4

γ Generalized Closed Mappings in Intuitionistic Fuzzy Topological Spaces

4.1 Introduction

Noiri (1973) has introduced semi-closed mappings which contain the class of closed mappings. Malghan (1982) has introduced generalized closed maps in topology. Ghosh (1990) has introduced semi closed mappings in fuzzy setting. Seok Jong Lee and Eun Pyo Lee (2000) have introduced intuitionistic fuzzy open mapping and intuitionistic fuzzy closed mapping in intuitionistic fuzzy topological spaces. Here in this chapter we have introduced intuitionistic fuzzy γ generalized closed mappings, intuitionistic fuzzy contra γ generalized open mappings, intuitionistic fuzzy almost γ generalized closed mappings, intuitionistic fuzzy M - γ generalized closed mappings, intuitionistic fuzzy contra M - γ generalized open mappings and intuitionistic fuzzy almost contra γ generalized closed mappings. The interrelations with other already existing closed mappings with our newly defined closed mappings have been established.

4.2 Intuitionistic fuzzy γ generalized closed mappings

In this section we have introduced intuitionistic fuzzy γ generalized closed mappings, intuitionistic fuzzy γ generalized open mappings, intuitionistic fuzzy M - γ generalized closed mappings and studied some of their properties.

Definition 4.2.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy γ generalized*** (IFγG) ***closed mapping*** if f (V) IFγGCS in Y for every IFCS V of X.

Example 4.2.2: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$, $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG closed mapping.

Proposition 4.2.3: Every IF closed mapping is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF closed mapping. Let A be an IFCS in X. Then f(A) is an IFCS in Y. Since every IFCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 4.2.4: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG closed mapping but not an IF closed mapping, since $G_1^c = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFCS in Y, as $cl(f(G_1^c)) = G_2^c \neq f(G_1^c)$.

Proposition 4.2.5: Every IF semi closed mapping is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF semi closed mapping. Let A be an IFCS in X. Then f(A) is an IFSCS in Y. Since every IFSCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 4.2.6: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG closed mapping but not an IF semi closed mapping, since $G_1^c = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFSCS in Y, as $int(cl(f(G_1^c))) = int(G_2^c) = G_2 \nsubseteq f(G_1^c)$.

Proposition 4.2.7: Every IF pre closed mapping is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF pre closed mapping. Let A be an IFCS in X. Then f(A) is an IFPCS in Y. Since every IFPCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 4.2.8: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩, G_2= ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3= ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG closed mapping but not an IF pre closed mapping, since G_1^c = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFCS in X but f (G_1^c) is not an IFPCS in Y, as cl(int(f (G_1^c))) = cl(G_3) = G_2^c ⊈ f (G_1^c).

Proposition 4.2.9: Every IFα closed mapping is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF α closed mapping. Let A be an IFCS in X. Then f(A) is an IFαCS in Y. Since every IFαCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 4.2.10: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG closed mapping but not an IFα closed mapping, since G_1^c = ⟨x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)⟩ is an IFCS in X but f(G_1^c) is not an IFαCS in Y, as cl(int(cl(f (G_1^c)))) = cl(int(G_2^c)) = cl(G_3) = G_2^c ⊈ f (G_1^c).

Proposition 4.2.11: Every IFγ closed mapping is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγ closed mapping. Let A be an IFCS in X. Then f(A) is an IFγCS in Y. Since every IFγCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

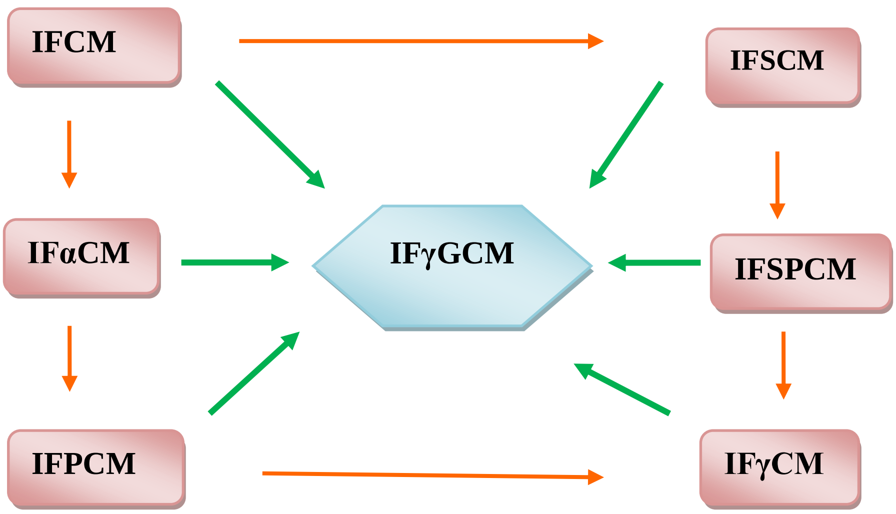
Example 4.2.12: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)⟩, G_2 = ⟨y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)⟩ and G_3 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG closed mapping but not an IFγ closed mapping, since G_1^c = ⟨x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)⟩ is an IFCS in X but f (G_1^c) is not an IFγCS in Y, as int(cl(f(G_1^c))) ∩ cl(int(f (G_1^c))) = $1_\sim$ ∩ $1_\sim$ = $1_\sim$ ⊈ f (G_1^c).

Proposition 4.2.13: Every IF semipre closed mapping is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF semipre closed mapping. Let A be an IFCS in X. Then f(A) is an IFSPCS in Y. Since every IFSPCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 4.2.14: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, $(0.4_a, 0.4_b)$, $(0.6_a, 0.6_b)$⟩, G_2 = ⟨y, $(0.5_u, 0.7_v)$, $(0.5_u, 0.3_v)$⟩ and G_2 = ⟨y, $(0.5_u, 0.6_v)$, $(0.5_u, 0.4_v)$⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$ G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG closed mapping but not an IF semipre closed mapping, since G_1^c is an IFCS in X but f(G_1^c) is not an IFSPCS in Y, as there exists no IFPCS B in Y such that int(B) ⊆ f (G_1^c) ⊆ B in Y.

The relation between various types of intuitionistic fuzzy closed mapping is given in the following diagram. In this diagram 'CM' means closed mappings. The reverse implications are not true in general in the below diagram.



Proposition 4.2.15: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following are equivalent if Y is an $IF\gamma_\gamma T_{1/2}$space:

(i) f is an IFγG closed mapping,

(ii) γcl(f(A)) ⊆ f(cl(A)) for each IFS A of X,

(iii) f^{-1}(γcl(B)) ⊆ cl(f^{-1}(B)) for every IFS B of Y.

Proof: (i) ⇒ (ii): Let A be an IFS in X. Then cl(A) is an IFCS in X. (i) implies that f(cl(A)) is an IFγGCS in Y. Since Y is an $IF\gamma_\gamma T_{1/2}$space, f(cl(A)) is an IFγCS in Y. Therefore γcl(f(cl(A))) = f(cl(A)). Now γcl(f(A)) ⊆ γcl(f(cl(A))) = f(cl(A)). Hence γcl(f(A))⊆f(cl(A)) for each IFS A of X.

(ii) ⇒ (i): Let A be any IFCS in X. Then cl(A) = A. (ii) implies that γcl(f(A)) ⊆ f(cl(A)) = f(A). But f(A) ⊆ γcl(f(A)). Therefore γcl(f(A)) = f(A). This implies f(A) is an IFγCS in Y. Since every IFγCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

(ii) ⇒ (iii): Let B be an IFS in Y. Then f^{-1}(B) is an IFS in X. Since f is onto, γcl(B) = γcl(f(f^{-1}(B))) and (ii) implies γcl(f(f^{-1}(B))) ⊆ f(cl(f^{-1}(B))). Therefore γcl(B) ⊆ f(cl(f^{-1}(B))). Now f^{-1}(γcl(B)) ⊆ f^{-1}(f(cl(f^{-1}(B)))) since f is one to one. Hence f^{-1}(γcl(B)) ⊆ cl(f^{-1}(B)).

(iii) ⇒ (ii): Let A be any IFS of X. Then f(A) is an IFS of Y. Since f is one to one, (iii) implies that f^{-1}(γcl(f(A))) ⊆ cl(f^{-1}(f(A))) = cl(A). Therefore f(f^{-1}(γcl(f(A)))) ⊆ f(cl(A)). Since f is onto, γcl(f(A)) = f(f^{-1}(γcl(f(A)))) ⊆ f(cl(A)).

Proposition 4.2.16: A bijective mapping f: (X, τ) → (Y, σ) is an IFγG closed mapping if and only if for every IFS B of Y and for every IFOS U containing f^{-1}(B), there is an IFγGOS A of Y such that B ⊆ A and f^{-1}(A) ⊆U.

Proof: Necessity: Let B be any IFS in Y. Let U be an IFOS in X such that f^{-1}(B) ⊆ U, then U^c is an IFCS in X. By hypothesis f(U^c) is an IFγGCS in Y. Let A = $(f(U^c))^c$, then A is an IFγGOS in Y and B ⊆ A, since for a bijective mapping $(f(U^c))^c = f(U)$. Now $f^{-1}(A) = f^{-1}(f(U^c))^c = (f^{-1}(f(U^c)))^c \subseteq U$.

Sufficiency: Let A be any IFCS in X, then A^c is an IFOS in X and $f^{-1}(f(A^c)) \subseteq A^c$. By hypothesis there exists an IFγGOS B in Y such that $f(A^c) \subseteq B$ and $f^{-1}(B) \subseteq A^c$. Hence $B^c \subseteq f(A) \subseteq f(f^{-1}(B))^c \subseteq (f(f^{-1}(B)))^c \subseteq B^c$. This implies that $f(A) = B^c$. Since B^c is an IFγGCS in Y, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Proposition 4.2.17: Let f: (X, τ) → (Y, σ) be a bijective mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an $IF\gamma_\gamma T_{1/2}$ space:

(i) f is an IFγG closed mapping,

(ii) $(int(cl(f(B))) \cap cl(int(f(B)))) \subseteq f(\gamma cl(B))$ for each IFCS B in X,

(iii) $f(\gamma int(B)) \subseteq (int(cl(f(B))) \cup cl(int(f(B))))$ for each IFOS B of X,

(iv) $f^{-1}(cl(int(A)) \cap int(cl(A))) \subseteq cl(f^{-1}(A))$ for each IFS A of Y.

Proof: (i) ⇒ (ii) Let B be an IFCS in X. Then f (B) is an IFγGCS in Y by hypothesis. Since Y is an $IF\gamma_\gamma T_{1/2}$ space, f(B) is an IFγCS in Y. Therefore $(int(cl(f(B))) \cap cl(int(f(B)))) \subseteq f(B) = f(\gamma cl(B))$.

