

1. INTRODUCTION

“The difference between the poet and the Mathematician is that the poet tries to gets his head into the heaven while the mathematician tries to get the heaven into his head”.

- G.K.Chesterton

Queueing theory is a branch of applied mathematics utilizing concepts from the field of stochastic processes. It has been developed for the purpose of better understanding of queueing systems and for the sake of taking appropriate decisions to maintain the system efficiency.

1.1 CLASSICAL QUEUEING SYSTEM

A classical queueing system can be described by the flow of units for service, waiting for service if it is not immediate and leaving the system after receiving service or sometimes even without receiving service. The general framework of a queueing system is shown in Fig. 1.1.

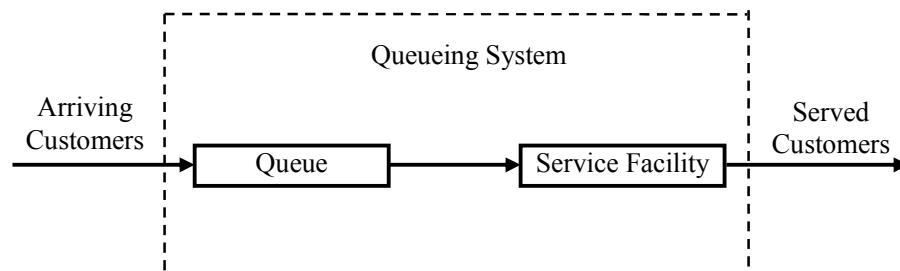


Fig. 1.1 Structure of Classical Queueing System

1.2 RETRIAL QUEUEING SYSTEM

In conventional queueing theory it is usually assumed that an arriving customer who cannot get service immediately either joins the waiting line or leaves the system forever. Sometimes impatient customers leave the queue but it is also assumed that they are leaving the system forever. Usually such

customers after some random period of time return to the system and try to get service. The standard queueing models do not consider the phenomenon of retrials and therefore cannot be applied in solving a number of practically important problems. Retrial queues have been introduced to meet this inadequacy.

Retrial queueing systems are characterized by the feature that arriving customers who cannot receive service immediately may join a virtual queue called orbit to try their request after some random time. Fig. 1.2 presents the general structure of a retrial queue.

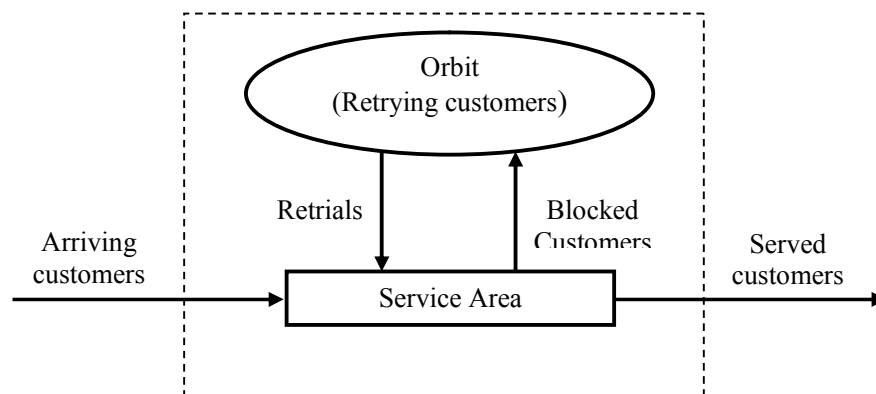


Fig. 1.2 General Structure of a Retrial Queue

1.3 CHARACTERISTICS OF QUEUEING SYSTEM

The basic characteristics of a queueing system which provide an adequate description are arrival pattern, service pattern, queue discipline, system capacity and number of service channels.

Arrival Pattern

Arrival pattern describes the manner in which the units arrive and join the system. The source from which the units come may be finite or infinite. A unit may arrive either singly or in a group. The arrival pattern is often measured in terms of the average number of arrivals per unit time.

Another factor to be considered regarding the arrival pattern is the reaction of the customers in the queue. If the queue is too long, a customer may decide not to enter it upon arrival and in this situation he is said to have balked. On the other hand, a customer may enter the queue, but after some time lose patience and decide to leave. In this case, he is said to have reneged. In the event that there are more than one queue, customers may switch from one to another, that is jockey for position.

The arrival pattern of customers that does not change with time is called stationary arrival pattern otherwise, it is called non-stationary.

Service Pattern

Service pattern describes the manner in which the service is rendered to the arrivals. Customers may be served either singly or in batches. The time required for serving a unit is called service time. The service pattern may be stationary or non-stationary with respect to time and state dependent or independent with respect to number of customers waiting for service.

