

Generalized semi-pre connectedness in intuitionistic fuzzy topological spaces

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Received 25 April 2011; Accepted 11 August 2011

ABSTRACT. In this paper we have introduced the intuitionistic fuzzy generalized semi-pre connected space, intuitionistic fuzzy generalized semi-pre super connected space and intuitionistic fuzzy generalized semi-pre extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy generalized semi-pre super connected space.

2010 AMS Classification: 54A99, 03E99

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy generalized semi-pre connected space, Intuitionistic fuzzy generalized semi-pre super connected space.

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1. INTRODUCTION

Zadeh [10] introduced the notion of fuzzy sets. Fuzzy topological space was introduced by Chang [2]. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Çoker [3] introduced the notion of intuitionistic fuzzy topological space. Connectedness in intuitionistic fuzzy special topological spaces was introduced by Ösçağ and Çoker [7]. Jun and Song [4] discussed intuitionistic fuzzy semi-pre openness and intuitionistic fuzzy semi-pre continuity.

In this paper we have introduced intuitionistic fuzzy generalized semi-pre connected space, intuitionistic fuzzy generalized semi-pre super connected space and intuitionistic fuzzy generalized semi-pre extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy generalized semi-pre super connected space.

2. PRELIMINARIES

Definition 2.1 ([1]). An intuitionistic fuzzy set (IFS for short) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

where the functions $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in X .

Definition 2.2 ([1]). Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$. Then

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
- (b) $A = B$ if and only if and only if $A \subseteq B$ and $B \subseteq A$;
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$;
- (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$;
- (e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle : x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle : x \in X \}$ are respectively the empty set and the whole set of X .

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$.

Definition 2.3 ([3]). An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms:

- (i) $0_\sim, 1_\sim \in \tau$;
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
- (iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X . The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X .

Definition 2.4 ([3]). An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be

- (i) intuitionistic fuzzy semi-pre closed set (IFSPCS for short) if there exists an intuitionistic fuzzy pre closed set (IFPCS for short) B such that $\text{int}(B) \subseteq A \subseteq B$;
- (ii) intuitionistic fuzzy semi-pre open set (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) B such that $B \subseteq A \subseteq \text{cl}(B)$.

Note that an IFS A is an IFSPCS if and only if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ [5].

Definition 2.5 ([5]). Let A be an IFS in an IFTS (X, τ) . Then the semi-pre interior and the semi-pre closure of A are defined as

$$\begin{aligned} \text{spint}(A) &= \cup \{G : G \text{ is an IFSPOS in } X \text{ and } G \subseteq A\}; \\ \text{spcl}(A) &= \cap \{K : K \text{ is an IFSPCS in } X \text{ and } A \subseteq K\}. \end{aligned}$$

Note that for any IFS A in (X, τ) , we have $\text{spcl}(A^c) = [\text{spint}(A)]^c$ and $\text{spint}(A^c) = [\text{spcl}(A)]^c$ [5].

Definition 2.6 ([8]). An IFS A is an intuitionistic fuzzy generalized closed set (IFGCS for short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS. The complement of an IFGCS is called an intuitionistic fuzzy generalized open set (IFGOS for short).

Definition 2.7 ([5]). An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS for short) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

Every IFCS, IFSPCS is an IFGSPCS but the separate converses may not be true in general [5].

Definition 2.8 ([5]). The complement A^c of an IFGSPCS A in an IFTS (X, τ) is called an intuitionistic fuzzy generalized semi-pre open set (IFGSPOS for short) in X .

Every IFOS, IFSPOS is an IFGSPOS but the separate converses may not be true in general [5].

Definition 2.9 ([5]). Let A be an IFS in an IFTS (X, τ) . Then the generalized semi-pre interior and the generalized semi-pre closure of A are defined as

$$\begin{aligned} \text{gspint}(A) &= \cup\{G : G \text{ is an IFGSPOS in } X \text{ and } G \subseteq A\}; \\ \text{gspcl}(A) &= \cap\{K : K \text{ is an IFGSPCS in } X \text{ and } A \subseteq K\}. \end{aligned}$$

Note that for any IFS A in an IFTS (X, τ) , we have $\text{gspcl}(A^c) = [\text{gspint}(A)]^c$ and $\text{gspint}(A^c) = [\text{gspcl}(A)]^c$ [5].

