresponding alternative characterized by criteria it only expresses truth value. Intuitionistic fuzzy sets (Atanassov and Gargov, 1989) which is represent truth membership T(x) as well as false membership function F(x), they satisfy the condition

$$T(x), F(x) \in [0, 1] \text{ and } 0 \leq T(x) + F(x) \leq 1$$

In Intuitionistic fuzzy set the determinate function is rest of truth and false functions 1-T(x)-F(x) that is indeterminate and inconsistence functions are not defined clearly. Atanassov proposed Intuitionistic single value fuzzy number which is represent truth function as well as false function of decision making expressions.

(iii) Interval Valued Intuitionistic Fuzzy Numbers

Atanassov and Gargov in 1989 extend intuitionistic single value set to interval valued intuitionistic fuzzy numbers in terms of truth and false membership functions. But it is unable to define indeterminate function expressively; here indeterminate value is rest of truth and false functions value. The intuitionistic single value fuzzy number and interval valued intuitionistic fuzzy number are unable to represent indeterminate and inconsistence data of decision maker's information clearly.

Smarandache in 1999 generalizes fuzzy set, Intuitionistic single value fuzzy number and interval valued intuitionistic fuzzy number so on., proposed a Neutrosophic set which is represent truth function, false function and indeterminate functions which are independent. Recently, neutrosophic set became interesting area of researcher to convert qualitative information into quantitative values which is express supporting, nondeterministic, rejection values in terms of neutrosophic values.

Neutrosophic sets grasped researchers attention in the field of scientific or engineering point view. Wang in 2010 developed single valued neutrosophic sets in 2013 given correlations coefficient and weighted correlation coefficient in single valued neutrosophic sets.

Wang in 2005 proposed interval neutrosophic sets in which the truth membership function, indeterminacy membership function, false membership functions were extended to interval values.

3.6.2 Classification of Neutrosophic Set and Systems

To handle the indeterminate information and inconsistent information which exist in commonly real situations, Smarandache firstly presented a neutrosophic set from philosophical point of view.

It is a powerful general formal framework and generalized the concept of the fuzzy set, interval valued fuzzy set, interval valued intuitionistic fuzzy set, paraconsistent set and tautological set. Neutrosophy is a new branch of philosophy that studies the origin, nature and scope of neutralities as well as their interactions with different spectra. The truth membership, an indeterminancy membership and a falsity membership are represented independently. Its function $T_A(x)$,$I_A(x)$ and $F_A(x)$ are real standard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: X⟶$]^-0, 1^+[$,$I_A(x)$: X ⟶ $]^-0, 1^+[$,and $F_A(x)$: X ⟶$]^-0,1^+[$.

CHAPTER 4

CHAPTER 4

CT AND MRI IMAGE FUSION USING SPATIAL FREQUENCY DISCRETE WAVELET TRANSFORM (HAAR) AND NEUTROSOPHIC SET

SECTION 4.1

PROPOSED METHOD

Block diagram of proposed spatial frequency discrete wavelet transform (Haar) neutrosophic method.