Queue Discipline

Queue discipline refers to the manner in which customers are selected for service from the queue. The most common disciplines based on the arrivals of customers into the system are first come first served (FCFS) and last come first served (LCFS). Customers may also be served randomly irrespective of their arrivals to the system called service in random order (SIRO).

Another discipline is priority queue discipline. There are two types in priority discipline, preemptive priority and non preemptive priority. In the preemptive case, the customer with high priority is allowed to enter service immediately suspending the service in progress to a customer with lower priority. In non preemptive case the higher priority goes to the head of the queue but gets into service only after the completion of service in progress.

System Capacity

In some queueing systems there is a physical limitation to the amount of waiting line. When the line reaches a certain length, no further customers are allowed to enter until space becomes available. These are referred to as finite queueing systems. A queue with limited waiting room can be viewed as one with forced balking.

Service Channels

The number of servers in a queueing model may be finite or infinite. The number of servers may be arranged in series, parallel or a combination of both, depending upon the nature of the services required. In parallel channels, all the channels provide identical services so that several customers may be served simultaneously. In series channels, a customer must pass through successively in several ordered channels before service is completed.

1.4 KENDALL'S NOTATION

A queueing system can be represented by the notation introduced by Kendall (1951) as follows : $A/B/X/Y/Z$, where A represents inter-arrival time distribution of the customers, B denotes the service time distribution, X is the number of parallel servers, Y represents the capacity of the system and Z denotes the queue discipline. If the queueing system has infinite capacity and the queue discipline is FIFO, then the system is denoted as $A/B/C$ without mentioning X and Y.

For example, a single server queueing system with bulk arrival Poisson process, general service time distribution, infinite capacity and first in first out queue discipline is denoted as $M^X/G/1$.

1.5 MARKOVIAN AND NON-MARKOVIAN QUEUEING MODELS

Queueing models are classified as Markovian queueing models and non-Markovian queueing models. The techniques generally adopted to solve these types of queueing models are given below.

Markovian Queueing Models

Queueing models with exponential inter-arrival time and exponential service time are called Markovian queueing models. Some of the techniques used to solve Markovian queueing models are :

- Difference-differential equation method
- Neuts matrix-geometric algorithm
- Continued fraction method

Non-Markovian Queueing Models

The exponential assumption on queueing models, although very convenient, is not always realistic. There is a practical need for models that do not depend on strict Markov assumptions. Queueing models having the inter-arrival times and/or service times which are not exponentially distributed are known as non-Markovian queueing models.

The techniques generally used to study non-Markovian queueing models are :

- Embedded Markov chain technique
- Supplementary variable technique

1.6 FUZZY SET THEORY

Fuzzy logic is a form of mathematical logic in which truth can assume a continuum of values between 0 and 1. This allows a given proposition to be partially true and partially false at the same time. Our traditional tools for formal modelling, reasoning and computing are crisp, deterministic and precise in character that is, yes or no type. Real situations are not crisp and deterministic and cannot be described precisely. The development of fuzzy logic was motivated in large measure by the need for a conceptual framework which can address the issue of uncertainty and lexical imprecision.

Fuzzy set theory is an efficient tool for modelling the kind of uncertainty associated with vagueness, with imprecision and with a lack of information regarding a particular element of the problem at hand.

Fuzzy Set

Let X be a nonempty set. A fuzzy set \tilde{A} in X is characterized by its membership function.

$$\mu_{\tilde{A}} : X \rightarrow [0, 1]$$

and $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of element x in fuzzy set \tilde{A} for each $x \in X$.

α -Cut

An α -cut of a fuzzy set \tilde{A} is a crisp set $A(\alpha)$ that contains all the elements of the universal set x that have a membership grade in A greater than or equal to the specified value of α . Thus

$$A(\alpha) = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}, \quad 0 \leq \alpha \leq 1.$$

Strong α -cut

The strong α -cut of a fuzzy set \tilde{A} is a crisp set $A(\alpha)$ that contains all the elements of the universal set x that have a membership grade in A greater than the specified value of α . Thus,

$$A(\alpha) = \{x \in X : \mu_{\tilde{A}}(x) > \alpha\}, \quad 0 \leq \alpha \leq 1.$$

Fuzzy Numbers

A fuzzy subset \tilde{A} of the real line R with membership function

$\mu_{\tilde{A}} : R \rightarrow [0, 1]$ is called a fuzzy number if

- (i) there exists an element $x_0 \in \tilde{A}$ such that $\mu_{\tilde{A}}(x_0) = 1$.
- (ii) $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda) x_2) \geq (\mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in \mathbb{R}$ and $\forall \lambda \in [0, 1]$.
- (iii) $\mu_{\tilde{A}}$ is upper semi continuous and
- (iv) $\text{sup } \tilde{A} = \{x \in \mathbb{R} : \mu_{\tilde{A}}(x) > 0\}$ is bounded.