Definition 2.10 ([5]). If every IFGSPCS in an IFTS (X, τ) is IFSPCS in (X, τ) , then the space can be called as an intuitionistic fuzzy semi-pre $T_{1/2}$ space (IFSP $T_{1/2}$ space for short).

Definition 2.11 ([6]). A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi-pre continuous (IFGSP continuous for short) mapping if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFCS V of (Y, σ) .

Definition 2.12 ([6]). A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi-pre irresolute (IFGSP irresolute for short) mapping if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFGSPCS V of (Y, σ) .

Definition 2.13 ([8]). Two IFSs A and B in X are said to be q-coincident (AqB for short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_A(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.14 ([8]). Two IFSs A and B in X are said to be not q-coincident (Aq^cB for short) if and only if $A \subseteq B^c$.

Definition 2.15 ([8]). An IFTS (X, τ) is said to be an IF $T_{1/2}$ space if every IFGCS in (X, τ) is an IFCS in (X, τ) .

Definition 2.16 ([9]). An IFTS (X, τ) is said to be an intuitionistic fuzzy C_5 -connected (IF C_5 -connected for short) space if the only IFSs which are both intuitionistic fuzzy open and intuitionistic fuzzy closed are 0_\sim and 1_\sim .

Definition 2.17 ([9]). An IFTS (X, τ) is said to be an intuitionistic fuzzy GO-connected (IFGO-connected for short) space if the only IFSs which are both intuitionistic fuzzy generalized open and intuitionistic fuzzy generalized closed are 0_{\sim} and 1_{\sim} .

3. INTUITIONISTIC FUZZY GENERALIZED SEMI-PRE CONNECTED SPACES

In this section we introduce intuitionistic fuzzy generalized semi-pre connected space and intuitionistic fuzzy generalized semi-pre super connected space. We investigate some of their properties. Also we provide a characterization theorem for an intuitionistic fuzzy generalized semi-pre super connected space.

Definition 3.1. An IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi-pre connected space if the only IFSs which are both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed are 0_{\sim} and 1_{\sim} .

Example 3.2. Let $X = \{a, b\}$ and $\tau = \{0_{\sim}, M, 1_{\sim}\}$ be an IFT on X , where $M = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$. Then (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space.

Theorem 3.3. Every intuitionistic fuzzy generalized semi-pre connected space is an intuitionistic fuzzy C_5 -connected space but not conversely.

Proof. Let (X, τ) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose (X, τ) is not an intuitionistic fuzzy C_5 -connected space, then there exists a proper IFS A which both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X, τ) . That is A is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) . This implies that (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space. This is a contradiction. Therefore (X, τ) is an intuitionistic fuzzy C_5 -connected space. \square

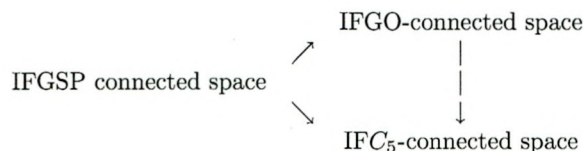
Example 3.4. Let $X = \{a, b\}$ and $\tau = \{0_{\sim}, M, 1_{\sim}\}$ be an IFT on X , where $M = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$. Then (X, τ) is an intuitionistic fuzzy C_5 -connected space but not an intuitionistic fuzzy generalized semi-pre connected space, since the IFS M in τ is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) .

Theorem 3.5. Every intuitionistic fuzzy generalized semi-pre connected space is an intuitionistic fuzzy GO-connected space but not conversely.