(ii) ⇒ (iii) can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let A ⊆ Y. Then $B = f^{-1}(A) \subseteq X$. Now $A = f(f^{-1}(A)) = f(B)$. Here $int(f^{-1}(A)) = int(B)$ is an IFOS in X. Then (iii) implies that $f(\gamma int(int(B))) \subseteq (int(cl(f(int(B))) \cup cl(int(f(int(B))))) \subseteq (int(cl(f(B))) \cup cl(int(f(B))))$. Now $(int(cl(A^c)) \cup cl(int(A^c)))^c \subseteq (int(cl(f(B)^c)) \cup cl(int(f(B)^c)))^c \subseteq (f(\gamma int(int(B^c))))^c$. Therefore $(cl(int(A)) \cap int(cl(A))) \subseteq f(\gamma cl(cl(B)))$. Now $f^{-1}(cl(int(A)) \cap int(cl(A))) \subseteq f^{-1}(f(\gamma cl(cl(B)))) \subseteq cl(B) = cl(f^{-1}(A))$.

(iv) ⇒ (i) Let B be any IFCS in X, then f(B) is an IFS in Y. By hypothesis $f^{-1}(cl(int(f(B))) \cap int(cl(f(B)))) \subseteq cl(f^{-1}(f(B))) \subseteq cl(B) = B$. Now $(cl(int(f(B))) \cap int(cl(f(B)))) \subseteq f(f^{-1}(cl(int(f(B))) \cap int(cl(f(B))))) \subseteq f(B)$. This implies f(B) is an IFγCS and hence it is an IFγGCS in Y. Thus f is an IFγG closed mapping.

Proposition 4.2.18: If f: (X, τ) → (Y, σ) is an IFCM and g: (Y, σ) → (Z, δ) is an IFγG closed mapping, then g o f : (X, τ) → (Z, δ) is an IFγG closed mapping.

Proof: Let A be an IFCS in X, then f(A) is an IFCS in Y, since f is an IF closed mapping. Since g is an IFγG closed mapping, g(f(A)) is an IFγGCS in Z. Therefore g o f is an IFγG closed mapping.

Proposition 4.2.19: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an $IF\gamma_\gamma T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγG closed mapping,

(ii) f(B) is an IFγGCS in Y for every IFCS B in X,

(iii) (int(cl(f(B))) ∩ cl(int(f(B)))) ⊆f(cl(B)) for every IFS B in X.

Proof: (i) ⇔ (ii) is obvious.

(ii) ⇒ (iii) Let B be an IFS in X, then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFγGCS in Y. Since Y is an $IF\gamma_\gamma T_{1/2}$ space, f(cl(B)) is an IFγCS in Y. Therefore f(cl(B)) = γcl(f(cl(B))) ⊇ f(cl(B)) ∪ (int(cl(f(cl(B)))) ∩ cl(int(f(cl(B))))) ⊇ (int(cl(f(cl(B)))) ∩ cl(int(f(cl(B))))) ⊇ (int(cl(f(B))) ∩ cl(int(f(B)))).

(iii) ⇒ (i) Let A be an IFCS in X. By hypothesis, f(cl(A)) = f(A) ⊇ (int(cl(f(A))) ∩ cl(int(f(A)))). This implies f(A) is an IFγCS in Y and hence an IFγGCS in Y. Therefore f is an IFγG closed mapping.

Proposition 4.2.20: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an $IF\gamma_c T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγG closed mapping,

(ii)f(B) is an IFγGCS in Y for every IFCS B in X,

(iii) int(cl(f(B))) ⊆ f(cl(B)) for every IFS B in X.

Proof: (i) ⇔ (ii) is obvious.

(ii) ⇒ (iii) Let B be an IFS in X, then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFγGCS in Y. Since Y is an IF$\gamma_c T_{1/2}$ space, f(cl(B)) is an IFCS in Y. Therefore f(cl(B)) = cl(f(cl(B))) ⊇ int(cl(f(cl(B)))) ⊇ int(cl(f(B))).

(iii) ⇒ (i) Let A be an IFCS in X. By hypothesis, f(cl(A)) = f(A) ⊇ int(cl(f(A))). This implies f(A) is an IFSCS in Y and hence an IFγGCS in Y. Therefore f is an IFγG closed mapping.

Definition 4.2.21: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy γ generalized*** (IFγG) ***open mapping*** if f(A) is an IFγGOS in Y for each IFOS A in X.

Proposition 4.2.22: For a bijective mapping f: (X, τ) → (Y, σ) the following are equivalent:

(i) f is an IFγG closed mapping,

(ii) f is an IFγG open mapping.

Proof: Proof is obvious as for a bijective mapping, $f(A^c) = (f(A))^c$.

Proposition 4.2.23: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following are equivalent if Y is an IF$\gamma_\gamma T_{1/2}$ space:

(i) f is an IFγG open mapping,

(ii) f(int(A)) ⊆ γint(f(A)) for each IFS A of X,

(iii) $int(f^{-1}(B)) \subseteq f^{-1}(\gamma int(B))$ for every IFS B of Y.

Proof: This Proposition can be easily proved by taking complement in Proposition 4.2.15 as the mapping is bijective.

Proposition 4.2.24: A mapping f: (X, τ) → (Y, σ) is an IFγG open mapping if f(γint(A)) ⊆ γint(f(A)) for every A ⊆ X.

Proof: Let A be an IFOS in X. Then int(A) = A. Now f(A) = f(int(A)) ⊆ f(γint(A)) ⊆ γint(f(A)), by hypothesis. But γint(f(A)) ⊆ f(A). Therefore f(A) is an IFγOS in Y and hence is an IFγGOS in Y. Thus f is an IFγG open mapping.

Proposition 4.2.25: A mapping f: (X, τ) → (Y, σ) is an IFγG open mapping if and only if $int(f^{-1}(B)) \subseteq f^{-1}(\gamma int(B))$ for every B ⊆Y, where Y is an $IF\gamma_\gamma T_{1/2}$ space.

Proof: Necessity: Let B ⊆ Y. Then $f^{-1}(B) \subseteq X$ and $int(f^{-1}(B))$ is an IFOS in X. By hypothesis, $f(int(f^{-1}(B)))$ is an IFγGOS in Y. Since Y is an $IF\gamma_\gamma T_{1/2}$ space, $f(int(f^{-1}(B))$ is an IFγOS in Y. Therefore $f(int(f^{-1}(B))) = \gamma int(f(int(f^{-1}(B)))) \subseteq \gamma int(B)$. This implies $int(f^{-1}(B)) \subseteq f^{-1}(f(int(f^{-1}(B)))) \subseteq f^{-1}(\gamma int(B))$.

Sufficiency: Let A be an IFOS in X. Therefore int(A) = A and f(A) ⊆ Y. By hypothesis $int(f^{-1}(f(A))) \subseteq f^{-1}(\gamma int(f(A)))$. Therefore $A = int(A) \subseteq int(f^{-1}(f(A))) \subseteq f^{-1}(\gamma int(f(A)))$. This implies $f(A) \subseteq f(f^{-1}(\gamma int(f(A)))) \subseteq \gamma int(f(A)) \subseteq f(A)$. Thus f(A) is an IFγOS in Y and hence an IFγGOS in Y. Thus f is an IFγG open mapping.

Proposition 4.2.26: Let (X, τ) be an IFTS where X is an $IF\gamma_\gamma T_{1/2}$ space. An IFS A is an IFγGOS in X if and only if A is an IFγN of $p_{(\alpha,\beta)}$ for each $p_{(\alpha,\beta)} \in A$.

Proof: Necessity: Let $p_{(\alpha,\beta)} \in A$. Let A be an IFγGOS in X. Since X is an $IF\gamma_\gamma T_{1/2}$ space, A is an IFγOS in X. Then clearly A is an IFγN of $p_{(\alpha,\beta)}$ as $p_{(\alpha,\beta)} \in A \subseteq A$.

Sufficiency: Let $p_{(\alpha,\beta)} \in A$. Since A is an IFγN of $p_{(\alpha,\beta)}$, there is an IFγOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$. Now $A = \cup\{p_{(\alpha,\beta)} / p_{(\alpha,\beta)} \in A\} \subseteq \cup\{B_{p(\alpha,\beta)} / p_{(\alpha,\beta)} \in A\} \subseteq A$. This implies $A = \cup\{B_{p(\alpha,\beta)} / p_{(\alpha,\beta)} \in A\}$. Since each B is an IFγOS, A is an IFγOS and hence is an IFγGOS in X.

Proposition 4.2.27: For any IFS A in an IFTS (X, τ) where X is an $IF\gamma_\gamma T_{1/2}$ space, A ∈ IFγGO(X) if and only if for every IFP $p_{(\alpha,\beta)} \in A$, there exists an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Proof: Necessity: If A ∈ IFγGO(X), then we can take B = A so that $p_{(\alpha,\beta)} \in B \subseteq A$ for every IFP $p_{(\alpha,\beta)} \in A$.

Sufficiency: Let A be an IFS in X and assume that there exists B ∈ IFγGO(X) such that $p_{(\alpha,\beta)} \in B \subseteq A$. Since X is an $IF\gamma_\gamma T_{1/2}$ space, B is an IFγOS of X. Then

$A = \bigcup_{p_{(\alpha,\beta)} \in A} \{ p_{(\alpha,\beta)} \} \subseteq \bigcup_{p_{(\alpha,\beta)} \in A} B \subseteq A$. Therefore A is an IFγOS and hence an IFγGOS in X. Thus A ∈ IFγGO(X).

Proposition 4.2.28: Let f: (X, τ) → (Y, σ) be a mapping that satisfies f (int(B)) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))) for every IFS B in X. Then f is an IFγG open mapping.

Proof: Let B be an IFOS in X. Then int(B) = B. By hypothesis f(B) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))). This implies f(B) is an IFγOS in Y. Therefore it is an IFγGOS in Y and hence f is an IFγG open mapping.

Proposition 4.2.29: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ_γ $T_{1/2}$ space. Then f is an IFγG open mapping if and only if for any IFP $p_{(\alpha,\beta)}$ ∈ Y and for any IFN B of $f^{-1}(p_{(\alpha,\beta)})$, there is an IFγN A of $p_{(\alpha,\beta)}$ ∈ A and $f^{-1}(A) \subseteq B$.

Proof: Necessity: Let $p_{(\alpha,\beta)}$ ∈ Y and B be an IFN of $f^{-1}(p_{(\alpha,\beta)})$. Then there is an IFOS C in X such that $f^{-1}(p_{(\alpha,\beta)}) \in C \subseteq B$. Since f is an IFγG open mapping, f(C) is an IFγGOS in Y. Since Y is an IFγ_γ $T_{1/2}$ space, f(C) is an IFγOS in Y and $p_{(\alpha,\beta)} \in f(f^{-1}(p_{(\alpha,\beta)})) \subseteq f(C) \subseteq f(B)$. Put A = f(C). Then A is an IFγN of $p_{(\alpha,\beta)}$ and $p_{(\alpha,\beta)} \in A \subseteq f(B)$. Thus $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq f^{-1}(f(B)) = B$.

Sufficiency: Let B ⊆ X be an IFOS. If f(B) = $0_\sim$ then there is nothing to prove. Suppose that $p_{(\alpha,\beta)} \in f(B)$ then $f^{-1}(p_{(\alpha,\beta)}) \in B$ and B is an IFN of $f^{-1}(p_{(\alpha,\beta)})$. By hypothesis there is an IFγN A of $p_{(\alpha,\beta)}$ such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$. Therefore there is an IFγOS C in Y such that $p_{(\alpha,\beta)} \in C \subseteq A = f(f^{-1}(A)) \subseteq f(B)$.