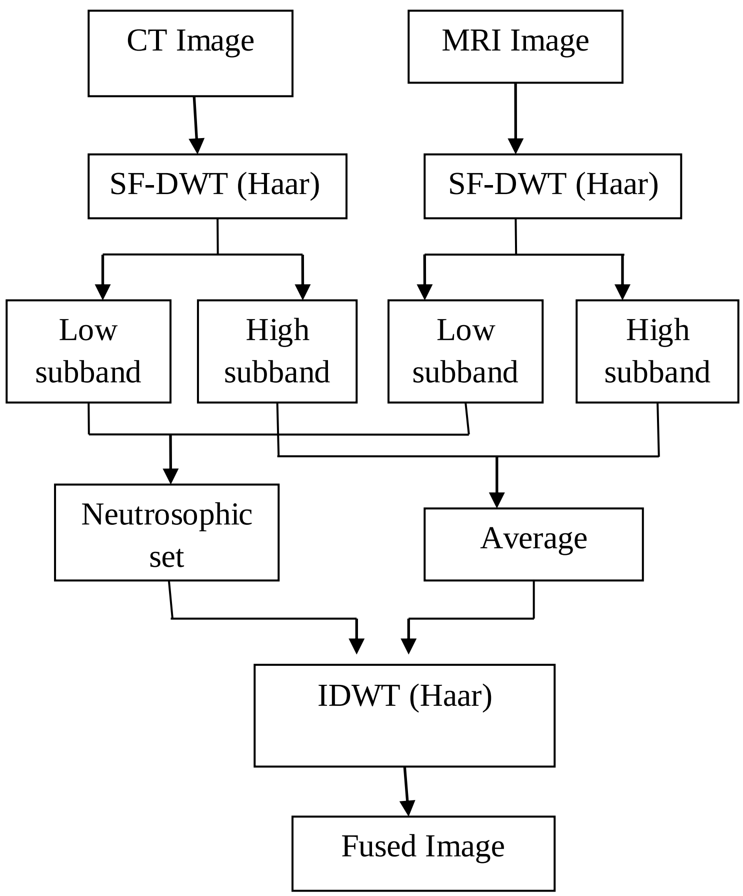


Fig.4.1

The steps involved in the proposed method are as follows:

1. Input- two images, (CT image (A) and MRI image (B)).
2. Decompose both input images using spatial frequency discrete wavelet transform (Haar).
3. Four images will be obtained approximate sub-image, Horizontal frequency subband, Vertical frequency Subband and Diagonal frequency subband.
4. Perform Neutrosophic set on the low-frequency subbands.
5. Apply average fusion rule for high-frequency subbands.
6. Apply inverse discrete wavelet transform (Haar) on the images to obtain a reconstructed fused image.

Section 4.2

Image Fusion using Spatial Frequency Discrete Wavelet Transform

Image fusion using spatial frequency discrete wavelet transform (Haar) and neutrosophic set is proposed. In this Section Spatial frequency evaluates the amount of frequency contents present in the image. It determines sharpness and spectral quality of the image. The spatial frequency discrete wavelet transform (Haar) will be more absolute in images with high frequency contents.

Spatial Frequency (SF) defined as M×N image F with gray value F(i,j) at position (i,j) is given by Spatial frequency, $SF = \sqrt{RF^2 + CF^2}$ where

Row frequency,

$$RF = \sqrt{\frac{1}{M * N}\sum_{i=1}^{M}\sum_{j=2}^{N}(F(i,j) - F(i,j-1))^2}$$

and Column frequency,

$$CF = \sqrt{\frac{1}{M * N}\sum_{j=1}^{N}\sum_{i=2}^{M}(F(i,j) - F(i-1,j))^2}$$

Where, F represents fused image, M and N denotes the dimensions of the fused image.

The proposed spatial frequency discrete wavelet transform (Haar) neutrosophic method consists of three steps: decomposition, fusion and reconstruction. The block diagram of the proposed spatial frequency discrete wavelet transform (Haar) neutrosophic method is shown in Fig.4.1.

4.2.1 Decomposition

In the proposed method, a two-level spatial frequency discrete wavelet transform (Haar) decomposition technique is employed to decompose the input images. In Discrete wavelet transform (Haar) decomposes the image into a low-level subband and high-level subbands. This is the first level of decomposition. The low-level subband is decomposed for the second time at second level to produce another set of low-level, high-level subbands. The four components selected for the fusion of the images are the approximate sub band, Horizontal detail subband, vertical subband and Diagonal subband. The decomposing procedure is defined as

$$[A_1, H_1, V_1, D_1] = \text{dwt2}(A, \text{'Haar'})$$

$$[A_2, H_2, V_2, D_2] = \text{dwt2}(B, \text{'Haar'})$$

Where A and B are source images, A_1, H_1, V_1, $D_1$and A_2, H_2, V_2, D_2 are decomposed coefficients of A and B images respectively. The obtained high-frequency and low-frequency subbands of the two images are fused using a fusion algorithm.

4.2.2 Fusion

(i) Fusion of Low-frequency Sub-images

A neutrosophic set is introduced by Florentin Smarandache in 1995. The neutrosophic logic deals with neutral values to determine the membership, non-membership and indeterministic values in a particular set. It is used to solve the problems of uncertainty.

Let U be a universe and $W \subseteq U$ which is composed by the bright pixels. A neutrosophic image P_{NS} is characterized by three membership sets T, I and F. A pixel P in the image is described as P(t, i, f) and belongs to W in the following way: it is t% true, i% indeterminate, and f% false, in the bright pixel set, where t varies in T, i varies in I and f varies in F.