Proof. Let (X, τ) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose (X, τ) is not an intuitionistic fuzzy GO-connected space, then there exists a proper IFS A which both intuitionistic fuzzy g -open and intuitionistic fuzzy g -closed in (X, τ) . That is A is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) . This implies that (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space. This is a contradiction. Therefore (X, τ) is an intuitionistic fuzzy GO-connected space. \square

Example 3.6. In Example 3.4, (X, τ) is an intuitionistic fuzzy GO-connected space but not an intuitionistic fuzzy generalized semi-pre connected space.

The relation among various types of intuitionistic fuzzy connectedness is given in the following diagram.



The reverse implications are not true in general in the above diagram.

Theorem 3.7. *An IFTS (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space if and only if there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets A and B in (X, τ) such that $A = B^c$.*

Proof. Necessity: Let A and B be two intuitionistic fuzzy generalized semi-pre open sets in (X, τ) such that $A \neq 0_{\sim} \neq B$ and $A = B^c$. Therefore B^c is an intuitionistic fuzzy generalized semi-pre closed set. Since $A \neq 0_{\sim}$, $B \neq 1_{\sim}$. This implies B is a proper IFS which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) . Hence (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space. But this is a contradiction to our hypothesis. Thus there exist no non-zero intuitionistic fuzzy semi-pre open sets A and B in (X, τ) such that $A = B^c$.

Sufficiency: Let A be both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) such that $1_{\sim} \neq A \neq 0_{\sim}$. Now let $B = A^c$. Then B is an intuitionistic fuzzy generalized semi-pre open set and $B \neq 1_{\sim}$. This implies $B = A^c \neq 0_{\sim}$, which is a contradiction to our hypothesis. Therefore (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space. \square

Theorem 3.8. *An IFTS (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space if and only if there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets A and B in (X, τ) such that $A = B^c$, $B = (\text{spcl}(A))^c$ and $A = (\text{spcl}(B))^c$.*

Proof. Necessity: Assume that there exist IFSs A and B such that $A \neq 0_{\sim} \neq B$, $B = A^c$, $B = (\text{spcl}(A))^c$ and $A = (\text{spcl}(B))^c$. Since $(\text{spcl}(A))^c$ and $(\text{spcl}(B))^c$ are intuitionistic fuzzy generalized semi-pre open sets in (X, τ) , A and B are intuitionistic fuzzy generalized semi-pre open sets in (X, τ) . This implies (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space, which is a contradiction. Therefore there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets A and B in (X, τ) such that $A = B^c$, $B = (\text{spcl}(A))^c$ and $A = (\text{spcl}(B))^c$.

Sufficiency: Let A be both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) such that $1_{\sim} \neq A \neq 0_{\sim}$. Now by taking $B = A^c$, we obtain a contradiction to our hypothesis. Hence (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space. \square

Definition 3.9. An IFTS (X, τ) is said to be an intuitionistic fuzzy semi-pre $T_{1/2}^*$ space if every intuitionistic fuzzy generalized semi-pre closed set is an intuitionistic fuzzy closed set in (X, τ) .

Example 3.10. In Example 3.2, the IFTS (X, τ) is an intuitionistic fuzzy semi-pre $T_{1/2}^*$ space.

Remark 3.11. Every intuitionistic fuzzy semi-pre $T_{1/2}^*$ space is an intuitionistic fuzzy semi-pre $T_{1/2}$ space but not conversely.

Proof. Let (X, τ) be an intuitionistic fuzzy semi-pre $T_{1/2}^*$ space. Let A be an intuitionistic fuzzy generalized semi-pre closed set in (X, τ) . By hypothesis A is an intuitionistic fuzzy closed set. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi-pre closed set, A is an intuitionistic fuzzy semi-pre closed set in (X, τ) . Hence (X, τ) is an intuitionistic fuzzy semi-pre $T_{1/2}$ space. \square

Example 3.12. In Example 3.4, (X, τ) is an intuitionistic fuzzy generalized semi-pre $T_{1/2}$ space, but not an intuitionistic fuzzy semi-pre $T_{1/2}^*$ space, since the IFS M is an intuitionistic fuzzy generalized semi-pre closed set in (X, τ) but not an intuitionistic fuzzy closed set in (X, τ) , since $\text{cl}(M) = M^c \neq M$.