Hence $f(B) = \cup\{ p_{(\alpha,\beta)} / p_{(\alpha,\beta)} \in f(B)\} \subseteq \cup\{C / p_{(\alpha,\beta)} \in f(B)\} \subseteq f(B)$. Thus $f(B) = \cup\{C / p_{(\alpha,\beta)} \in f(B)\}$. Since each C is an IFγOS, f(B) is also an IFγOS and hence is an IFγGOS in Y. Therefore f is an IFγG open mapping.

Proposition 4.2.30: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ_γ $T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγG closed mapping,

(ii) f(B) is an IFγGOS in Y for every IFOS B in X,

(iii) f(int(B)) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))) for every IFS B in X.

Proof: (i) ⇔ (ii) is obvious as $f(A^c) = (f(A))^c$ for a bijective mapping.

(ii) ⟹ (iii) Let B be an IFS in X, then int(B) is an IFOS in X. By hypothesis f(int(B)) is an IFγGOS in Y. Since Y is an $IF\gamma_\gamma T_{1/2}$ space, f(int(B)) is an IFγOS in Y. Therefore f(int(B)) = γint(f(int(B))) ⊆ f(int(B)) ∩ (int(cl(f(int(B))) ∪ cl(int(f(int(B)))) ⊆ int(cl(f(int(B))) ∪ cl(int(f(int(B)))) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))).

(iii) ⟹ (i) Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, $f(int(A^c)) = f(A^c) \subseteq (int(cl(f(A^c))) \cup cl(int(f(A^c))))$. That is (cl(int(f(A))) ∩ int(cl(f(A)))) ⊆ f(A). This implies f(A) is an IFγCS and hence an IFγGCS. Therefore f is an IFγG closed mapping.

Proposition 4.2.31: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following conditions are equivalent if X and Y are $IF\gamma_\gamma T_{1/2}$ spaces:

(i) f is an IFγG closed mapping,

(ii) f (B) is an IFγGOS in Y for each IFOS B in X,

(iii) for each IFP $p_{(\alpha,\beta)}$ in Y and for every IFOS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$, there exists an IFγOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$.

Proof: (i) ⇔ (ii) is obvious as $f(A^c) = (f(A))^c$ for a bijective mapping.

(ii) ⇒ (iii) Let B be any IFOS in X and let $p_{(\alpha,\beta)} \in Y$. Given $f^{-1}(p_{(\alpha,\beta)}) \in B$. By hypothesis f(B) is an IFγGOS in Y. As Y is an $IF\gamma_\gamma T_{1/2}$ spaces, f(B) is an IFγOS in Y. Take A = f(B). Then $p_{(\alpha,\beta)} \in f(B) = A$ and $f^{-1}(A) = f^{-1}(f(B)) \subseteq B$.

(iii) ⇒ (i) Let A be an IFCS in X. Then its complement, say B is an IFOS in X. Let $p_{(\alpha,\beta)} \in Y$ and $f^{-1}(p_{(\alpha,\beta)}) \in B$. By hypothesis there exists an IFγOS C in Y such that $p_{(\alpha,\beta)} \in C$ and $f^{-1}(C) \subseteq B$. This implies $p_{(\alpha,\beta)} \in C \subseteq f(f^{-1}(C)) \subseteq f(B)$. That is $p_{(\alpha,\beta)} \in f(B)$. Since C is an IFγOS, C = γint(C) ⊆ γint(f(B)). Therefore $p_{(\alpha,\beta)} \in$ γint(f(B)) ⊆ f(B) and $f(B) = \bigcup_{p(\alpha,\beta) \in f(B)} \{p_{(\alpha,\beta)}\} \subseteq$ γint(f(B)) ⊆ f(B). Hence f(B) is an IFγOS in Y and is an IFγGOS in Y. Thus f(A) is an IFγGCS in Y and f is an IFγG closed mapping.

Proposition 4.2.32: A bijective mapping f: (X, τ) → (Y, σ) is an IFγG open mapping if (int(cl(f(A))) ∪ cl(int(f(A)))) ⊆ f (cl(A)) for every IFS A in X.

Proof: Let A be an IFOS in X then A^c is an IFCS in X. By hypothesis, (int(cl(f(A^c))) ∩ cl(int(f(A^c)))) ⊆ f (cl(A^c)) = f (A^c). Now $(\text{int}(\text{cl}(f(A))) \cup \text{cl}(\text{int}(f(A))))^c$ = (int(cl(f(A^c))) ∩ cl(int(f(A^c)))) ⊆ f (A^c) = $(f\ (A))^c$. This implies f (A) ⊆ (int(cl(f(A))) ∪ cl(int(f(A)))). Hence f(A) is an IFγOS and hence it is an IFγGOS. Therefore f is an IFγG open mapping.

Proposition 4.2.33: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IF$\gamma_c T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγG closed mapping,

(ii) f(B) is an IFγGOS in Y for every IFOS B in X,

(iii) f(int(B)) ⊆ cl(int(f(B))) for every IFS B in X.

Proof: (i) ⟺ (ii) is obvious as f(A^c) = $(f(A))^c$ for a bijective mapping.

(ii) ⟹ (iii) Let B be an IFS in X, then int(B) is an IFOS in X. By hypothesis f(int(B)) is an IFγGOS in Y. Since Y is an IF$\gamma_c T_{1/2}$ space, f(int(B)) is an IFOS in Y. Therefore f(int(B)) = int(f(int(B))) ⊆ cl(int(f(int(B)))) ⊆ cl(int(f(B))).

(iii) ⟹ (i) Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, f(int(A^c)) = f(A^c) ⊆ cl(int(f(A^c)))). That is int(cl(f(A))) ⊆ f(A). This implies f(A) is an IFSCS in Y and hence an IFγGCS in Y. Therefore f is an IFγG closed mapping.

Definition 4.2.34: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy M-γ generalized*** (IFM-γG) ***closed mapping*** if f(V) is an IFγGCS in Y for every IFγGCS V of X.

Example 4.2.35: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩, and G_2 = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)⟩. Then τ = {0~ , G_1, 1~} and σ = {0~ , G_2, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v Then f is an IFM-γG closed mapping.

Proposition 4.2.36: Every IFM-γG closed mapping is an IFγG closed mapping but not conversely in general.

Proof: Let f be an IFM-γG closed mapping. Let V be any IFCS in X. Then V is an IFγGCS and by hypothesis f(V) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 4.2.37: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩, G_2 = ⟨y, (0.6_u, 0.8_v), (0.4_u, 0.2_v)⟩ and G_3 = ⟨y, (0.5_u, 0.5_v), (0.4_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG closed mapping but not an IFM-γG closed mapping. Since the IFS A = ⟨x, (0.5_a, 0.5_b), (0.5_a, 0.4_b)⟩ is an IFγGCS in X but f(A) = ⟨y, (0.5_u, 0.5_v), (0.5_u, 0.4_v)⟩ is not an IFγGCS in Y, as γcl(f(A)) = $1_\sim$ $\nsubseteq$ G_2, G_3 where as f(A) $\subseteq$ G_2, G_3.

Proposition 4.2.38: The composition of two IFM-γG closed mappings is an IFM-γG closed mapping in general.

Proof: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be any two IFM-γG closed mappings. Let V be an IFγGCS in X. Then f(V) is an IFγGCS in Y, by hypothesis. Since g is an IFM-γG closed mapping, g(f(V)) is an IFγGCS in Z. Hence g o f is an IFM-γG closed mapping.

Proposition 4.2.39: Let f: (X, τ) → (Y, σ) be an IFγG closed mapping and g: (Y, σ) → (Z, δ) is an IFM-γG closed mapping then g o f : (X, τ) → (Z, δ) is an IFγG closed mapping.

Proof: Let V be an IFCS in X. Then f(V) is an IFγGCS in Y. Since g is an IFM-γG closed mapping, g(f(V)) is an IFγGCS in Z. Hence g o f is an IFγG closed mapping.

Proposition 4.2.40: If f: (X, τ) → (Y, σ) is a bijective mapping, then the following are equivalent:

(i) f is an IFM-γG closed mapping,

(ii) f(A) is an IFγGCS in Y for every IFγGCS A in X,

(iii) f(A) is an IFγGOS in Y for every IFγGOS A in X.

Proof: As $f(A^c) = (f(A))^c$ is true for a bijective mapping the proof is obvious.

Proposition 4.2.41: If f: $(X, \tau) \to (Y, \sigma)$ is a bijective mapping and Y is an IFγ_γ $T_{1/2}$ space then the following are equivalent:

(i) f is an IFM-γG closed mapping,

(ii) f(A) is an IFγGOS in Y for every IFγGOS A in X,

(iii) for every IFP $p_{(\alpha,\beta)} \in Y$ and for every IFγGOS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$, there exists an IFγGOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$.

Proof: (i) ⟺ (ii) is obvious by Proposition 4.2.39.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in Y$ and let B be an IFγGOS in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$. This implies $p_{(\alpha,\beta)} \in f(B)$. By hypothesis, f(B) is an IFγGOS in Y. Let A = f(B). Therefore $p_{(\alpha,\beta)} \in f(B) = A$ and $f^{-1}(A) = f^{-1}(f(B)) \subseteq B$.

(iii) ⇒ (i) Let B be an IFγGCS in X. Then B^c is an IFγGOS in X. Let $p_{(\alpha,\beta)} \in Y$ and $f^{-1}(p_{(\alpha,\beta)}) \in B^c$. This implies $p_{(\alpha,\beta)} \in f(B^c)$. By hypothesis there exists an IFγGOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B^c$, then $A = f(f^{-1}(A)) \subseteq f(B^c)$. Hence by Proposition 4.2.26, $f(B^c)$ is an IFγGOS in Y. As f is a bijective mapping, $f(B^c) = (f(B))^c$. Therefore f(B) is an IFγGCS in Y. Thus f is an IFM-γG closed mapping.

Proposition 4.2.42: If f: $(X, \tau) \to (Y, \sigma)$ is a bijective mapping, where X and Y are IFγ_γ $T_{1/2}$ space, then the following are equivalent:

(i) f is an IFM-γG closed mapping,

(ii) f(A) is an IFγGOS in Y for every IFγGOS A in X,

(iii) $f(\gamma int(B)) \subseteq \gamma int(f(B))$ for every IFS B in X,

(iv) $\gamma cl(f(B)) \subseteq f(\gamma cl(B))$ for every IFS B in X.

Proof: (i) ⟺ (ii) is obvious.

(ii) $\Longrightarrow$ (iii) Let B be any IFS in X. Since γint(B) is an IFγOS, it is an IFγGOS in X. Then by hypothesis, f(γint(B)) is an IFγGOS in Y. Since Y is an $IF\gamma_\gamma$ $T_{1/2}$ space, f(γint(B)) is an IFγOS in Y. Therefore f(γint(B)) = γint(f(γint(B))) ⊆ γint(f(B)).