The pixel P(i,j) in the image domain is transformed into the neutrosophic domain and is given by, P_{NS}(i,j) ={T(i,j),I(i,j),F(i,j)}. T(i,j), I(i,j) and F(i,j) are the membership values defined as

$$\mathrm{T(i,j)} = \frac{\bar{g}(i,j) - \bar{g}_{min}}{\bar{g}_{max} - \bar{g}_{min}}$$

$$\bar{g}(\mathrm{i,j}) = \frac{1}{w \times w} \sum_{m=i-w/2}^{i+w/2} \sum_{n=j-w/2}^{j+w/2} g(m,n)$$

$$I(i,j) = \frac{\delta(i,j) - \delta_{min}}{\delta_{max} - \delta_{min}}$$

$$\delta(i,j) = abs\big(g(i,j) - \bar{g}(\mathrm{i,j})\big)$$

$$\mathrm{F(i,j) = 1 - T(i,j)}$$

Where g(i,j) is the intensity value of the pixel (i,j), $\bar{g}(\mathrm{i,j})$ is the local mean value of g(i,j), $\delta(i,j)$ is the absolute value of the difference between intensity g(i,j) and its local mean value $\bar{g}(\mathrm{i,j})$. We assume w=3, T, I and F denotes the approximate subbands of the images (A_1, A_2).

According to the information theory, a neutrosophic set entropy is calculated based on the following formula,

$$En_{NS} = En_T + En_I + En_F$$

$$En_T = -\sum_{i=\min\{T\}}^{\max\{T\}} P_T\,(i)\; ln\; P_T(i)$$

$$En_I = -\sum_{i=\min\{T\}}^{\max\{T\}} P_I\,(i)\; ln\; P_I(i)$$

$$En_F = -\sum_{i=\min\{T\}}^{\max\{T\}} P_F\,(i)\; ln\; P_F(i)$$

where En_T , En_I and En_F are the entropies of the sets T, I and F, respectively and $P_T(i)$, $P_I(i)$ and $P_F(i)$ are the probabilities of the elements in T, I and F, respectively.

Based on the entropy, corresponding values of the subbands are chosen to be present in the final image.

$$A_3 = \begin{cases} A_1, & if\ E_1 \geq E_2 \\ A_2, & otherwise \end{cases}$$

Where E_1 and E_2 are the entropy of the approximate subbands of the images A and B respectively.

(ii) Fusion of High-frequency sub-images

The High-frequency subbands are fused using the averaging filter. CT and MRI image are fused along with its each detailed subbands. Hence, three fused sub-images are obtained by the process,

$$H_3 = \text{average}\ (H_1 + H_2)$$

$$V_3 = \text{average}\ (V_1 + V_2)$$

$$D_3 = \text{average}\ (D_1 + D_2)$$

4.2.3 Reconstruction

According to the fusion algorithms, the four sub-images are fused and the inverse transformation is applied on the obtained four sub-images. Reconstruction is the inverse process of up-sampling of images. A rescaling filter is applied to the low frequency subband and wavelet filter is used for the high-frequency subbands. The reconstruction of the final image as follows:

$$F = \text{idwt2}\ (A_3, H_3, V_3, D_3, \text{'Haar'})$$

where F represents the final image.

1. Performance Measures

To evaluate the performance of the fused image, subjective and objective measures are used. The subjective measure relates the evaluation of visual perception and objective analysis of the fused image is done using various measures such as Peak Signal to Noise (PSNR), Root Mean Square Error (RMSE), and Correlation (CORR). Let us consider a source image S(i,j) and the fused image F(i,j) of size M∗N.

(i) Peak Signal to Noise Ratio (PSNR)

Peak Signal to Noise Ratio is used to access the improvement in the quality of the fused image. It is defined by,

$$\text{PSNR} = 10\, log_{10}\left(\frac{MAX^2}{\frac{1}{M*N}\sum_{i=1}^{M}\sum_{j=1}^{N}[S(i,j)-F(i,j)]^2}\right)$$

Here M∗N, S(i,j) and F(i,j) denotes the size of the image, source image and fused image respectively. MAX is the maximum value of an image. A higher PSNR value determines a better quality of the fused image.

(ii) Root Mean Square Error (RMSE)

Root Mean Square Error between the source image and the fused image can be calculated as

$$RMSE = \sqrt{\frac{1}{M*N}\sum_{i=1}^{M}\sum_{j=1}^{N}[S(i,j)-F(i,j)]^2}$$

RMSE value approaches zero whenever the reference and fused images are similar and it will increase when the similarity decreases.