Remark 3.13. Every intuitionistic fuzzy semi-pre $T_{1/2}^*$ space is an intuitionistic fuzzy $T_{1/2}$ space but not conversely.

Proof. Let (X, τ) be an intuitionistic fuzzy semi-pre $T_{1/2}^*$ space. Let A be an intuitionistic fuzzy generalized closed set in (X, τ) . Since every intuitionistic fuzzy generalized closed set is an intuitionistic fuzzy generalized semi-pre closed set, A is an intuitionistic fuzzy generalized semi-pre closed set in (X, τ) . By hypothesis, A is an intuitionistic fuzzy closed set. Hence (X, τ) is an intuitionistic fuzzy $T_{1/2}$ space. \square

Example 3.14. In Example 3.4, the IFTS (X, τ) is an intuitionistic fuzzy $T_{1/2}$ space, but not an intuitionistic fuzzy semi-pre $T_{1/2}^*$ space, since the IFS M is an intuitionistic fuzzy generalized semi-pre closed set in (X, τ) but not an intuitionistic fuzzy closed set in (X, τ) , since $\text{cl}(M) = M^c \neq M$.

Theorem 3.15. Let (X, τ) be an intuitionistic fuzzy semi-pre $T_{1/2}^*$ space, then the following are equivalent.

- (i) (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space.
- (ii) (X, τ) is an intuitionistic fuzzy GO-connected space.
- (iii) (X, τ) is an intuitionistic fuzzy C_5 -connected space.

Proof. (i) \Rightarrow (ii) is obvious from Theorem 3.5.

(ii) \Rightarrow (iii) is obvious from [8].

(iii) \Rightarrow (i) Let (X, τ) be an intuitionistic fuzzy C_5 -connected space. Suppose (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space, then there exists a proper IFS A in (X, τ) which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) . But since (X, τ) is an intuitionistic fuzzy semi-pre $T_{1/2}^*$ space, A is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X, τ) . This implies that (X, τ) is not an intuitionistic fuzzy C_5 -connected space, which is a contradiction to our hypothesis. Therefore (X, τ) is intuitionistic fuzzy generalized semi-pre connected space. \square

Theorem 3.16. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy semi-pre continuous surjection and (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space, then (Y, σ) is an intuitionistic fuzzy C_5 -connected space.*

Proof. Let (X, τ) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose (Y, σ) is not an intuitionistic fuzzy C_5 -connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (Y, σ) . Since f is an intuitionistic fuzzy semi-pre continuous mapping, $f^{-1}(A)$ is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) . But this is a contradiction to hypothesis. Hence (Y, σ) is an intuitionistic fuzzy C_5 -connected space. \square

Theorem 3.17. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy semi-pre irresolute surjection and (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space, then (Y, σ) is also an intuitionistic fuzzy generalized semi-pre connected space.*

Proof. Suppose (Y, σ) is not an intuitionistic fuzzy generalized semi-pre connected space, then there exists a proper IFS A which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (Y, σ) . Since f is an intuitionistic fuzzy semi-pre irresolute mapping, $f^{-1}(A)$ is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) . But this is a contradiction to hypothesis. Hence (Y, σ) is an intuitionistic fuzzy generalized semi-pre connected space. \square

Definition 3.18. An IFTS (X, τ) is called intuitionistic fuzzy C_5 -connected between two IFSs A and B if there is no intuitionistic fuzzy open set E in (X, τ) such that $A \subseteq E$ and $E q^c B$.

Definition 3.19. An IFTS (X, τ) is called intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B if there is no intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E$ and $E q^c B$.

Example 3.20. Let $X = \{a, b\}$ and $\tau = \{0_\sim, M, 1_\sim\}$ be an IFT on X , where $M = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.1_b) \rangle$. Then (X, τ) is an intuitionistic fuzzy generalized semi-pre connected between the two IFSs $A = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.3_b) \rangle$ and $B = \langle x, (0.5_a, 0.2_b), (0.5_a, 0.5_b) \rangle$.