Triangular Fuzzy Number

A triangular fuzzy number \tilde{A} is a fuzzy number specified by 3-tuples (a_1, a_2, a_3) such that $a_1 \leq a_2 \leq a_3$, with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ (x - a_1) / (a_2 - a_1) & \text{if } a_1 \leq x \leq a_2 \\ (x - a_3) / (a_2 - a_3) & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{if } x \geq a_3 \end{cases}$$

The membership function of a triangular fuzzy number can be represented diagrammatically as in Fig. 1.3.

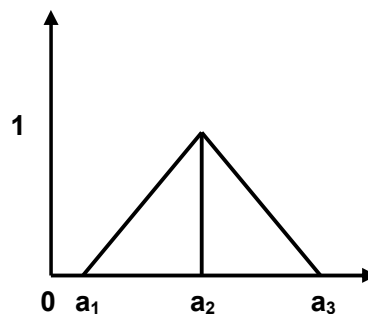


Fig. 1.3 Membership Function of Triangular Fuzzy Number

Trapezoidal Fuzzy Number

A trapezoidal fuzzy number \tilde{A} is a fuzzy number fully specified by 4-tuples (a_1, a_2, a_3, a_4) such that $a_1 \leq a_2 \leq a_3 \leq a_4$, with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - a_1) / (a_2 - a_1) & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ (x - a_4) / (a_3 - a_4) & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

Fig. 1.4 represents the membership function of trapezoidal fuzzy number.

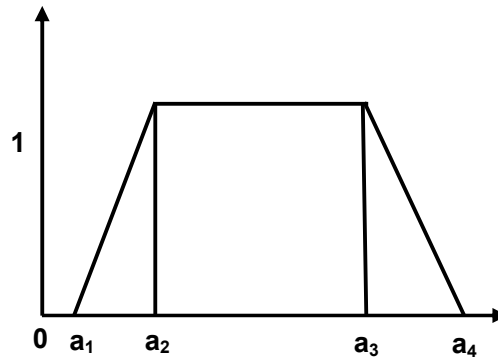


Fig. 1.4 Membership Function of Trapezoidal Fuzzy Number

Zadeh's Extension Principle

Let X be a Cartesian product of universes $X = X_1 \times X_2 \times \dots \times X_r$ and $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_r$ be r fuzzy sets in X_1, X_2, \dots, X_r respectively. f is a mapping from X to a universe Y , $y = f(x_1, x_2, \dots, x_r)$. Then the extension principle allows us to define a fuzzy set \tilde{B} in Y by

$$\tilde{B} = \{(y, \mu_{\tilde{B}}(y)) \mid y = f(x_1, x_2, \dots, x_r), (x_1, x_2, \dots, x_r) \in X\}$$

where

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup_{(x_1, x_2, \dots, x_r) \in f^{-1}(y)} \min \{\mu_{\tilde{A}_1}(x_1), \mu_{\tilde{A}_2}(x_2), \dots, \mu_{\tilde{A}_r}(x_r)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

Defuzzification

Defuzzification is the conversion of a fuzzy quantity to a precise quantity, just as fuzzification is the conversion of a precise quantity to a fuzzy quantity.

There is no unique way to perform the operation defuzzification. There are several existing methods for defuzzification depending on the shape of the clipped fuzzy numbers. The most popular defuzzification methods are centre of area method, centre of sums method, mean of maximum method, graded mean integration method, height defuzzification method, weighted average method and Yager index methods. Among all the above methods Yager ranking index method is suitable to analyse queueing system with fuzzy parameter.

Yager (1981) proposed a procedure for ordering fuzzy sets based on the concept of area compensation. Area compensation possesses the properties of linearity. A ranking index $\tilde{Y}(\tilde{P})$ is calculated for the convex fuzzy number \tilde{P} from its α -cut $P(\alpha) = [P_\alpha^L, P_\alpha^U]$ according to the following formula :

$$\tilde{Y}(\tilde{P}) = \frac{1}{2} \int_0^1 (P_\alpha^L + P_\alpha^U) d\alpha \text{ which is the centre of the mean value of } \tilde{P}.$$

1.7 LITERATURE SURVEY

Queueing theory has advancement in many disciplines since its inception. Johannsen's "Waiting Times and Number of Calls", an article published in 1907 and reprinted in Post Office Electrical Engineers Journal, London, 1910 seems to be the seed paper on the subject. The Theory of Probabilities and Telephone Conversations of Erlang (1909) has historic importance in Queueing Theory due to its exact mathematical treatment. Following Erlang, Molina (1927) and Thornton (1928) published papers on applications of the theory of probability.