(iii) $\Longrightarrow$ (iv) can easily proved by taking complement in (iii).

(iv) $\Longrightarrow$ (i) Let A be an IFγGCS in X. By hypothesis, γcl(f(A)) ⊆ f(γcl(A)). Since X is an $IF\gamma_\gamma$ $T_{1/2}$space, A is an IFγCS in X. Therefore, γcl(f(A)) ⊆ f(γcl(A)) = f(A) ⊆ γcl(f(A)). Hence f(A) is an IFγCS in Y and hence an IFγGCS in Y. Thus f is an IFM-γG closed mapping.

Proposition 4.2.43: Let f: (X, τ) → (Y, σ) be a bijective mapping, where X is an $IF\gamma_\gamma$ $T_{1/2}$ space. If f is an IFM-γG closed mapping, then for each IFP $p_{(\alpha, \beta)}$ ∈ Y and every IFγN A of $f^{-1}(p_{(\alpha, \beta)})$, there exists an IFγGOS B inY such that $p_{(\alpha, \beta)}$ ∈ B ⊆ f(A).

Proof: Let $p_{(\alpha, \beta)}$ ∈ Y and let A be the IFγN of $f^{-1}(p_{(\alpha, \beta)})$. Then there exists an IFγOS C in X such that $f^{-1}(p_{(\alpha, \beta)})$ ∈ C ⊆ A. Since every IFγOS is an IFγGOS, C is an IFγGOS in X. Then by hypothesis, f(C) is an IFγGOS in Y. Now $p_{(\alpha, \beta)}$ ∈ f(C) ⊆ f(A). Put B = f(C). This implies $p_{(\alpha, \beta)}$ ∈ B ⊆ f(A).

Proposition 4.2.44: Let f: (X, τ) → (Y, σ) be a bijective mapping, where X is an $IF\gamma_\gamma$ $T_{1/2}$ space. If f is an IFM-γG closed mapping, then for each IFP $p_{(\alpha, \beta)}$ ∈ Y and every IFγN A of $f^{-1}(p_{(\alpha, \beta)})$, there exists an IFγGOS B in Y such that $p_{(\alpha, \beta)}$ ∈ B and $f^{-1}(B)$ ⊆ A.

Proof: Let $p_{(\alpha, \beta)}$ ∈ Y and let A be the IFγN of $f^{-1}(p_{(\alpha, \beta)})$. Then there exists an IFγOS B in X such $p_{(\alpha, \beta)}$ ∈ B ⊆ f(A). Now $f^{-1}(B)$ ⊆ $f^{-1}(f(A))$ = A. That is $f^{-1}(B)$ ⊆ A.

4.3 Intuitionistic fuzzy contra γ generalized open mappings

In this section we have introduced intuitionistic fuzzy contra γ generalized open mappings and investigated some of their properties.

Definition 4.3.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy contra γ generalized*** (IF contra γG) ***open mapping*** if f(V) is an IFγGCS in (Y, σ) for every IFOS V of (X, τ).

Example 4.3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG open mapping.

Proposition 4.3.3: Every IF contra open mapping is an IF contra γG open mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra open mapping. Let A be an IFOS in X. Then f(A) is an IFCS in Y. Since every IFCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IF contra γG open mapping.

Example 4.3.4: Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG open mapping but not an IF contra open mapping, as G_1 is an IFOS in X where as f (G_1) is not an IFCS in Y, since $cl(f(G_1)) = G_2^c \neq f(G_1)$.

Proposition 4.3.5: Every IF contra semi open mapping is an IF contra γG open mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra semi open mapping. Let A be an IFOS in X. Then f(A) is an IFSCS in Y. Since every IFSCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IF contra γG open mapping.

Example 4.3.6: Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG open mapping but not an IF contra semi open mapping, as G_1 is an IFOS in X whereas f (G_1) is not an IFSCS in Y, since $int(cl(f(G_1))) = int(G_2^c) = G_3 \not\subseteq f(G_1)$.

Proposition 4.3.7: Every IF contra pre open mapping is an IF contra γG open mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra pre open mapping. Let A be an IFOS in X. Then f(A) is an IFPCS in Y. Since every IFPCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IF contra γG open mapping.

Example 4.3.8: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩, G_2= ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3= ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_1 = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFOS in X. Then f is an IF contra γG open mapping but not an IF contra pre open mapping, as G_1 is an IFOS in X whereas f (G_1) is not an IFPCS in Y, since cl(int(f(G_1))) = cl(G_3) = $G_2^c \nsubseteq$ f(G_1).

Proposition 4.3.9: Every IF contra α open mapping is an IF contra γG open mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra α open mapping. Let A be an IFOS in X. Then f(A) is an IFαCS in Y. Since every IFαCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IF contra γG open mapping.

Example 4.3.10: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG open mapping, but not an IF contra α open mapping, as G_1 is an IFOS in X whereas f (G_1) is not an IFαCS in Y, since cl(int(cl(f(G_1)))) = cl(int(G_2^c)) = cl(G_3) = $G_2^c \nsubseteq$ f (G_1).

Proposition 4.3.11: Every IF contra γ open mapping is an IF contra γG open mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra γ open mapping. Let A be an IFOS in X. Then f(A) is an IFγCS in Y. Since every IFγCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IF contra γG open mapping.

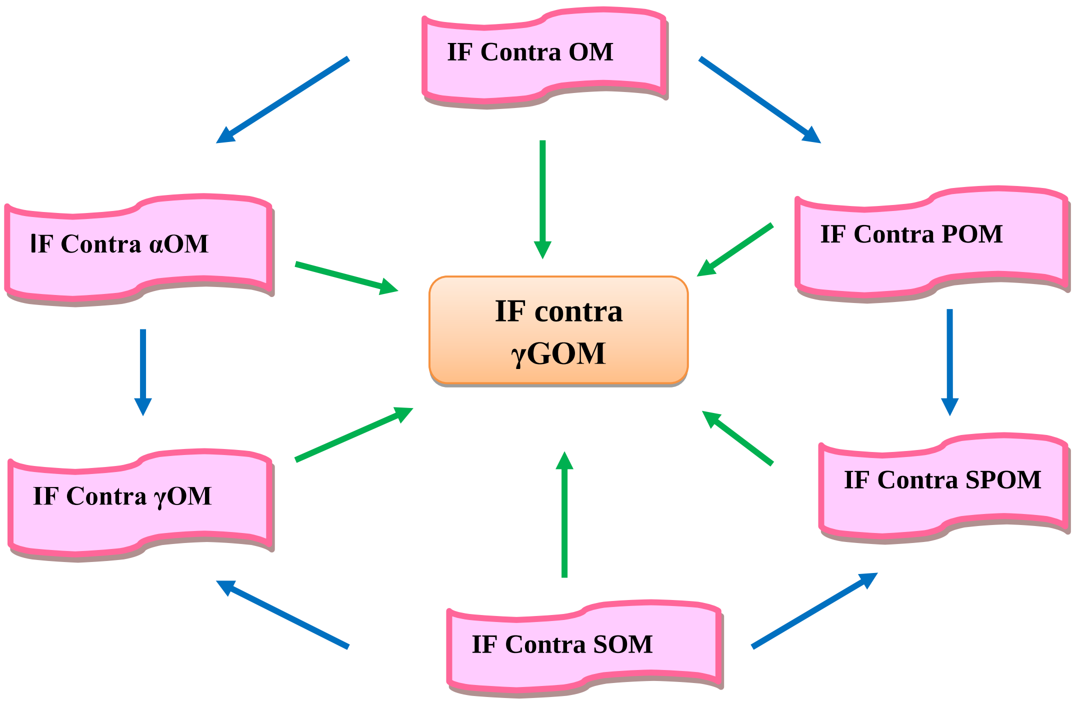
Example 4.3.12: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)⟩, G_2= ⟨y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)⟩, G_3= ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) →(Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG open mapping but not an IF contra γ open mapping, as G_1 is an IFOS in X whereas f (G_1) is not an IFγCS in Y, since int(cl(f(G_1))) ∩ cl(int(f(1_3))) = $1_\sim$ ⊈ f (G_1).

Proposition 4.3.13: Every IF contra semipre open mapping is an IF contra γG open mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra semipre open mapping. Let A be an IFOS in X. Then f(A) is an IFSPCS in Y. Since every IFSPCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IF contra γG open mapping.

Example 4.3.14: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.2_b), (0.5_a, 0.8_b)⟩, G_2= ⟨y, (0.5_u, 0.3_v), (0.5_u, 0.7_v)⟩, G_2 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$ G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) →(Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG open mapping but not an IF contra semipre open mapping, as G_1 is an IFOS in X whereas f(G_1) is not an IFSPCS in Y, since there exists no IFPCS B in X such that int(B) ⊆ f(G_1) ⊆ B in X. Therefore and f is.

The relation between various types of intuitionistic fuzzy contra open mappings and IF contra γG open mapping is given in the following diagram. In this diagram 'OM' means open mappings. The reverse implications are not true in general in the below diagram.



Definition 4.3.15: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy contra γ generalized*** (IF contra γG) ***closed mapping*** if f(V) is an IFγGOS in (Y, σ) for every IFCS V of (X, τ).

Proposition 4.3.16: For a bijective mapping f: (X, τ) → (Y, σ) the following are equivalent:

(i) f is an IF contra γG open mapping
(ii) f is an IF contra γG closed mapping

Proof: As $f(A^c) = (f(A))^c$ for a bijective mapping, the proposition is obviously true.

Proposition 4.3.17: Let f: (X, τ) → (Y, σ) be a bijective mapping, suppose that one of the following properties hold:

(i) f (cl(B)) ⊆ int(γcl(f(B))) for each IFS B in X,

(ii) cl(γint(f(B))) ⊆ f(int(B)) for each IFS B in X,

(iii) f^{-1}(cl(γint(A))) ⊆ int(f^{-1}(A)) for each IFS A in Y,

(iv) f^{-1}(cl(A)) ⊆ int(f^{-1}(A)) for each IFγOS A in Y.

Then f is an IF contra γG open mapping.

Proof: (i) ⇒ (ii) is obvious by taking complement in (i).

(ii) ⇒ (iii) Let A ⊆ Y. Put B = f^{-1}(A) in X. This implies A = f(f^{-1}(A)) = f(B) in Y. Now cl(γint(A)) = cl(γint(f(B))) ⊆ f(int(B)) by (ii). Therefore f^{-1}(cl(γint(A))) ⊆ f^{-1}(f(int(B))) = int(B) = int(f^{-1}(A)).

(iii) ⇒ (iv) Let A ⊆ Y be an IFγOS. Then γint(A) = A. By hypothesis, f^{-1}(cl(γint(A))) ⊆ int(f^{-1}(A)). Therefore f^{-1}(cl(A)) = f^{-1}(cl(γint(A))) ⊆ int(f^{-1}(A)).