Theorem 3.21. *If an IFTS (X, τ) is intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B , then it is intuitionistic fuzzy C_5 -connected between two IFSs A and B but the converse may not be true in general.*

Proof. Suppose (X, τ) is not intuitionistic fuzzy C_5 -connected between A and B , then there exists an intuitionistic fuzzy open set E in (X, τ) such that $A \subseteq E$ and $E q^c B$. Since every intuitionistic fuzzy open set is intuitionistic fuzzy generalized semi-pre open set, there exists an intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E$ and $E q^c B$. This implies (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B , a contradiction to our hypothesis. Therefore (X, τ) is intuitionistic fuzzy C_5 -connected between A and B . \square

Example 3.22. Let $X = \{a, b\}$ and $\tau = \{0_{\sim}, M, 1_{\sim}\}$ be an IFT on X , where $M = \langle x, (0.4_a, 0.3_b), (0.2_a, 0.3_b) \rangle$. Then (X, τ) is an intuitionistic fuzzy C_5 -connected between the IFSs $A = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b) \rangle$ and $B = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$. But (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected between A and B , since the IFS $E = \langle x, (0.4_a, 0.4_b), (0.5_a, 0.5_b) \rangle$ is an intuitionistic fuzzy generalized semi-pre open set such that $A \subseteq E$ and $E \subseteq B^c$.

Theorem 3.23. An IFTS (X, τ) is intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B if and only if there is no intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed set E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Proof. Necessity: Let (X, τ) be intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B . Suppose that there exists an intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed set E in (X, τ) such that $A \subseteq E \subseteq B^c$, then $E q^c B$ and $A \subseteq E$. This implies (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B , by Definition 3.19. A contradiction to our hypothesis. Therefore there is no intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed set E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Sufficiency: Suppose that (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B . Then there exists an intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E$ and $E q^c B$. This implies that there is no intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E \subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A and B . \square

Theorem 3.24. If an IFTS (X, τ) is intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B , $A \subseteq A_1$ and $B \subseteq B_1$, then (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A_1 and B_1 .

Proof. Suppose that (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A_1 and B_1 , then by Definition 3.19, there exists an intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A_1 \subseteq E$ and $E q^c B_1$. This implies $E \subseteq B_1^c$ and $A_1 \subseteq E$ implies $A \subseteq A_1 \subseteq E$. That is $A \subseteq E$. Now let us prove that $E \subseteq B^c$, that is let us prove $E q^c B$. Suppose that $E q B$, then by Definition 2.12, there exists an element x in X such that $\mu_E(x) > \nu_B(x)$ and $\nu_E(x) < \mu_B(x)$. Therefore $\mu_E(x) > \nu_B(x) > \nu_{B_1}(x)$ and $\nu_E(x) < \mu_B(x) < \mu_{B_1}(x)$, since $B \subseteq B_1$. Thus $E q B_1$. But $E \subseteq B_1$. That is $E q^c B_1$, which is a contradiction. Therefore $E q^c B$. That is $E \subseteq B^c$. Hence (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B , which is a contradiction to our hypothesis. Thus (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A_1 and B_1 . \square

Theorem 3.25. Let (X, τ) be an IFTS and A and B be IFSs in (X, τ) . If $A q B$, then (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A and B .

Proof. Suppose (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B . Then there exists an intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E$ and $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is

$Aq^c B$. But this is a contradiction to our hypothesis. Therefore (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A and B . \square

Remark 3.26. The converse of the above theorem may not be true in general.

Example 3.27. In Example 3.20, (X, τ) is intuitionistic fuzzy generalized semi-pre connected between the IFSs A and B but not q-coincident with B , since $\mu_A(x) < \nu_B(x)$ and $\mu_B(x) < \nu_A(x)$.