(iii) Correlation (CORR)

The standard value is one when the ideal and fused images are similar and is less than one whenever dissimilarity increases.

$$\text{CORR} = \frac{2C_{rf}}{C_r + C_f}$$

Here $C_r = \sum_{i=1}^{M}\sum_{j=1}^{N} S(i,j)^2$,$C_f = \sum_{i=1}^{M}\sum_{i=1}^{N} F(i,j)^2$ and $C_{rf} = \sum_{i=1}^{M}\sum_{j=1}^{N} S(i,j)F(i,j)$

CHAPTER 5

CHAPTER 5

PERFORMANCE MEASURES OF PROPOSED METHOD

SECTION 5.1

COMPARISON RESULTS

Comparison of SF-DWT (Haar) Neutrosophic and DWT (Haar) Neutrosophic Image Fusion Techniques Interms of PSNR, RMSE and CORR are given in the following table:

Table 5.1

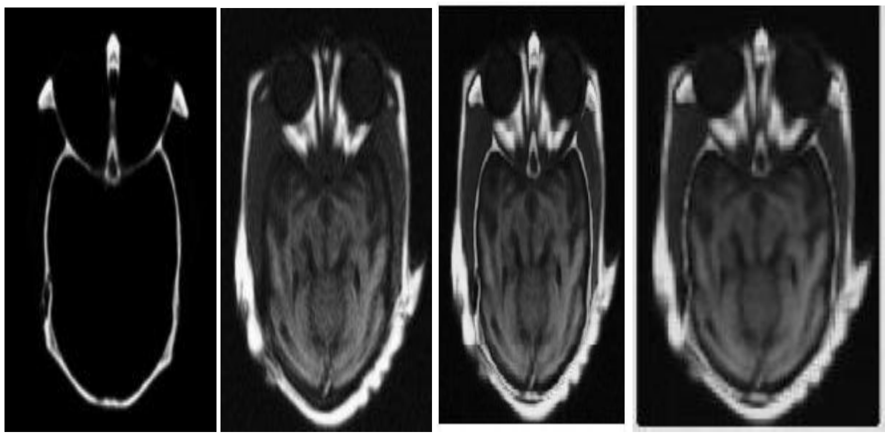
IMAGE	METHODS	PSNR	RMSE	CORR
Dataset 1	SFDWT(Haar)-Neutrosophic	46.7173	2.0259	0.9906
	DWT(Haar)-Neutrosophic	45.4826	2.4381	0.9140
Dataset 2	SFDWT(Haar)-Neutrosophic	44.6702	2.1196	0.9401
	DWT(Haar)-Neutrosophic	43.8110	2.1266	0.8189

Dataset 3	SFDWT(Haar)-Neutrosophic	46.4998	1.4558	0.9820
	DWT(Haar)-Neutrosophic	45.5284	2.8402	0.9361
Dataset 4	SFDWT(Haar)-Neutrosophic	44.3079	1.5331	0.9395
	DWT(Haar)-Neutrosophic	42.8203	3.3966	0.8661
Dataset 5	SFDWT(Haar)-Neutrosophic	44.3810	1.9066	0.9524
	DWT(Haar)-Neutrosophic	42.9735	2.5490	0.7190
Dataset 6	SFDWT(Haar)-Neutrosophic	46.2149	1.9915	0.9798
	DWT(Haar)-Neutrosophic	44.8189	2.1129	0.9444