Important contributors in the next two decades are Crommelin, Feller, Jensen, Khintchine, Kolmogorov, Palm and Pollaczek. A detailed account of the investigations made by these authors may be found in the books by Saaty (1960), Kleinrock (1975), Gross and Harris (1985) and Takagi (1991, 1993a, 1993b).

Retrial Queues

Retrial queueing systems are characterised by the fact that an arriving customer who finds the server occupied is obliged to join a group of blocked customers (called orbit) and reapply after random intervals of time to obtain the service. Retrial queueing systems are basically motivated by the applications to telephone switching systems, telecommunication networks and computer systems.

One of the earliest papers in retrial queues was on the influence of repeated calls in the theory of probabilities of blocking by Kosten (1947). Cohen (1957) analysed the basic problems of telephone traffic theory and the influence of repeated calls. Keilson et al. (1968) published the first article on general service retrial queue using the technique of supplementary variables. The first investigation with general retrial time was done by Kapyrin (1977), in which each customer in the orbit generates a stream of repeated attempts that are independent of the customers in the orbit and the state of the server.

Two extensive survey articles in retrial queues are due to Yang and Templeton (1987) and Falin (1990), covering respectively, the developments upto mid 80's and late 80's. The only monograph on this topic is by Falin and Templeton (1997) which provides an excellent scenario of retrial queues. Gomez-Corral (1999) discussed widely about a single server retrial queueing system with general retrial times. Accessible bibliographies on this topic are provided by Artalejo (1999a, 1999b, 2010).

Busy period of the M/G/1 retrial queue was analysed by Artalejo and Lopez-Herrero (2000) and Gomez-Corral and Ramalhoto (2000). Stochastic

comparisons of Markovian retrial queues were given by Shin and Kim (2000). A comparative analysis of standard and retrial queueing systems was presented in Artalejo and Falin (2002). Artalejo and Lopez-Herrero (2005) proposed an information theoretic approach for the estimation of the main performance characteristics of the M/G/1 retrial queue. Krishnamoorthy et al. (2005) and Wu et al. (2005) considered retrial queueing system with impatient customers. Waiting time probabilities of M/G/1 retrial queue was developed by Nobel and Tijms (2006). An algorithmic approach of retrial queues was done by Artalejo (2009).

The batch arrival retrial queue was first considered by Falin (1976) with the following operating rule : “If the server is busy at the arrival epoch, then whole batch joins the retrial group, whereas if the server is free, then one of the arriving units starts its service and the rest join the retrial group”. Choi et al. (1992) discussed its applicability connected with the performance evaluation of Local Area Networks operating under transmission protocols like the Carrier Sense Multiple Access with Collision Detection (CSMA / CD). Recently many authors, like, Artalejo and Atencia (2004), Dudin et al. (2004), Falin (2010) and Deepak et al. (2013) discussed retrial queueing situations with batch arrivals.

Vacation Queueing Models

Queueing systems are broadly classified into two types – queueing systems with and without vacations. Non availability of the server for primary customers at occasional intervals of time is called vacation. Queueing models with vacations occur in many real life situations where the server may be used for other secondary jobs. Allowing server to take vacations makes the queueing model more realistic and flexible. Applications arise naturally in call centres with multi task employees, customized manufacturing firms, telecommunication and computer networks etc.

Miller (1964) was the first to study a queueing system in which the server goes idle and is unavailable for random intervals of time. Levy and Yechiali (1975) included several types of vacations in classical M/G/1 queueing system. Since Levy and Yechiali, models of similar specifications have been reported by a number of authors.

Bernoulli vacation schedule was proposed by Keilson and Servi (1986). It is characterized by the feature that if the queue is not empty then the server begins service with specified probability or takes a vacation with complementary probability. Doshi (1986), Shanthikumar (1988), Ramaswami and Servi (1988), Cramer (1989), Shomrony and Yechiali (2001) and Tadj et al. (2006a) are a few among many authors who have studied queueing systems with server vacations.

Numerous researchers, including Baba (1986), Lee and Srinivasan (1989), Teghem (1990), Rosenberg and Yechiali (1993), Lee et al (1994a, 1994b, 1995), Krishna Reddy et al. (1998), Choudhury and Borthakur (2000), Choudhury (2000, 2002a, 2002b), Ke and Chu (2006), Tadj et al. (2006b) and many others analysed batch arrival vacation queues under different vacation policies because of its interdisciplinary character.