Suppose (iv) holds. Let A be an IFOS in X. Then f(A) is an IFS in Y and γint(f(A)) is an IFγOS in Y. Hence by hypothesis, f^{-1}(cl(γint(f(A)))) ⊆ int(f^{-1}(γint(f(A)))) ⊆ int(f^{-1}(f(A))) = int(A) = A. Therefore cl(γint(f(A))) = f(f^{-1}(cl(γint(f(A))))) ⊆ f(A). Now cl(int(f(A))) ⊆ cl(γint(f(A))) ⊆ f(A). This implies f(A) is an IFPCS in Y and hence an IFγGCS in Y. Thus f is an IF contra γG open mapping.

Proposition 4.3.18: Let f: (X, τ) → (Y, σ) be a bijective mapping. Suppose that one of the following properties hold:

(i) f^{-1}(γcl(A)) ⊆ int(f^{-1}(A)) for each IFS A in Y,

(ii) γcl(f(B)) ⊆ f(int(B)) for each IFS B in X,

(iii) f (cl(B)) ⊆ γint(f (B)) for each IFS B in X.

Then f is an IF contra γG closed mapping.

Proof: (i) ⇒ (ii) Let B ⊆ X. Then f(B) is an IFS in Y. By hypothesis, f^{-1}(γcl(f(B))) ⊆ int(f^{-1}(f(B))) = int(B). Now γcl(f(B)) = f(f^{-1}(γcl(f(B)))) ⊆ f(int(B)).

(ii) ⇒ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in X. Then cl(A) = A and f (A) is an IFS in Y. Now f(A) = f(cl(A)) ⊆ γint(f(A)) ⊆ f (A), by hypothesis. This implies f(A) is an IFγOS in Y and hence an IFγGOS in Y. Therefore f is an IF contra γG closed mapping.

Proposition 4.3.19: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then f is an IF contra γG closed mapping if cl(f^{-1}(A)) ⊆ f^{-1}(γint(A)) for every IFS A in Y.

Proof: Let A be an IFCS in X. Then cl(A) = A and f(A) is an IFS in Y. By hypothesis cl(f^{-1}(f(A))) ⊆ f^{-1}(γint(f(A))). Since f is bijective, f^{-1}(f(A)) = A. Therefore A = cl(A) = cl(f^{-1}(f(A))) ⊆ f^{-1}(γint(f(A))). Now f (A) ⊆ f(f^{-1}(γint(f(A)))) = γint(f(A)) ⊆ f(A). Hence f(A) is an IFγOS in Y and hence an IFγGOS in Y. Thus f is an IF contra γG closed mapping.

Proposition 4.3.20: If f: (X, τ) → (Y, σ) is an IF contra γG open mapping, where Y is an IF$\gamma_\gamma T_{1/2}$ space, then the following conditions hold:

(i) γcl(f(B)) ⊆ f(int(γcl(B))) for every IFOS B in X,

(ii) f(cl(γint(B))) ⊆ γint(f(B)) for every IFCS B in X.

Proof: (i) Let B ⊆ X be an IFOS. By hypothesis f (B) is an IFγGCS in Y. Since Y is an IF$\gamma_\gamma T_{1/2}$ space, f(B) is an IFγCS in Y. This implies γcl(f(B)) = f(B) = f(int(B)) ⊆ f(int(γcl(B))).

(ii) can be proved easily by taking the complement in (i).

Remark 4.3.21: The composition of two IF contra γG open mappings is not an IF contra γG open mapping in general as seen in the following example.

Example 4.3.22: Let X = {a, b}, Y = {u, v}, Z = {p, q}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)\rangle$, $G_3 = \langle z, (0.6_p, 0.8_q), (0.4_p, 0.2_q)\rangle$ and $G_4 = \langle z, (0.5_p, 0.5_q), (0.4_p, 0.4_q)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$, $\sigma = \{0_\sim, G_2, 1_\sim\}$ and $\delta = \{0_\sim , G_3, G_4, 1_\sim\}$ are IFTs on X, Y and Z respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v and g: (Y, σ) → (Z, δ) by g(u) = p and g(v) = q. Here f and g are IF contra γG open mappings but their composition g o f : (X, τ) → (Z, δ) defined by g(f(a)) = p and g(f(b)) = q is not an IF contra γG open mapping, since $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ is an IFOS in X, but $g(f(G_1))$ is not an IFγGCS in Z, as $g(f(G_1)) = \langle z, (0.5_p, 0.6_q), (0.5_p, 0.4_q)\rangle \subseteq G_3$ whereas $\gamma cl(g(f(G_1))) = 1_\sim \nsubseteq G_3$.

Proposition 4.3.23: If f: (X, τ) → (Y, σ) is an IF open mapping and g: (Y, σ) → (Z, δ) is an IF contra γG open mapping then g o f: (X, τ) → (Z, δ) is an IF contra γG open mapping.

Proof: Let V be an IFOS in X. Then f(V) is an IFOS in Y, since f is an IF open mapping. Since g is an IF contra γG open mapping, g(f(V)) is an IFγGCS in Z. Therefore g o f is an IF contra γG open mapping.

Proposition 4.3.24: For a mapping f: (X, τ) → (Y, σ), where Y is an $IF\gamma_\gamma T_{1/2}$ space, the following are equivalent:

(i) f is an IF contra γG closed mapping,

(ii) For every IFCS A in X and for every IFP $p_{(\alpha,\beta)} \in Y$, if $f^{-1}(p_{(\alpha,\beta)})_q$ A then $p_{(\alpha,\beta)\ q}\ \gamma int(f(A))$.

Proof: (i) ⇒ (ii) Let f be an IF contra γG closed mapping. Let A ⊆ X be an IFCS and let $p_{(\alpha,\beta)} \in Y$. Also let $f^{-1}(p_{(\alpha,\beta)})_q$ A then $p_{(\alpha,\beta)\ q}$ f (A). By hypothesis f (A) is an IFγGOS in Y. Since Y is an $IF\gamma_\gamma T_{1/2}$space, f(A) is an IFγOS in Y and γint(f(A)) = f(A). This implies $p_{(\alpha,\beta)\ q}\ \gamma int(f(A))$.

(ii) ⇒ (i)Let A ⊆ X be an IFCS then f(A) is an IFS in Y. Let $p_{(\alpha,\beta)} \in Y$ and let $f^{-1}(p_{(\alpha,\beta)})_q$ A then $p_{(\alpha,\beta)\ q}$ f(A). By hypothesis this implies $p_{(\alpha,\beta)\ q}\ \gamma int(f(A))$. Therefore f(A)

$\subseteq$ γint(f (A)). But γint(f(A)) $\subseteq$ f(A) and hence γint(f(A)) = f(A). Thus f(A) is an IFγOS in Y and hence an IFγGOS in Y**.** This implies f is an IF contra γG closed mapping.

Proposition 4.3.25: A mapping f: (X, τ) → (Y, σ) is an IF contra γG closed mapping, where Y is an IF$\gamma_\gamma T_{1/2}$ space if and only if f(γcl(B)) $\subseteq$ γint(f(cl(B))) for every IFS B in X.

Proof: Necessity: Let B $\subseteq$ X be an IFS. Then cl(B) is an IFCS in X. By hypothesis, f(cl(B)) is an IFγGOS in Y. Since Y is an IF$\gamma_\gamma T_{1/2}$ space, f(cl(B)) is an IFγOS in Y. Therefore f(γcl(B)) $\subseteq$ f(cl(B)) = γint(f(cl(B))).

Sufficiency: Let B $\subseteq$ X be an IFCS. Then cl(B) = B. By hypothesis, f(γcl(B)) $\subseteq$ γint(f(cl(B))) = γint(f(B)). But γcl(B) = B. Therefore f(B) = f(γcl(B)) $\subseteq$ γint(f(B)) $\subseteq$ f(B). This implies f(B) is an IFγOS in Y and hence an IFγGOS in Y. Hence f is an IF contra γG closed mapping.

Proposition 4.3.26: A mapping f: (X, τ) → (Y, σ) is an IF contra γG closed mapping if f(γcl(B)) $\subseteq$ int(f(B)) for every IFS B in X.

Proof: Let B $\subseteq$ X be an IFCS. Then cl(B) = B. Since every IFCS is an IFγCS, γcl(B) = B. Now by hypothesis, f(B) = f(γcl(B)) $\subseteq$ int(f(B)) $\subseteq$ f(B). This implies f(B) = int(f(B)). Therefore f (B) is an IFOS in Y and hence is an IFγGOS. Thus f is an IF contra γG closed mapping.

Proposition 4.3.27: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following conditions are equivalent if Y is an IF$\gamma_c T_{1/2}$ space:

(i) f is an IF contra γG closed mapping,

(ii) f is an IF contra γG open mapping,

(iii) int(cl(f(A))) $\subseteq$ (f(A)) for every IFOS A in X.

Proof: (i) ⟺ (ii) is obviously true.

(ii) ⇒ (iii): Let A be an IFOS in X. Then f(A) is an IFγGCS in Y. Since Y is an $IF\gamma_cT_{1/2}$ space, f(A) is an IFCS in Y. Therefore cl(f(A)) = f(A). This implies int(cl(f(A))) = int(f(A)) ⊆ f(A).

(iii) ⇒ (i): Let A be an IFCS in X. Then its complement A^c is an IFOS in X. By hypothesis, $int(cl(f(A^c))) \subseteq f(A^c)$. Hence $f(A^c)$ is an IFSCS in Y. Since every IFSCS is an IFγGCS, $f(A^c)$ is an IFγGCS in Y. Therefore f(A) is an IFγGOS in Y. Hence f is an IF contra γG closed mapping.

Proposition 4.3.28: If f: (X, τ) → (Y, σ) is a bijective mapping where Y is an $IF\gamma_\gamma T_{1/2}$ space, then the following are equivalent:

(iv) f is an IF contra γG open mapping,

(v) for each IFP $p_{(\alpha,\beta)} \in Y$ and for each IFCS B containing $f^{-1}(p_{(\alpha,\beta)})$, there exists an IFγOS A ⊆ Y and $p_{(\alpha,\beta)} \in A$ such that A⊆ f(B),

(vi) For each IFP $p_{(\alpha,\beta)} \in Y$ and for each IFCS B containing $f^{-1}(p_{(\alpha,\beta)})$, there exists an IFγOS A ⊆ Y and $p_{(\alpha,\beta)} \in A$ such that $f^{-1}(A) \subseteq B$.

Proof: (i) ⇒ (ii) Let B be an IFCS in X. Let $p_{(\alpha,\beta)}$ be an IFP in Y such that $f^{-1}(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f(f^{-1}(p_{(\alpha,\beta)}) \in f(B)$. By hypothesis f(B) is an IFγGOS in Y. Since Y is an $IF\gamma_\gamma T_{1/2}$ space, f(B) is an IFγOS in Y. Now let A = γint(f(B)) ⊆ f(B). Therefore A ⊆ f(B).

(ii) ⇒ (iii) Let B be an IFCS in X. Let $p_{(\alpha,\beta)}$ be an IFP in Y such that $f^{-1}(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f(f^{-1}(p_{(\alpha,\beta)}) \in f(B)$. By hypothesis f(B) is an IFγGOS in Y. Since Y is an $IF\gamma_\gamma T_{1/2}$ space, f(B) is an IFγOS in Y and A ⊆ f(B). This implies $f^{-1}(A) \subseteq f^{-1}(f(B)) = B$.