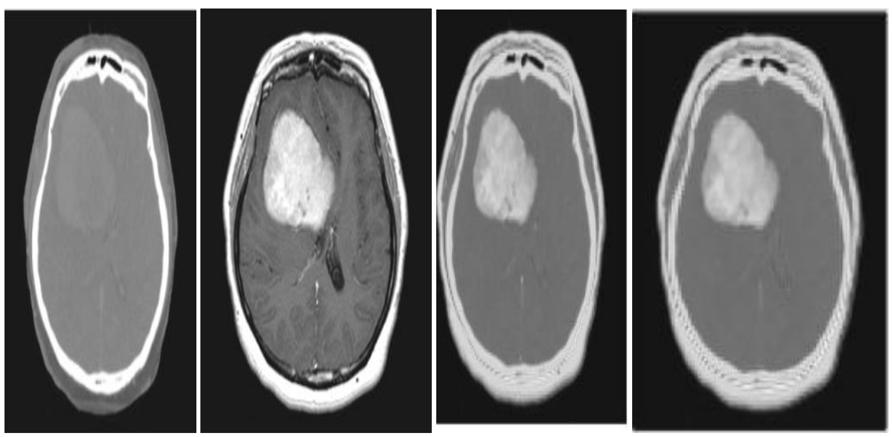
Dataset 7	SFDWT(Haar)-Neutrosophic	45.3570	1.8940	0.9616
	DWT(Haar)-Neutrosophic	44.6028	2.6298	0.9028
Dataset 8	SFDWT(Haar)-Neutrosophic	47.8409	1.3458	0.9857
	DWT(Haar)-Neutrosophic	46.3079	3.0190	0.9702
Dataset 9	SFDWT(Haar)-Neutrosophic	46.9878	1.3011	0.9817
	DWT(Haar)-Neutrosophic	46.6798	3.0357	0.9604
Dataset 10	SFDWT(Haar)-Neutrosophic	46.4100	1.4862	0.9802
	DWT(Haar)-Neutrosophic	46.0587	3.0674	0.9562

The efficiency of the proposed method can be seen from the following picture by comparing the obtained images with the previous images.

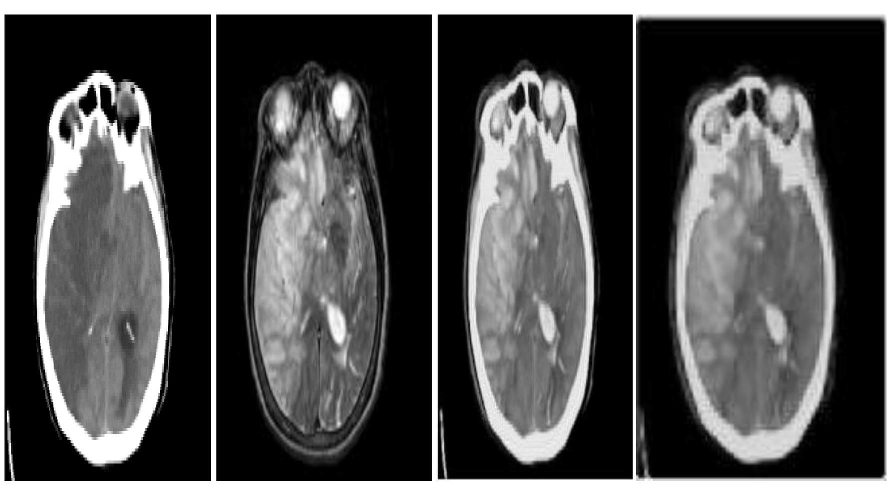
CT Images	MRI Images	DWT (Haar)-Neutrosophic	SFDWT (Haar)-Neutrosophic