Choi and Park (1990) considered single server general service retrial queue with Bernoulli vacation. Artalejo (1997) analysed M/G/1 queue with constant repeated attempts and server vacations. Krishna Kumar and Arivudainambi (2002) studied M/G/1 retrial queue with Bernoulli vacation and verified stochastic decomposition property. Atencia and Moreno (2005) and Arivudainambi et al. (2009) investigated single server priority retrial queue with server vacation. Boualem et al. (2009) derived stochastic inequalities of M/G/1 retrial queue with vacations and constant retrial policy. Ebenesar Anna Bagyam and Udaya Chandrika (2010) studied single server retrial queue with emergency and multiple vacation. Boualem et al. (2011) considered M/G/1 retrial queue with exhaustive service and server vacations.

Batch arrival retrial queues with vacation were analysed by many researchers. Aissani (2000) considered bulk arrival queue with exhaustive vacation. Krishna Kumar et al. (2002a) studied $M^X/G/1$ retrial queue with Bernoulli schedule. Krishna Kumar and Pavai Madheswari (2003) investigated a batch arrival retrial queueing model with multiple vacation and starting failures. Aissani (2009) analysed $M^X/G/1$ energetic retrial queue with vacation. Chang and Ke (2009) discussed a batch arrival retrial queue under modified vacation policy. Ke et al. (2010) developed bulk arrival retrial queueing models with N policy and at most J vacations.

Unreliable Queueing Models

Breakdown is an remarkable and unavoidable phenomenon in the service facility of a queueing system. Queues with servers subject to breakdowns and repairs are often encountered in many practical applications.

The study of queueing models with service interruptions goes back to 1950s. The articles presented by Gaver (1962), Avi-Itzhak and Naor (1963), Thiruvengadan (1963) and Mitrany and Avi-Itzhak (1968) are some early papers on queues with server breakdown.

Retrial queues with server failures and repairs were investigated by Aissani (1988, 1994), Kulkarni and Choi (1990), Aissani and Artalejo (1998), Wang et al. (2001), Djellab (2002), Almasi et al. (2005), Moreno (2006), Atencia et al. (2006), Wang (2006), Wang et al. (2008), Sherman et al. (2009), Wang and Zhang (2009), Geni Gupur (2010), Sumitha and Udaya Chandrika (2012) and many others.

Li and et al. (2006) considered batch arrival retrial queue with server subject to breakdown and repair. Jain and Mishra (2008) examined the influence of bulk arrival and state dependent arrival rates on a single server retrial queueing model with server breakdown by supplementary variable method and obtained reliability indices. Jain and Charu Bhargava (2008)

investigated bulk arrival retrial queue with unreliable server and priority subscriber.

In all the above cited articles, the repair of the breakdown server starts instantaneously at the epoch of failure. But this assumption is unrealistic. In several real life situations the system has to wait for the repair to start. This time is known as setup time or delay time. Prakash Rani et al. (2009) analysed retrial queueing models with server breakdown and delayed repair. Ke and Huang (2010) derived system size distributions for bulk arrival queueing system with vacation, breakdown and delayed repair at random epoch, at departure epoch and at the initiation epoch of a busy period. Khalaf et al. (2011, 2012) investigated bulk arrival queue with server vacation, random breakdown and delay times. Choudhury et al. (2011) investigated unreliable single server queue with Bernoulli vacation under N policy and delaying repair. Choudhury and Ke (2012) considered a batch arrival retrial queue with general retrial time, Bernoulli vacation, unreliable server and delayed repair. Ebenesar Anna Bagyam et al. (2013) studied bulk arrival retrial queueing model with two types of heterogeneous service, server breakdown, delayed repair and reserved time.

Due to the complex interaction between the many components of modern electromechanical and computer or communication networks, the failure of one component may cause a sequence of failures, and so a breakdown of the service mechanism may require several stages of repair. Hsieh et al. (1995) studied a model where the server is subject to several types of breakdowns and each type has two possible stages of repair. Gray et al. (2004) considered a general queueing model in which the server may experience several different types of breakdowns. Each type of breakdown requires a finite random number of stages of repair. The authors obtained queue length distribution by matrix geometric method and derived explicit expression for its mean. Kernane (2009) analysed the equilibrium distribution of the system and obtain the generating functions of the limiting

distribution for a single server retrial queue with various types of interruption of the server.

Two Phase Queueing Models

Recently there have been several contributions considering queueing systems with two phases of service. As soon as the first phase service is completed, customers may leave the system or immediately go for the second phase service. This type of queueing system occurs in many real life situations. For example in a manufacturing process, all the arriving customers require the main service and only some of them opt the subsidiary service.

Two phase queueing models were studied by many authors including Madan (2000), Madan and Anabosi (2003), Artalejo and Choudhury (2004) and Madan and Choudhury (2005). Choudhury (2003), Choudhury and Madan (2004, 2005), Madan et al. (2005), Choudhury and Paul (2006) and Choudhury et al. (2007) investigated bulk arrival two phase queueing models.