(iii) ⇒ (i) Let B be an IFCS in X and let $p_{(\alpha,\beta)} \in Y$. Let $f^{-1}(p_{(\alpha,\beta)}) \in B$. By hypothesis, there exists an IFγOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$. This implies $p_{(\alpha,\beta)} \in A \subseteq f(f^{-1}(A)) \subseteq f(B)$. That is $p_{(\alpha,\beta)} \in f(B)$. Since A is an IFγOS, A = γint(A) ⊆ γint(f(B)). Therefore $p_{(\alpha,\beta)} \in \gamma int(f(B)) \subseteq f(B)$. But $f(B) = \bigcup_{p(\alpha.\beta)\in f(B)} \{p_{(\alpha,\beta)}\} \subseteq \gamma int(f(B)) \subseteq f(B)$.

Hence f(B) is an IFγOS in Y and hence f(B) is an IFγGOS in Y. Thus f is an IF contra γG open mapping.

Definition 4.3.29: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy M- γ generalized*** (IFM-γG) ***open mapping*** if f(V) is an IFγGOS in (Y, σ) for every IFγGOS V of (X, τ).

Definition 4.3.30: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy contra M- γ generalized*** (IF contra M-γG) ***open mapping*** if f(V) is an IFγGCS in (Y, σ) for every IFγGOS V of (X, τ).

Example 4.3.31: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, G_2 =⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1,$1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra M-γG open mapping.

Proposition 4.3.32: A bijective mapping f: (X, τ) → (Y, σ) is an IF contra M-γG open mapping if and only if the image of each IFγGCS in X is an IFγGOS in Y.

Proof: Obvious as $f(A^c) = (f(A))^c$ for a bijective mapping.

Proposition 4.3.33: Every IF contra M-γG open mapping is an IF contra γG open mapping but not conversely in general.

Proof: Let A be an IFOS in X. Since every IFOS is an IFγGOS, then A is an IFγGOS in X. By hypothesis, f(A) is IFγGCS in Y. Hence f is IF contra γG open mapping.

Example 4.3.34: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Therefore f is an IF contra γG open mapping but not an IF contra M-γG open mapping. As A= ⟨y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)⟩ is an IFγGOS in X, but f(A) = ⟨y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)⟩ is not an IFγGCS in Y, as f(A) ⊆ G_2 where as γcl(A) = $1_\sim$ ⊈ G_2.

Proposition 4.3.35: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be any two mappings. Then

(i) g o f : (X, τ) → (Z, δ) is an IF contra γG open mapping if f is an IF contra M-γG open mapping and g is an IFM-γG closed mapping.

(ii) g o f : (X, τ) → (Z, δ) is an IF contra γG open mapping if f is an IFγG open mapping and g is an IF contra M-γG open mapping.

(iii) g o f : (X, τ) → (Z, δ) is an IFγG open mapping if f is an IF contra γG open mapping and g is an IF contra M-γG closed mapping.

Proof: (i) Let A be an IFOS in X. Then A is an IFγGOS in X. By hypothesis, f(A) is an IFγGCS in Y. Since g is an IFM-γG closed mapping, g(f(A)) is an IFγGCS in Z. Hence g o f is an IF contra γG open mapping.

(ii) Let A be an IFOS in X. Then by hypothesis, f(A) is an IFγGOS in Y. Since g is an IF contra M-γG open mapping, g(f(A)) is an IFγGCS in Z. Hence g o f is an IF contra γG open mapping.

(iii) Let A be an IFOS in X. Since f is an IF contra γG open mapping, f(A) is an IFγGCS in Y. Since g is an IF contra M-γG open mapping, g(f(A)) is an IFγGOS in Z. Hence g o f is an IFγG open mapping.

4.4 Intuitionistic fuzzy almost γ generalized closed mappings

In this section we have introduced intuitionistic fuzzy almost γ generalized closed mappings and investigated some of their properties.

Definition 4.4.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy almost γ generalized*** (IF almost γG) ***closed mapping*** if f(V) is an IFγGCS in Y for every IFRCS V of X.

Example 4.4.2: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG closed mapping.

Proposition 4.4.3: Every IF closed mapping is an IF almost γG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFCS in Y by hypothesis. Since every IFCS is an IFγGCS, f(V) is an IFγGCS in Y. Hence f is an IF almost γG closed mapping.

Example 4.4.4: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG closed mapping but not an IF closed mapping, since G_1^c = ⟨x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)⟩ is an IFCS in X but f (G_1^c) is not an IFCS in Y, as cl(f(G_1^c)) = $1_\sim \neq$ f(G_1^c).

Proposition 4.4.5: Every IF semi closed mapping is an IF almost γG closed mapping in (X, τ) but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF semi closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFSCS in Y by hypothesis. Since every IFSCS is an IFγGCS, f(V) is an IFγGCS in Y. Hence f is an IF almost γG closed mapping.

Example 4.4.6: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG closed mapping not an IF semi closed mapping, since G_1^c = ⟨x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)⟩ is an IFCS in X but f (G_1^c) is not an IFSCS in Y, as int(cl(f(G_1))) = $1_\sim \nsubseteq$ f (G_1^c).

Proposition 4.4.7: Every IF pre closed mapping is an IF almost γG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF pre closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFPCS in Y by hypothesis. Since every IFPCS is an IFγGCS, f (V) is an IFγGCS in Y. Hence f is an IF almost γG closed mapping.

Example 4.4.8: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩, G_2 = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩, G_3= ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩, and G_4= ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩ . Then τ = {$0_\sim$, G_1, G_2 $1_\sim$} and σ = {$0_\sim$, G_3, G_4, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG closed mapping, but not an IF pre closed mapping since G_1^c is an IFCS in X but f (G_1^c) is not an IFPCS in Y, as cl(int(f(G_1^c))) = cl(G_4) = G_3^c ⊈ f (G_1^c).

Proposition 4.4.9: Every IFα closed mapping is an IF almost γG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFαCS in Y by hypothesis. Since every IFαCS is an IFγGCS, f(V) is an IFγGCS in Y. Hence f is an IF almost γG closed mapping.

Example 4.4.10: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩ and G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG closed mapping but not an IFα closed mapping, since G_1^c = ⟨x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)⟩ is an IFCS in X but f (G_1^c) is not an IFαCS in Y, as cl(int(cl(f(G_1^c)))) = $1_\sim$ ⊈ f (G_1^c).

Proposition 4.4.11: Every IFγ closed mapping is an IF almost γG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγ closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFγCS in Y by hypothesis. Since every IFγCS is an IFγGCS, f(V) is an IFγGCS in Y. Hence f is an IF almost γG closed mapping.

Example 4.4.12: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)⟩, G_2 = ⟨x, (0.6_u, 0.6_v), (0.4_u, 0.4_v)⟩, G_3= ⟨y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)⟩ and G_4= ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, G_4, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG closed mapping but not an IFγ closed mapping, since G_1^c = ⟨x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)⟩ is an IFCS in X but f (G_1^c) is not an IFγCS in Y, as int(cl(f(G_1^c))) ∩ cl(int(f(G_1^c))) = $1_\sim$ ∩ $1_\sim$ = $1_\sim$ ⊈ f(G_1^c).

Proposition 4.4.13: Every IF semi pre closed mapping is an IF almost γG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF semi pre closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFSPCS in Y by hypothesis. Since every IFSPCS is an IFγGCS, f(V) is an IFγGCS in Y. Hence f is an IF almost γG closed mapping.

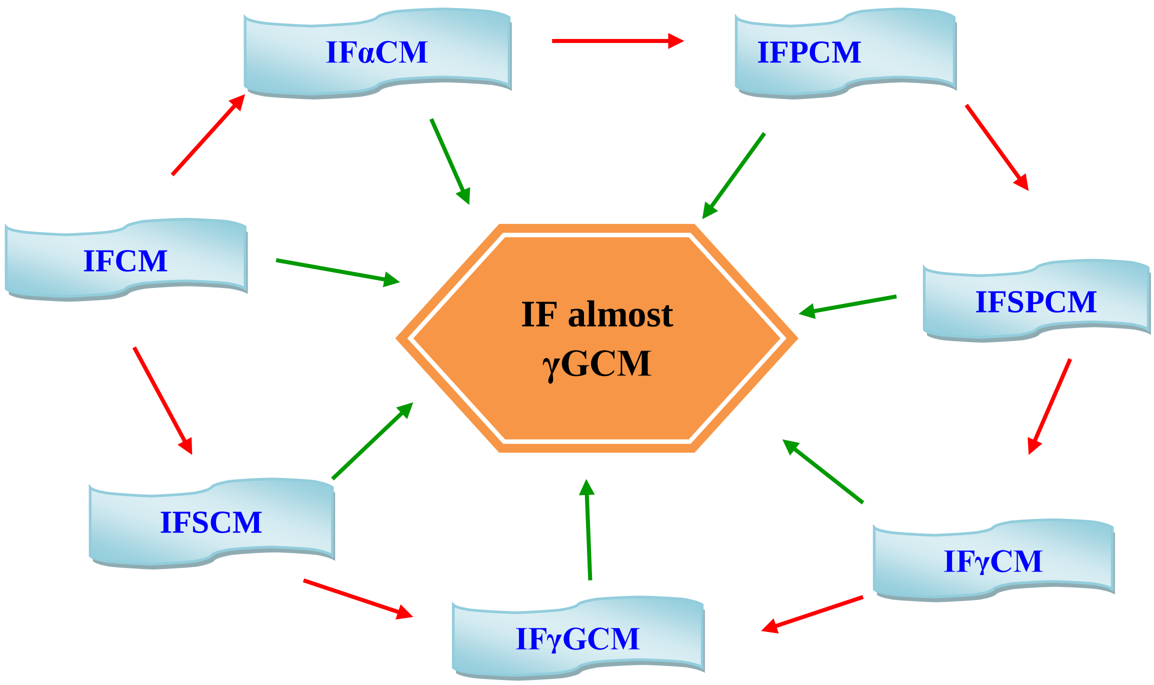
Example 4.4.14: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.2_b), (0.5_a, 0.8_b)⟩, G_2= ⟨y, (0.5_u, 0.3_v), (0.5_u, 0.7_v)⟩ and G_3= ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG closed mapping but not an IFSP closed mapping, since G_1^c is an IFCS in X but f(G_1^c) is not an IFSPCS in Y, as there exists no IFPCS B in Y such that int(B) ⊆ f(G_1^c) ⊆ B in Y.

Proposition 4.4.15: Every IFγG closed mapping is an IF almost γG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγG closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFγGCS in Y by hypothesis. Hence f is an IF almost γG closed mapping.

Example 4.4.16: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.2_a, 0.2_b), (0.5_a, 0.8_b)⟩, G_2 = ⟨x, (0.2_u, 0.2_v), (0.5_u, 0.6_v)⟩ and G_3 = ⟨y, (0.5_u, 0.8_v), (0.2_u, 0.2_v)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG closed mapping but not an IFγG closed mapping, since G_1^c = ⟨x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)⟩ is an IFCS in X but f (G_1^c) is not an IFγGCS in Y, as f(G_1^c) ⊆ G_3 where as γcl(f(G_1^c)) = $1_\sim$ ⊈ G_3.