Definition 3.28. An intuitionistic fuzzy generalized semi-pre open set A is called an intuitionistic fuzzy regular generalized semi-pre open set if $A = \text{gspint}(\text{gspcl}(A))$. The complement of an intuitionistic fuzzy regular generalized semi-pre open set is called an intuitionistic fuzzy regular generalized semi-pre closed set.

Definition 3.29. An IFTS (X, τ) is called an intuitionistic fuzzy generalized semi-pre super connected space if there exists no intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) .

Theorem 3.30. Let (X, τ) be an IFTS, then the following are equivalent.

- (i) (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space.
- (ii) For every non-zero intuitionistic fuzzy regular generalized semi-pre open set A , $\text{gspcl}(A) = 1_{\sim}$.
- (iii) For every intuitionistic fuzzy regular generalized semi-pre closed set A with $A \neq 1_{\sim}$, $\text{gspint}(A) = 0_{\sim}$.
- (iv) There exists no intuitionistic fuzzy regular generalized semi-pre open sets A and B in (X, τ) such that $A \neq 0_{\sim} \neq B$, $A \subseteq B^c$.
- (v) There exists no intuitionistic fuzzy regular generalized semi-pre open sets A and B in (X, τ) such that $A \neq 0_{\sim} \neq B$, $B = (\text{gspcl}(A))^c$, $A = (\text{gspcl}(B))^c$.
- (vi) There exists no intuitionistic fuzzy regular generalized semi-pre closed sets A and B in (X, τ) such that $A \neq 1_{\sim} \neq B$, $B = (\text{gspint}(A))^c$, $A = (\text{gspint}(B))^c$.

Proof. (i) \Rightarrow (ii) Assume that there exists an intuitionistic fuzzy regular generalized semi-pre open set A in (X, τ) such that $A \neq 0_{\sim}$ and $\text{gspcl}(A) \neq 1_{\sim}$. Now let $B = \text{gspint}(\text{gspcl}(A))^c$. Then B is a proper intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) . But this is a contradiction to the fact that (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space. Therefore $\text{gspcl}(A) = 1_{\sim}$.

(ii) \Rightarrow (iii) Let $A \neq 1_{\sim}$ be an intuitionistic fuzzy regular generalized semi-pre closed set in (X, τ) . If $B = A^c$, then B is an intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) with $B \neq 0_{\sim}$. Hence $\text{gspcl}(B) = 1_{\sim}$. This implies $(\text{gspcl}(B))^c = 0_{\sim}$. That is $\text{gspint}(B^c) = 0_{\sim}$. Hence $\text{gspint}(A) = 0_{\sim}$.

(iii) \Rightarrow (iv) Let A and B be two intuitionistic fuzzy regular generalized semi-pre open sets in (X, τ) such that $A \neq 0_{\sim} \neq B$, $A \subseteq B^c$. Since B^c is an intuitionistic fuzzy regular generalized semi-pre closed set in (X, τ) and $B \neq 0_{\sim}$ implies $B^c \neq 1_{\sim}$, $B^c = \text{gspcl}(\text{gspint}(B^c))$ and we have $\text{gspint}(B^c) = 0_{\sim}$. But $A \subseteq B^c$. Therefore $0_{\sim} \neq A = \text{gspint}(\text{gspcl}(A)) \subset \text{gspint}(\text{gspcl}(B^c)) = \text{gspint}(\text{gspcl}(\text{gspint}(B^c))) = \text{gspint}(\text{gspcl}(\text{gspint}(B^c))) = \text{gspint}(B^c) = 0_{\sim}$. A contradiction arises. Therefore (iv) is true.

(iv) \Rightarrow (i) Let $0_{\sim} \neq A \neq 1_{\sim}$ be an intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) . If we take $B = (\text{gspcl}(A))^c$, then B is an intuitionistic fuzzy regular generalized semi-pre open set, since $\text{gspint}(\text{gspcl}(B)) = \text{gspint}(\text{gspcl}(\text{gspcl}(A))^c) = \text{gspint}(\text{gspint}(\text{gspcl}(A)))^c = \text{gspint}(A^c) = (\text{gspcl}(A))^c = B$. Also we get $B \neq 0_{\sim}$, since otherwise, we have $B = 0_{\sim}$ and this implies $(\text{gspcl}(A))^c = 0_{\sim}$. This is $\text{gspcl}(A) = 1_{\sim}$. Hence $A = \text{gspint}(\text{gspcl}(A)) = \text{gspint}(1_{\sim}) = 1_{\sim}$. This is $A = 1_{\sim}$, which is a contradiction. Therefore $B \neq 0_{\sim}$ and $A \subseteq B^c$. But this is a contradiction to (iv). Therefore (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space.