Retrial queues with two phase service were discussed by Krishna Kumar et al. (2002c), Atencia and Moreno (2006), Artalejo and Gomez-Corral (2007), Srinivasan et al. (2007), Wang and Zhao (2007), Choudhury (2008b), Choudhury and Deka (2008), Prakash Rani et al. (2008), Udaya Chandrika et al. (2008), Udaya Chandrika and Ebenesar Anna Bagyam (2008), Wang and Li (2008, 2009), Choudhury and Kalidas (2009) and Choudhury (2009). Senthil Kumar and Arumuganathan (2009, 2010) investigated single server retrial queue with impatient customers, two phase heterogeneous service and vacation. Dimitriou and Langaris (2008) presented a paper on retrial queue with two phases of service and server vacation. Dimitriou and Langaris (2010) obtained system characteristics for a repairable two phase queueing model with start-up times. Ramanath and Kalidas (2010) analysed two phase retrial queueing system with non-persistent customers and vacation. Ebenesar Anna Bagyam and Udaya Chandrika (2012) obtained availability of the server and failure frequency for two phase retrial queueing

system with impatient customers, breakdown, delayed repair and reserved time.

Choudhury (2007) and Senthil Kumar and Arumuganathan (2008) investigated a single server batch arrival retrial queueing system with two phases of heterogeneous service and Bernoulli vacation. Choudhury and Deka (2009) analysed $M^X/G/1$ unreliable retrial queue with two phase service and Bernoulli admission mechanism. Sumitha et al. (2012) studied $M^X/G/1$ two phase retrial queueing system with orbital search, impatient customers and different types of server vacations. Choudhury and Deka (2013) analysed various decomposition properties for batch arrival retrial queue with two phases of service and Bernoulli schedule. Maurya (2013) presented a paper on maximum entropy analysis of $M^X/(G_1, G_2)/1$ retrial queue with second phase optional service and Bernoulli vacation schedule.

Multi Phase Queueing Models

Shahkar and Badamchi Zadeh (2006) analysed multi phase queueing model with vacation. Salehirad and Badamchi Zadeh (2009) and Abdollahi and Salehirad (2012) studied the multi phase $M/G/1$ queueing system with random feedbacks. Ebenesar Anna Bagyam and Udaya Chandrika (2013b) analysed the above articles by introducing the concept of repeated attempts. Ebenesar Anna Bagyam and Udaya Chandrika (2013a) investigated multi stage retrial queueing model with active server breakdown, delayed repair and reserved time.

Feedback Queueing Models

The feedback phenomenon is another important tool for communication systems. When the service of a customer is unsatisfied, the customer may retry again and again until the service is completed successfully. Takacs (1963) was the first to study such a model, where the customers who completed their service feedback instantaneously to the tail of

the queue with certain probability or depart from the system forever with complementary probability.

Feedback retrial queue is mainly applied in computer networks and telecommunication systems. For example, in multiple access telecommunication systems, where messages turned out as errors are sent again can be modeled as retrial queues with feedback.

Many authors including Krishna Kumar et al. (2002b, 2009, 2010), Mokaddis et al. (2007) and Lee and Jang (2009) analysed retrial queueing systems with feedback. Unreliable feedback retrial queueing model with phase repair and modified vacation was developed by Jain and Charu Bhargava (2009). Ramanath and Lakshmi (2011) studied M/G/1 retrial queue with second multi-optional service and feedback. A batch arrival retrial queueing model with feedback and multi-optional vacation was investigated by Arivudainambi and Godhandaram (2012).

Admission Control Queueing Models

In many systems with batch arrival there is a restriction such that not all batches are allowed to join the system at all time. This policy is named as restricted admissibility. Madan and Abu-Dayyeh (2002a, 2002b) proposed restricted admissibility of arriving batches and analysed $M^X/G/1$ queueing system with Bernoulli schedule server vacation. Madan and Choudhury (2004), Alnowibet and Tadj (2007) and Choudhury (2008a) presented articles on the admission control concepts. Badamchi Zadeh (2009) studied a batch arrival queueing system with two phases of heterogeneous service, restricted admissibility and server vacation. Madan (2010) investigated a queue with restricted admissibility of arriving batches in which the server provides two stages of heterogeneous service with each customer having the option to choose one of the two types of first stage service followed by one of the two types of second stage service.

Artalejo et al. (2005) developed a discrete time retrial queueing model with admission control. Wang and Zhou (2010) analysed the impact of the parameters on the system performance for a batch arrival retrial queue with starting failures, feedback and admission control. Various kinds of admission control policies was recently studied by some authors namely Ayyappan et al. (2013) and Upadhyaya (2013a).