The relation between various types of intuitionistic fuzzy closed mappings is given in the following diagram. In this diagram 'CM' means closed mappings. The reverse implications are not true in general in the below diagram.



Definition 4.4.17: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy almost γ generalized*** (IF almost γG) ***open mapping*** if f(A) is an IFγGOS in Y for each IFROS A in X.

Proposition 4.4.18: A bijective mapping f: (X, τ) → (Y, σ) is an IF almost γG closed mapping if and only if the image of each IFROS in X is an IFγGOS in Y.

Proof: Necessity: Let A be an IFROS in X. This implies A^c is an IFRCS in X. Then $f(A^c)$ is an IFγGCS in Y, by hypothesis. Since for a bijective mapping $f(A^c) = (f(A))^c$, f(A) is an IFγGOS in Y.

Sufficiency: Let A be an IFRCS in X. Then A^c is an IFROS in X. By hypothesis $f(A^c)$ is IFγGOS in Y. Since $f(A^c) = (f(A))^c$, $(f(A))^c$ is an IFγGOS in Y. Therefore f(A) is an IFγGCS in Y. Hence f is an IF almost γG closed mapping.

Proposition 4.4.19: Let $p_{(\alpha,\beta)}$ be an IFP in X. A mapping f: X → Y is an IF almost γG open mapping if for every IFOS A in X with $f^{-1}(p_{(\alpha,\beta)}) \in A$, there exists an IFOS B in Y with $p_{(\alpha,\beta)} \in B$ such that f(A) is IFD in B.

Proof: Let A be an IFROS in X. Then A is an IFOS in X. Let $f^{-1}(p_{(\alpha,\beta)}) \in A$, then there exists an IFOS B in Y such that $p_{(\alpha,\beta)} \in B$ and cl(f(A)) = B. Since B is an IFOS, cl(f(A)) = B is also an IFOS in Y. Therefore int(cl(f(A))) = cl(f(A)). Now f(A) ⊆ cl(f(A)) = int(cl(f(A))) ⊆ cl(A). This implies f(A) is an IFPOS in Y and hence an IFγGOS in Y. Thus f is an IF almost γG open mapping.

Proposition 4.4.20: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following are equivalent:

(i) f is an IF almost γG open mapping,

(ii) f is an IF almost γG closed mapping,

(iii) f^{-1} is an IF almost γG continuous mapping.

Proof: (i) ⇔ (ii) is obvious as for a bijective mapping, $f(A^c) = f(A)^c$.

(ii) ⇒ (iii) Let A ⊆ X be an IFRCS. Then by hypothesis, f(A) is an IFγGCS in Y. That is $(f^{-1})^{-1}(A)$ is an IFγGCS in Y. This implies f^{-1} is an IF almost γG continuous mapping.

(iii) ⇒ (ii) Let A ⊆ X be an IFRCS. Then by hypothesis, $(f^{-1})^{-1}(A)$ is an IFγGCS in Y. That is f(A) is an IFγGCS in Y. Hence f is an IF almost γG closed mapping.

Proposition 4.4.21: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IF$\gamma_\gamma T_{1/2}$ space. Then the following are equivalent:

(i) f is an IF almost γG closed mapping,

(ii) γcl(f(A)) ⊆ f(cl(A)) for every IFγOS A in X,

(iii) γcl(f(A)) ⊆ f(cl(A)) for every IFSOS A in X,

(iv) f(A) ⊆ γint(f(int(cl(A)))) for every IFPOS A in X.

Proof: (i) ⇒ (ii) Let A be an IFγOS in X. Then cl(A) is an IFRCS in X. By hypothesis, f(cl(A)) is an IFγGCS in Y and hence is an IFγCS in Y, since Y is an IF$\gamma_\gamma T_{1/2}$ space. This implies γcl(f(cl(A))) = f(cl(A)). Now γcl(f(A)) ⊆ γcl(f(cl(A))) = f(cl(A)). Thus γcl(f(A)) ⊆ f(cl(A)).

(ii) ⇒ (iii) Since every IFSOS is an IFγOS, the proof directly follows.

(iii) ⇒ (i) Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X. By hypothesis, γcl(f(A)) ⊆ f(cl(A)) = f(A) ⊆ γcl(f(A)). Hence f(A) is an IFγCS and hence is an IFγGCS in Y. Thus f is an IF almost γG closed mapping.

(i) ⇒ (iv) Let A be an IFPOS in X. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFγGOS in Y. Since Y is an IF$\gamma_\gamma T_{1/2}$ space f(int(cl(A))) is an IFγOS in Y. Therefore f(A) ⊆ f(int(cl(A))) = γint(f(int(cl(A)))).

(iv) ⇒ (i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) ⊆ γint(f(int(cl(A)))) = γint(f(A)) ⊆ f(A). This implies f(A) is an IFγOS in Y and hence f(A) is an IFγGOS in Y. Therefore f is an IF almost γG open mapping and hence by Proposition 4.4.18, f is an IF almost γG closed mapping.

Proposition 4.4.22: Let f: (X, τ) → (Y, σ) be a mapping. If f is an IF almost γG closed mapping, then γgcl(f(A)) ⊆ f(cl(A)) for every IFγOS A in X.

Proof: Let A be an IFγOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFγGCS in Y. Then γgcl(f(cl(A))) = f(cl(A)). Now γgcl(f(A)) ⊆ γgcl(f(cl(A))) = f(cl(A)).

Corollary 4.4.23: Let f: (X, τ) → (Y, σ) be a mapping. If f is an IF almost γG closed mapping, then γgcl(f(A)) ⊆ f(cl(A)) for every IFSOS A in X.

Proof: Since every IFSOS is an IFγOS, the proof directly follows from the Proposition 4.4.22.

Corollary 4.4.24: Let f: (X, τ) → (Y, σ) be a mapping. If f is an IF almost γG closed mapping, then γgcl(f(A)) ⊆ f(cl(A)) for every IFPOS A in X.

Proof: Since every IFPOS is an IFγOS, the proof directly follows from the Proposition 4.4.22.

Proposition 4.4.25: Let f: (X, τ) → (Y, σ) be a mapping. If f is an IF almost γG closed mapping, then γgcl(f(A)) ⊆ f(cl(γint(A))) for every IFγOS A in X.

Proof: Let A be an IFγOS in X. Therefore γint(A) = A and cl(A) is an IFRCS in X. By hypothesis, f(cl(A)) is an IFγGCS in Y. Then γgcl(f(A)) ⊆ γgcl(f(cl(A))) = f(cl(A)) = f(cl(γint(A))).

Corollary 4.4.26: Let f: (X, τ) → (Y, σ) be a mapping. If f is an IF almost γG closed mapping, then γgcl(f(A)) ⊆ f(cl(γint(A))) for every IFSOS A in X.

Proof: Since every IFSOS is an IFγOS, the proof directly follows from the Proposition 4.4.25.

Corollary 4.4.27: Let f: (X, τ) → (Y, σ) be a mapping. If f is an IF almost γG closed mapping, then γgcl(f(A)) ⊆ f(cl(γint(A))) for every IFPOS A in X.

Proof: Since every IFPOS is an IFγOS, the proof directly follows from the Proposition 4.4.25.

Proposition 4.4.28: Let f: (X, τ) → (Y, σ) be a mapping. If f(γint(B)) ⊆ γint(f(B)) for every IFS B in X, then f is an IF almost γG open mapping.

Proof: Let B ⊆ X be an IFROS. By hypothesis, f(γint(B)) ⊆ γint(f(B)). Since B is an IFROS, it is an IFγOS in X. Therefore γint(B) = B. Hence f(B) = f(γint(B)) ⊆ γint(f(B)) ⊆

f(B). This implies f(B) is an IFγOS and hence an IFγGOS in Y. Thus f is an IF almost γG open mapping.

Proposition 4.4.29: Let f: (X, τ) → (Y, σ) be a mapping. If γcl(f(B)) ⊆ f(γcl(B)) for every IFS B in X, then f is an IF almost γG closed mapping.

Proof: Let B ⊆ X be an IFRCS. By hypothesis, γcl(f(B)) ⊆ f(γcl(B)). Since B is an IFRCS, it is an IFγCS in X. Therefore γcl(B) = B. Hence f(B) = f(γcl(B)) ⊇ γcl(f(B)) ⊇ f(B). This implies f(B) is an IFγCS and hence an IFγGCS in Y. Thus f is an IF almost γG closed mapping.

Proposition 4.4.30: Let f: (X, τ) → (Y, σ) be an IF almost γG open mapping where Y is an IF$\gamma_\gamma T_{1/2}$ space, then for each IFP $p_{(\alpha,\beta)}$ in Y and each IFROS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in$ B, cl(f(cl(B))) is an IFγN of $p_{(\alpha,\beta)}$ in Y.

Proof: Let $p_{(\alpha,\beta)} \in$ Y and let B be an IFROS in X such that $f^{-1}(p_{(\alpha,\beta)}) \in$ B. That is $p_{(\alpha,\beta)} \in$ f(B). By hypothesis, f(B) is an IFγGOS in Y. Since Y is an IF$\gamma_\gamma T_{1/2}$ space, f(B) is an IFγOS in Y. Now $p_{(\alpha,\beta)} \in$ f(B) ⊆ f(cl(B)) ⊆ cl(f(cl(B))). Hence cl(f(cl(B))) is an IFγN of $p_{(\alpha,\beta)}$ in Y.

Proposition 4.4.31: The following are equivalent for a bijective mapping f: (X, τ) → (Y, σ) where Y is an IF$\gamma_\gamma T_{1/2}$ space:

(i) f is an IF almost γG closed mapping,

(ii) γcl(f(A)) ⊆ f(αcl(A)) for every IFγOS A in X,

(iii) γcl(f(A)) ⊆ f(αcl(A)) for every IFSOS A in X,

(iv) f(A) ⊆ γint(f(scl(A))) for every IFPOS A in X.

Proof: (i) ⇒ (ii) Let A be an IFγOS in X. Then cl(A) is an IFRCS in X and cl(int(cl(A))) = cl(A). By hypothesis, f(A) is an IFγGCS in Y and hence is an IFγCS in Y, as Y is an IF$\gamma_\gamma T_{1/2}$ space. This implies γcl(f(cl(A))) = f(cl(A)). Now γcl(f(A)) ⊆ γcl(f(cl(A))) = f(cl(A)) = f(cl(int(cl(A)))) ⊆ f(A ∪ cl(int(cl(A)))) = f(αcl(A)). Hence γcl(f(A)) ⊆ f(αcl(A)).

(ii) ⇒ (iii) Let A be an IFSOS in X. Since every IFSOS is an IFγOS, the proof is obvious.

(iii) ⇒ (i) Let A be an IFRCS in X. Then A = cl(int(A)) and therefore A is an IFSOS in X. By hypothesis, γcl(f(A)) ⊆ f(αcl(A)) ⊆ f(cl(A)) = f(A) ⊆ γcl(f(A)). Hence f(A) is an IFγCS and hence is an IFγGCS in Y. Thus f is an IF almost γG closed mapping.

