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τ^*-Generalized-semi-closed sets in topological spaces

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Abstract. In this paper, we introduce a new class of sets called τ^*-generalized-semi-closed sets and τ^*-generalized-semi-open sets in topological spaces and study some of their properties.

Keywords: τ^*-gs-closed; τ^*-gs-open set.

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1 Introduction

Levine[3][4] introduced the concept of generalized closed sets and semi-open sets in topological spaces and studied most fundamental properties. Dunham[2] introduced the concept of the closure operator cl^* and a new topology τ^* and studied some of their properties. Arya[1] introduced and investigated generalized semi closed sets in topological spaces. Pushpalatha et al[5] introduced a new generalization of closed set in the weaker topological space (X, τ^*). The aim of this paper is to introduce the concepts of τ^*-gs-closed sets in the weaker topological space (X, τ^*).

2 Preliminaries

Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, $int(A)$, $int^*(A)$, $cl(A)$, $cl^*(A)$, $scl^*(A)$ and A^c denote the interior, interior*, closure, closure*, semi closure* and complement of A respectively.

Definition 2.1. A subset A of a topological space (X, τ) is called a generalized closed[3] if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is open in X.

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Note. For a subset A of a topological space (X, τ), the generalized closure of A denoted by $cl^*(A)$[2] is defined as the intersection of all g-closed sets containing A.

Definition 2.2. Let (X, τ) be a topological space. Then the collection $\{G : cl^*(G^c) = (G^c)\}$ is a topology on X and is denoted as τ^*[4] that is $\tau^* = \{G : cl^*(G^c) = (G^c)\}$.

Remark 2.3. For a subset A of X

$$cl^*(A) = \cap\{K : K \supset A, K^c \in \tau^*\},$$
$$int^*(A) = \cup\{G : G \subset A, G \in \tau^*\}.$$

Definition 2.4. A subset A of a topological space X is said to be τ^*-semi-closed set (briefly τ^*-s-closed) if $int^*(cl^*(A)) \subseteq A$. the complement of a τ^*-s-closed set is called the τ^*-semi-open set (briefly τ^*-s-open).

Definition 2.5. For a subset A of a topological space (X, τ^*), the generalized semi closure of A denoted as $scl^*(A)$ is defined as the intersection of all semi closed sets in τ^* containing A.

3 τ^*-Generalized-Semi-Closed Sets in Topological Spaces

In this section, we introduce the concept of τ^*-generalized-semi-closed sets in topological spaces.

Definition 3.1. A subset A of a topological space X is said to be τ^*-generalized-semi-closed set (briefly τ^*-gs-closed) if $scl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. The complement of a τ^*-gs-closed set is called the τ^*-generalized-semi-open set (briefly τ^*-gs-open).

Theorem 3.2. Every closed set in X is τ^*-gs-closed.

Proof. Let A be a closed set. Let $A \subseteq G$ where G is τ^*-open. Since A is closed, $cl(A) = A \subseteq G$. But $scl^*(A) \subseteq cl(A)$. Hence we have $scl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. Therefore A is τ^*-gs-closed. □

Theorem 3.3. Every τ^*-closed set in X is τ^*-gs-closed.

Proof. Let A be a τ^* closed set. Let $A \subseteq G$ where G is τ^*-open. Since A is τ^*- closed, $cl^*(A) = A \subseteq G$. But $scl^*(A) \subseteq cl^*(A)$. Thus, we have $scl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. Therefore A is τ^*-gs-closed. □

Theorem 3.4. Every τ^*-g-closed set in X is τ^*-gs-closed.

Proof. Let A be a τ^*-g-closed set. Let $A \subseteq G$ where G is τ^*-open. Since A is τ^*-g-closed, $cl^*(A) \subseteq G$. But $scl^*(A) \subseteq cl^*(A)$. Thus, we have $scl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. Therefore A is τ^*-gs-closed. □

The converse of the above theorems need not be true as seen from the following examples.

Example 3.1. Consider the topological spaces $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. Then the set $\{b\}$ is τ^*-gs-closed but not closed, τ^*-closed and τ^*-g-closed.

Theorem 3.5. If a subset A of X is τ^*-gs-closed and $A \subseteq B \subseteq scl^*(A)$ then B is τ^*-gs-closed.

Proof. Let A be a τ^*-gs-closed set and $A \subseteq B \subseteq scl^*(A)$. Let U be a τ^*-open set of X such that $B \subseteq U$. Since A is τ^*-gs-closed, we have $scl^*(A) \subseteq U$. Now $scl^*(A) \subseteq scl^*(B) \subseteq scl^*[scl^*(A)] = scl^*(A) \subseteq U$. That is $scl^*(B) \subseteq U$. whenever U is τ^*-open. Therefore B is τ^*-gs-closed. □

Theorem 3.6. Let A be an τ^*-gs-closed set. Then $scl^*(A) - A$ contains no non empty τ^*-closed set in X.

Proof. Given A is a τ^*-gs-closed set. Let F be a non empty τ^*-closed subset of $scl^*(A) - A$. Now $F \subseteq scl^*(A) - A$. Then $F \subseteq scl^*(A) \cap A^c$, since $scl^*(A) - A = scl^*(A) \cap A^c$. Therefore $F \subseteq scl^*(A)$ and $F \subseteq A^c$. Therefore $A \subseteq F^c$. Since F^c is a τ^*-open set and A is τ^*-gs-closed, $scl^*(A) \subseteq F^c$. That is $F \subseteq [scl^*(A)]^c$. Hence $F \subseteq scl^*(A) \cap [scl^*(A)]^c = \emptyset$. That is $F = \emptyset$, a contradiction. Thus $scl^*(A) - A$ contains no non empty τ^*-closed set in X. □

Corollary 3.7. Let A be a τ^*-gs-closed set. Then A is τ^*-s-closed if and only if $scl^*(A) - A$ is τ^*-closed.

Proof. Given A is a τ^*-gs-closed set. If A is τ^*-s-closed, then we have $scl^*(A) - A = \emptyset$ which is τ^*-closed set. Conversely, let $scl^*(A) - A$ be τ^*-closed. Then, by theorem 3.5, $scl^*(A) - A$ does not contain any non empty τ^*-closed subset and since $scl^*(A) - A$ is τ^*-closed subset of itself, then $scl^*(A) - A = \emptyset$. This implies that $A = scl^*(A)$ and so A is τ^*-s-closed set. □

Theorem 3.8. The union of two τ^*-gs-closed sets is again a τ^*-gs-closed.

Proof. Assume that A and B are τ^*-gs-closed sets in X. Let G be an τ^*-open set in X such that $A \cup B \subseteq G$. Then $A \subseteq G$ and $B \subseteq G$. Since A and B are τ^*-gs-closed, $scl^*(A) \subseteq G$ and $scl^*(B) \subseteq G$. Hence $scl^*(A \cup B) = scl^*(A) \cup scl^*(B) \subseteq G$. Therefore $A \cup B$ is τ^*-gs-closed. □

Theorem 3.9. For each $x \in X$ either the set $X - \{x\}$ is τ^*-gs-closed in X or τ^*-open.

Proof. Suppose $X - \{x\}$ is not τ^*-open. Then X is the only τ^*-open set containing $X - \{x\}$. This implies $scl^*(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is a τ^*-gs-closed in X. □

Remark 3.10. From the above observations we obtain the following implications.

$$\text{closed} \rightarrow \tau^*\text{-closed} \rightarrow \tau^*\text{-g-closed} \rightarrow \tau^*\text{-gs-closed}$$

The reverse implication is not true.

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