Fuzzy Queueing Models

Fuzzy queueing models are more practical and realistic than deterministic queueing models. Within the context of traditional queueing theory the inter-arrival times and service times are required to follow certain distributions. However, in many practical situations, the arrival pattern and service pattern are described by linguistic quantifiers such as fast, slow or moderate, rather than by probability distributions.

The concept of fuzzy sets was first introduced by Zadeh (1965) in his paper work entitled "Fuzzy sets". One of the most important tools in fuzzy set theory is Zadeh's extension principle. Following Zadeh, many researchers have considered the problem of fuzzy queueing systems. Li and Lee (1989) and Buckley (1990) discussed the problem of fuzzy queues by using Zadeh's extension principle, the possibility concept and fuzzy Markov chain. Negi and Lee (1992) introduced α -cut and two variable simulation approaches to analyze fuzzy queues. Using the parametric programming technique Kao et al. (1999) obtained the membership functions of the performance measures for fuzzy queues – M/F/1, F/M/1, F/F/1 and FM/FM/1, where F denotes fuzzy time and FM denotes fuzzified exponential distributions.

Chen (2004) studied a fuzzy finite capacity queue with fuzzified exponential arrival rate and service rate. Chen (2005, 2006) analysed bulk service fuzzy queueing system and bulk arrival queueing model with fuzzy parameters. Ke and Lin (2006) analysed fuzzy queueing system with server breakdown using non linear programming approach. Ke et al. (2006)

investigated batch arrival queueing system with vacation policies having fuzzy parameters and obtained system characteristics. Pardo and Fuente (2007) optimized two fuzzy queueing models, one with non-preemptive priority discipline and another with preemptive priority discipline. Chen (2007) calculated the α -cuts of the fuzzy minimal expected total cost per unit time for fuzzy queueing decision model. Lin et al. (2008) studied batch arrival queueing system with setup time and fuzzy parameter. Ke et al. (2008) obtained the mean time to failure and the availability of the system for a two-unit repairable system with imperfect coverage, reboot and fuzzy parameters. Aydin and Apaydin (2008) analysed the multi channel fuzzy queueing systems and constructed different membership functions.

Nagoor Gani and Ashok Kumar (2008) analysed finite capacity fuzzy queue by employing parametric non-linear programming technique. Using simulation techniques, Wu (2009) obtained the α -level closed intervals of the system performance measures. Nagoor Gani and Ashok Kumar (2009a, 2009b, 2009c) investigated bulk arrival queueing model with fuzzy parameters, multi-server batch arrival queue with fuzzy parameters and batch arrival queue with set up, uncertain parameters and fuzzy varying batch sizes respectively. Wang et al. (2010) applied the parametric non-linear programming technique to investigate infinite capacity fuzzy N policy queue. Ashok Kumar (2011) derived the membership function of the steady state performance measures for Erlang service queueing model.

Barak and Fallahnezhad (2012) performed sensitivity analysis to compare fuzzy queueing models of M/M/1 and M/E₂/1. Lin et al. (2012) constructed the membership function of the mean time to failure of a repairable system with uncertain parameters. Ritha and Sreelekha (2012) analysed a single server bulk service fuzzy queueing system in which the service facility suffers time homogenous random breakdown.

Yang and Chang (2013) investigated F policy fuzzy queue with startup time. They applied α -cuts approach and Zadeh's extension principle to

transform fuzzy F policy queues into a family of crisp F policy queues and constructed the membership function for the expected number of customers in the system. Jeeva and Rathnakumari (2014) presented an article entitled fuzzy cost computation of M/M/1 and M/G/1 queueing models.

In recent years, interest is growing in analysing retrial queue with fuzzy parameters. Ke et al. (2007) constructed the membership function of system characteristics for a retrial queueing model with fuzzy arrival, retrial and service rates. Kalyanaraman et al. (2010) analysed unreliable single server fuzzy retrial queue and derived mean system size and mean orbit size. Noora et al. (2011) studied fuzzy discrete-time multi server retrial queue with finite population. Stephen Vincent and Bhuvaneshwari (2011) analysed retrial queue with priority subscribers having fuzzified exponential arrival, retrial and service rates. Khodadadi and Jolai (2012) analysed a single server retrial queue with vacation by introducing fuzzy based threshold policy to control the server in order to minimize the total cost of the system. Jeeva and Rathnakumari (2012) constructed membership function of expected number of customers in the orbit and expected waiting time in the orbit for a two phase batch arrival retrial queue with Bernoulli vacation. Upadhyaya (2013b) investigated $M^X/G/1$ retrial queueing system with Bernoulli vacation and obtained fuzzy logic based system characteristics.