(i) ⇒ (iv) Let A be an IFPOS in X. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFγGOS in Y. Since Y is an IF$\gamma_\gamma T_{1/2}$ space, f(int(cl(A))) is an IFγOS in Y. Therefore f(A) ⊆ f(int(cl(A))) = γint(f(int(cl(A)))) ⊆ γint(f(A ∪ int(cl(A)))) = γint(f(scl(A))). That is f(A) ⊆ γint(f(scl(A))).

(iv) ⇒ (i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) ⊆ γint(f(scl(A))). This implies f(A) ⊆ γint(f(A ∪ int(cl(A)))) = γint(f(A ∪ A)) = γint(f(A)) ⊆ f(A). Therefore f(A) is an IFγOS in Y and hence is an IFγGOS in Y. Thus f is an IF almost γG closed mapping, by Proposition 4.4.18.

Proposition 4.4.32: Let f: (X, τ) → (Y, σ) be a mapping where Y is an IF$\gamma_\gamma T_{1/2}$ space. If f is an IF almost γG closed mapping, then int(cl(int(f(B)))) ⊆ f(γcl(B)) for every IFRCS B in X.

Proof: Let B ⊆ X be an IFRCS. By hypothesis, f(B) is an IFγGCS in Y. Since Y is an IF$\gamma_\gamma T_{1/2}$space, f(B) is an IFγCS in Y. Therefore γcl(f(B)) = f(B). Now int(cl(int(f(B)))) ⊆ f(B) ∪ int(cl(int(f(B)))) ⊆ γcl(f(B)) = f(B) ⊆ f(γcl(B)). Hence int(cl(int(f(B)))) ⊆ f(γcl(B)).

Proposition 4.4.33: Let f: (X, τ) → (Y, σ) be a mapping where Y is an IF$\gamma_\gamma T_{1/2}$ space. If f is an IF almost γG closed mapping, then f(γint(B)) ⊆ cl(int(cl(f(B)))) for every IFROS B in X.

Proof: This Proposition can be easily proved by taking complement in Proposition 4.4.32.

Definition 4.4.34: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy almost contra γ generalized*** (IF almost contra γG) ***open mapping*** if f(A) is an IFγGCS in Y for every IFROS A in X.

Example 4.4.35: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$, $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost contra γG open mapping.

Proposition 4.4.36: Every IF contra γG open mapping is an IF almost contra γG open mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra γG open mapping. Let A be an IFROS in X. Since every IFROS is an IFOS, A is an IFOS in X. Then f(A) is an IFγGCS in Y, by hypothesis. Therefore f is an IF almost contra γG open mapping.

Example 4.4.37: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.7_b), (0.2_a, 0.3_b)\rangle$, $G_3 = \langle y, (0.9_u, 0.8_v), (0.1_u, 0.2_v)\rangle$ and $G_4 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, G_4, 1\sim\}$ be IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost contra γG open mapping. But f is not an IF contra γG open mapping, as G_2 is an IFOS in X but $f(G_2)$ is not an IFγGCS in Y as $f(G_2) \subseteq G_3$ whereas $\gamma cl(f(G_2)) = 1\sim \nsubseteq G_3$.

Proposition 4.4.38: If f: (X, τ) → (Y, σ) is a bijective mapping, where Y is an $IF\gamma_\gamma T_{1/2}$ space, then the following are equivalent:

(i) f is an IF almost contra γG open mapping,

(ii) f(A) ∈ IFγGO(Y) for every A ∈ IFRC(X),

(iii) For each IFP $p_{(\alpha,\beta)} \in Y$ and for each IFRCS A in X containing $f^{-1}(p_{(\alpha,\beta)})$, there exists an IFγGOS B in Y containing $p_{(\alpha,\beta)}$ such that $f^{-1}(B) \subseteq A$,

(iv) f(int(cl(G))) ∈ IFγGC(Y) for every IFOS G ⊆ X,

(v) f(cl(int(G))) ∈ IFγGO(Y) for every IFCS H ⊆ X.

Proof: (i) ⇔ (ii) is obvious.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in Y$ and let $A \subseteq X$ be any IFRCS. Let $f^{-1}(p_{(\alpha,\beta)}) \in A$. Then $p_{(\alpha,\beta)} \in f(A)$. By hypothesis f(A) is an IFγGOS in Y. Let B = f(A). Then $p_{(\alpha,\beta)} \in B$ and $f^{-1}(B) = f^{-1}(f(A)) \subseteq A$.

(iii) ⇒ (ii) Let $A \subseteq X$ be an IFRCS. Let $p_{(\alpha,\beta)} \in Y$ and $f^{-1}(p_{(\alpha,\beta)}) \in A$. Then $p_{(\alpha,\beta)} \in f(A)$. By hypothesis there exists an IFγGOS B in Y that is an IFγOS in Y, as Y is an IFγ_γ $T_{1/2}$ space, such that $p_{(\alpha,\beta)} \in B$ and $f^{-1}(B) \subseteq A$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f(A)$. This implies $f(A) = \cup_{p(\alpha,\beta)} \in f(A) \subseteq B$. Since each of B is an IFγOS, f(A) is also an IFγOS and hence an IFγGOS in Y.

(i) ⇒ (iv) Let G be any IFOS in X. Then int(cl(G)) is an IFROS in X. By hypothesis, f(int(cl(G))) is an IFγGCS in Y. Hence f(int(cl(G))) ∈ IFγGC(Y).

(iv) ⇒ (i) Let A be any IFROS in X. Then A is an IFOS in X. By hypothesis, f(int(cl(A))) ∈ IFγGC(Y). That is f(A) ∈ IFγGC(Y), since int(cl(A)) = A. Hence f is an IF almost contra γG open mapping.

(ii) ⇔ (v) is similar as (i) ⇔ (iv).

Proposition 4.4.39: Let f: (X, τ) → (Y, σ) be a bijective mapping that satisfies f(int(cl(B))) ⊆ cl(int(f(B))) for every IFS B in Y. Then f is an IF almost contra γG open mapping.

Proof: Let B be an IFRCS in X. Then cl(int(B)) = B and by hypothesis f(B) = f(cl(int(B)) ⊆ cl(int(f(B))). This implies f(B) is an IFSOS in Y. Therefore it is an IFγGOS in Y. Hence f is an IF almost contra γG open mapping by Proposition 4.4.38.

Proposition 4.4.40: A mapping f: (X, τ) → (Y, σ) is an IF almost contra γG open mapping if f(γcl(B)) ⊆ int(f(B)) for every IFS B in X.

Proof: Let B ⊆ X be an IFRCS. Since every IFRCS is an IFγCS, γcl(B) = B. Now by hypothesis, f(B) = f(γcl(B)) ⊆ int(f(B)) ⊆ f(B). Therefore f(B) is an IFOS in Y and hence an IFγGOS in X. Then f is an IF almost contra γG open mapping by Proposition 4.4.38.

Proposition 4.4.41: If f: (X, τ) → (Y, σ) is an IF almost contra γG open mapping and g: (Y, σ) → (Z, δ) is an IF contra M-γG closed mapping then g o f : (X, τ) → (Z, σ) is an IF almost γG open mapping.

Proof: Let A be an IFROS in X. Then f(A) is an IFγGCS in Y, since f is an IF almost contra γG open mapping. Since g is an IF contra M-γG open mapping, g(f(A)) is an IFγGOS in Z. Therefore g o f is an IF almost γG open mapping.

Proposition 4.4.42: For a bijective mapping f: (X, τ) → (Y, σ), where X is an IFγ$_{\gamma}$ $T_{1/2}$ space, the following are equivalent:

(i) f is an IF almost contra γG open mapping,

(ii) For every IFRCS A in X and for every IFP $p_{(\alpha,\beta)}$ ∈ Y, if $f^{-1}(p_{(\alpha,\beta)})$ $_q$ A then $p_{(\alpha,\beta)}$ $_q$ γint(f(A)),

(iii) For every IFRCS in X and for any IFP $p_{(\alpha,\beta)}$ ∈ Y, if $f^{-1}(p_{(\alpha,\beta)})$ $_q$ A then there exists an IFγGOS B such that $p_{(\alpha,\beta)}$ $_q$ B and $f^{-1}(B)$ ⊆ A.

Proof: (i) ⇒ (ii) Let f be an IF almost contra γG open mapping. Let A ⊆ X be an IFRCS and let $p_{(\alpha,\beta)}$ ∈ Y. Also let $f(p_{(\alpha,\beta)})$ $_q$ A then $p_{(\alpha,\beta)}$ $_q$ f(A). By hypothesis f(A) is an IFγGOS in Y. Since Y is an IFγ$_{\gamma}$$T_{1/2}$ space**,** f(A) is an IFγOS in Y. Hence γint(f(A)) = f(A). This implies $p_{(\alpha,\beta)}$ $_q$ γint(f(A)).

(ii) ⇒ (i) Let A ⊆ X be an IFRCS then f(A) is an IFS in Y. Let $p_{(\alpha,\beta)}$ ∈ Y and let $f^{-1}(p_{(\alpha,\beta)})$ $_q$ A then $p_{(\alpha,\beta)}$ $_q$ f(A). By hypothesis this implies $p_{(\alpha,\beta)}$ $_q$ γint(f(A)). That is f(A) ⊆ γint(f(A)). But γint(f(A)) ⊆ f(A). Therefore γint(f(A)) = f(A). Thus f(A) is an IFγOS in Y and hence an IFγGOS in Y**.** This implies f is an IF almost contra γG open mapping by Proposition 4.4.38.

(ii) ⇒ (iii) Let A ⊆ X be an IFRCS then f(A) is an IFS in Y. Let $p_{(\alpha,\beta)} \in$ Y. Also let $f^{-1}(p_{(\alpha,\beta)})_{q}$ A then $p_{(\alpha,\beta)\ q}$ f(A). By hypothesis this implies $p_{(\alpha,\beta)\ q}$ γint(f(A)). That is f(A) ⊆ γint(f (A)). But γint(f(A)) ⊆ f(A). Therefore γint(f(A)) = f(A). Thus f(A) is an IFγOS in Y and hence an IFγGOS in Y**.** Let f(A) = B. Therefore $p_{(\alpha,\beta)\ q}$ B and $f^{-1}(B) = f^{-1}(f(A)) \subseteq A$.

(iii) ⇒ (ii) Let A ⊆ X be an IFRCS, then f(A) is an IFS in Y. Let $p_{(\alpha,\beta)} \in$ Y. Also let $f^{-1}(p_{(\alpha,\beta)})_{q}$ A then $p_{(\alpha,\beta)\ q}$ f(A). By hypothesis there exists an IFγGOS B in Y such that $p_{(\alpha,\beta)\ q}$ B and $f^{-1}(B) \subseteq A$. Let B = f(A). Since Y is an IFγ$_{\gamma}$T$_{1/2}$ space, f(A) is an IFγOS in Y and γint(f(A)) = f(A). Therefore $p_{(\alpha,\beta)\ q}$ γint(f(A)).