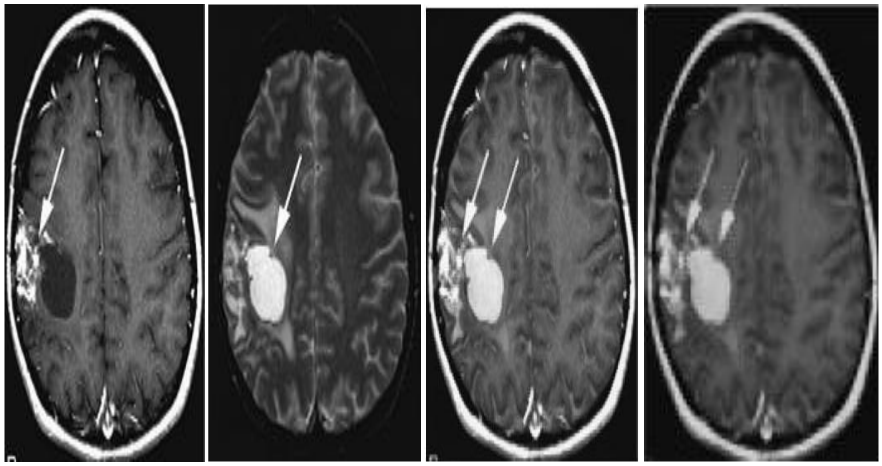
Dataset 1



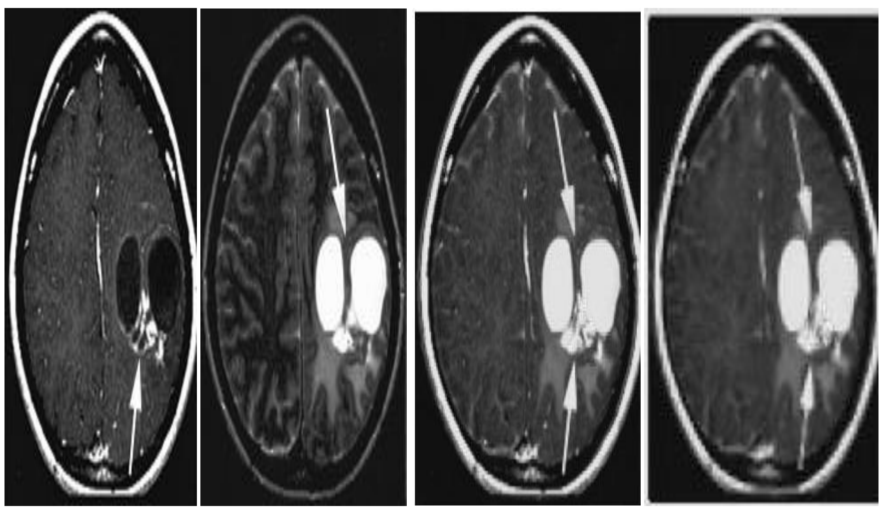
Dataset 2



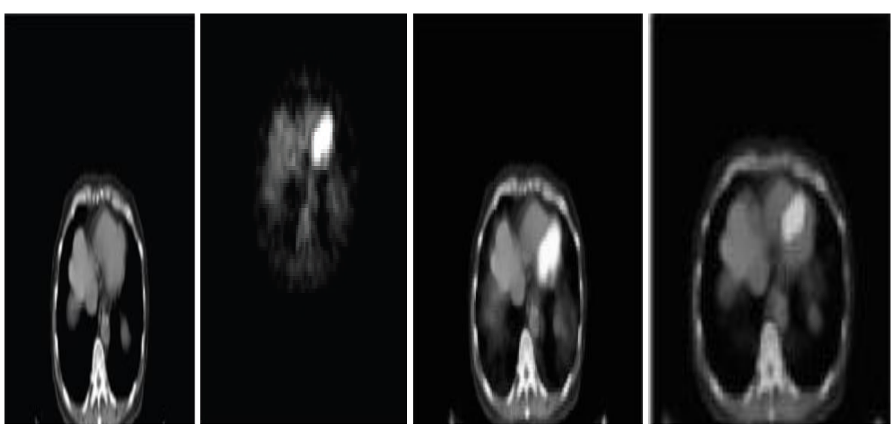
Dataset 3



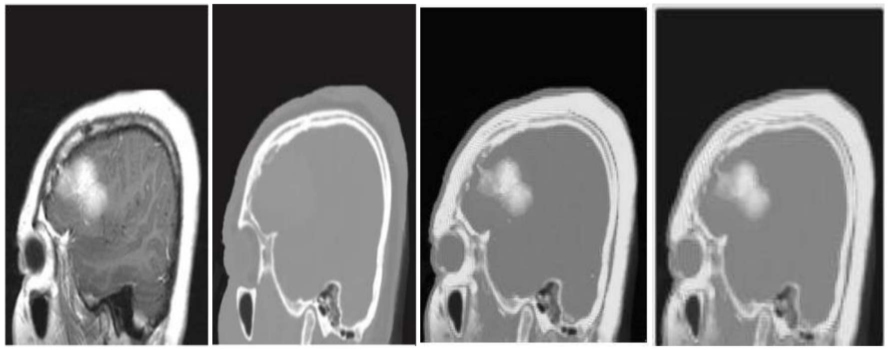
Dataset 4



Dataset 5



Dataset 6

Dataset 7


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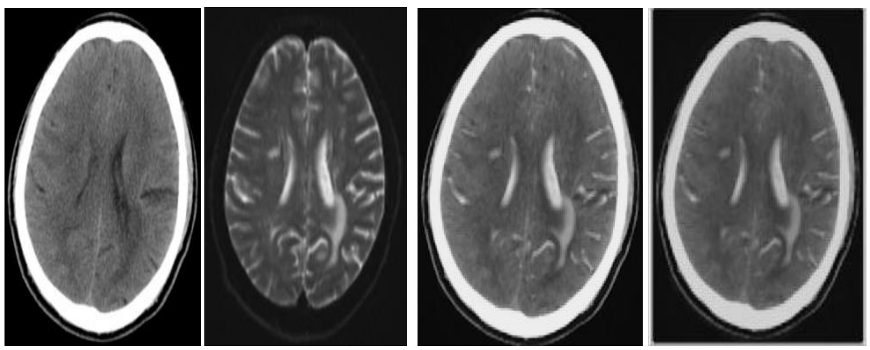
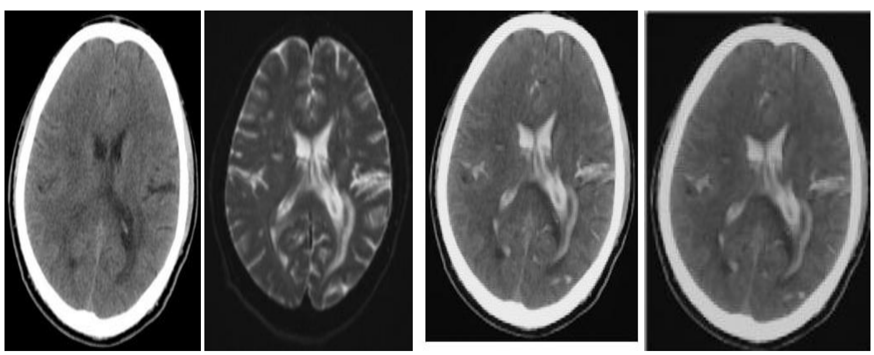
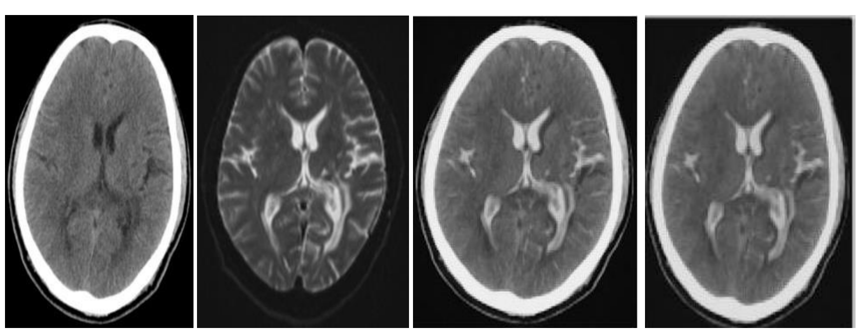


Dataset 9


Dataset 10


(i) Subjective Analysis:

The dataset 1 to 5 consists of CT - MRI brain images of different patients, the dataset 6 consists of CT- MRI abdomen image of different patients, the dataset 7 consists of CT-MRI Head image of different patients and dataset 8 to 10 consists of CT-MRI brain images of same patient with different modalities are used as input images for experimental purpose. Fused images are arranged as discrete wavelet transform (Haar) neutrosophic set and proposed spatial frequency discrete wavelet transform (Haar) neutrosophic set system. From the evaluation it is observed that the visibility of image is increased when compared to other methods. The proposed method gives better visualization because of the most important features is chosen based on the higher value of neutrosophic entropy to fuse the low subbands coefficients. The entropy gives the texture information of the image and it is important factor for the fusion of images. Hence the proposed method gives better result than the existing methods.

(ii) Objective Analysis:

Using subjective analysis, we cannot determine the completely fused image. Thus, objective analysis is results in fused images with measures mentioned in the previous section performance evaluation measures. For each measure, the result obtained from the proposed spatial frequency discrete wavelet transform (Haar) neutrosophic set is compared with discrete wavelet transform (Haar) neutrosophic set. The comparative analysis of different performance evaluation measures is tabulated in Table 5.1. From the result, one can observe that the proposed spatial frequency discrete wavelet transform (Haar) neutrosophic set produces a better result than the other existing fusion methods.

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

In **Chapter 1**, the theory of fusion techniques and medical imaging are discussed.

In **Chapter 2**, review of literature which provides a summary of the work done by various image fusion techniques using medical images are discussed.

In **Chapter 3**, the preliminary definitions that are needed for the study are discussed.

In **Chapter 4**, CT and MRI image fusion using spatial frequency discrete wavelet transform (Haar) and neutrosophic set are investigated.

In **Chapter 5**, the performance of medical image fusion in terms of Peak Signal to Noise Ratio, Root Mean Square Error and Correlation are obtained.

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