Thesis Organization

Schematic representation of thesis is presented in Figure 1.5.

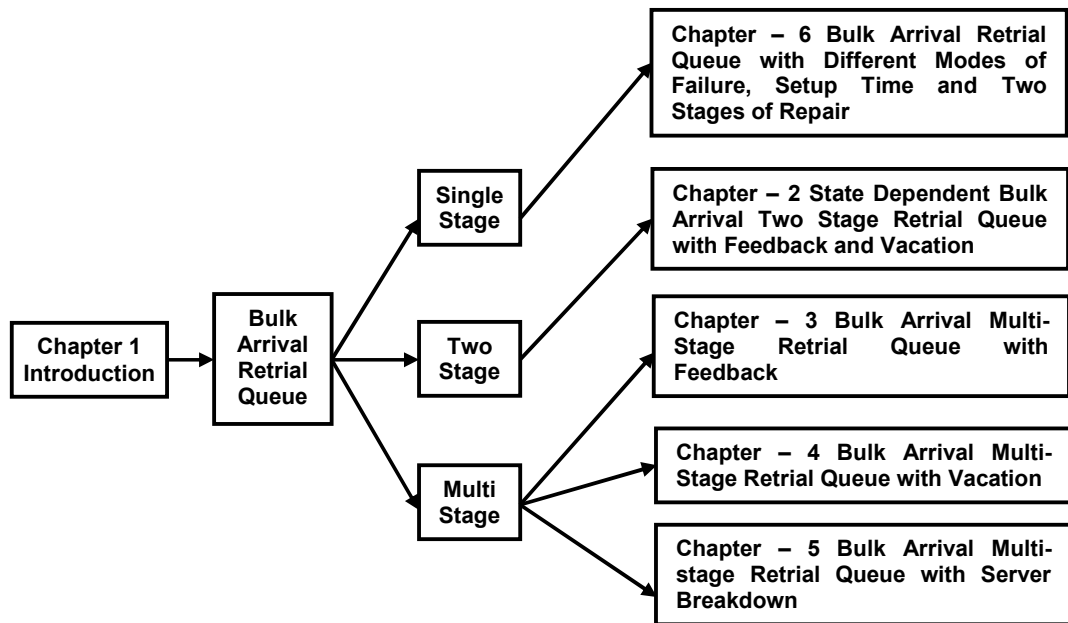


Fig. 1.5 Organisation of the Thesis

Chapter two deals with single server batch arrival retrial queue with state dependent admission, two phases of heterogeneous service, feedback and multi-optional Bernoulli vacation. The arrival rate of a batch depends upon the state of the server. All the customers demand the first essential service whereas only some of them opt for second multi-optional service. After the completion of essential service, if the customer is dissatisfied with the service he can immediately join the orbit as a feedback customer. After providing service to a customer, the server may either wait for a new customer or take one of the optional vacations. The service time, retrial time and vacation time are generally distributed.

In Chapter three bulk arrival retrial queueing system with multi-stages of service and feedback is considered. The server provides M stages of heterogeneous service in succession. If the server is free one of the arriving customers receives the first stage service immediately. Otherwise, all the customers enter a retrial orbit. After receiving service from one stage, the customer may move to next stage or feedback to the retrial orbit or leave the system. After completing the final stage, the customer may feedback to the

retrial group or leave the system. The service time and retrial time are assumed to follow general distribution.

Bulk arrival retrial queueing model with multi-stages of service and non exhaustive server vacation is analysed in Chapter four. The vacation time, service time and retrial time are assumed to be generally distributed.

Chapter five investigates batch arrival multi-stages retrial queueing system with active breakdown, setup time and reserved time. The server is subject to breakdown while it is working. The repair of the failed server starts after a random amount of time. After repair the server continues the service of the interrupted customer or waits for the same customer. The service time, retrial time, setup time and repair time are arbitrarily distributed.

Performance analysis of single server bulk arrival retrial queue with different modes of failure, setup time and reserved time is presented in Chapter six. The server is prone to different modes of random breakdown. The failed server undergoes two phases of repair. The first phase is essential and varies according to the mode of failure. The second phase is optional and common to all types of failure. The service time, retrial time, setup time and repair time are generally distributed.

For all the models analytical expressions of mean orbit size, mean system size, expected system size at a departure epoch, expected number of customers in the orbit due to retrial, availability of the server and failure frequency are obtained. Stochastic decomposition is verified and some special cases are mentioned. The effect of several parameters on the proposed models are discussed numerically.

The models are further analysed under fuzzy environment. The concept of α -cut and Zadeh's extension principle are applied to the models discussed in this thesis to construct membership function of system characteristic. Numerical study is carried out by selecting a set of parameters as fuzzy numbers.