(i) \Rightarrow (v) Let A and B be two intuitionistic fuzzy regular generalized semi-pre open sets in (X, τ) such that $A \neq 0_{\sim} \neq B$, $B = (\text{gspcl}(A))^c$ and $A = (\text{gspcl}(B))^c$. Now we have $\text{gspint}(\text{gspcl}(A)) = \text{gspint}(B^c) = (\text{gspcl}(B))^c = A$, $A \neq 0_{\sim}$ and $A \neq 1_{\sim}$, since if $A = 1_{\sim}$, then $1_{\sim} = (\text{gspcl}(B))^c \Rightarrow \text{gspcl}(B) = 0_{\sim} \Rightarrow B = 0_{\sim}$. But $B \neq 0_{\sim}$. Therefore $A \neq 1_{\sim} \Rightarrow A$ is proper intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) , which is a contradiction to (i). Hence (v) is true.

(v) \Rightarrow (i) Let A be an intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) such that $A = \text{gspnit}(\text{gspcl}(A))$ and $0_{\sim} \neq A \neq 1_{\sim}$. Now take $B = (\text{gspcl}(A))^c$. In this case we get $B \neq 0_{\sim}$ and B is intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) , $B = (\text{gspcl}(A))^c$ and $(\text{gspcl}(B))^c = (\text{gspcl}(\text{gspcl}(A))^c)^c = \text{gspint}(\text{gspcl}(A))^c = \text{gspint}(\text{gspcl}(A)) = A$. But this is a contradiction to (v). Therefore (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space.

(v) \Rightarrow (vi) Let A and B be two intuitionistic fuzzy regular generalized semi-pre closed sets in (X, τ) such that $A \neq 1_{\sim} \neq B$, $B = (\text{gspint}(A))^c$ and $A = (\text{gspint}(B))^c$. Taking $C = A^c$ and $D = B^c$, C and D become intuitionistic fuzzy regular generalized semi-pre open sets in (X, τ) with $C \neq 0_{\sim} \neq D$, $D = (\text{gspcl}(C))^c$ and $C = (\text{gspcl}(D))^c$, which is a contradiction to (v). Hence (vi) is true.

(vi) \Rightarrow (v) can be easily proved by the similar way as in (v) \Rightarrow (vi). □

Definition 3.31. An IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi-pre extremally disconnected if the generalized semi-pre closure of every intuitionistic fuzzy generalized semi-pre open set in (X, τ) is an intuitionistic fuzzy generalized semi-pre open set.

Theorem 3.32. Let (X, τ) be an intuitionistic fuzzy semi-pre $T_{1/2}$ space, then the following are equivalent.

- (i) (X, τ) is an intuitionistic fuzzy generalized semi-pre extremally disconnected space.
- (ii) For each intuitionistic fuzzy generalized semi-pre closed set A , $\text{gspint}(A)$ is an intuitionistic fuzzy generalized semi-pre closed set.
- (iii) For each intuitionistic fuzzy generalized semi-pre open set A , $\text{gspcl}(A) = (\text{gspcl}(\text{gspcl}(A))^c)^c$.
- (iv) For each intuitionistic fuzzy generalized semi-pre open sets A and B with $\text{gspcl}(A) = B^c$, $\text{gspcl}(A) = (\text{gspcl}(B))^c$.

Proof. (i) \Rightarrow (ii) Let A be any intuitionistic fuzzy generalized semi-pre closed set. Then A^c is an intuitionistic fuzzy generalized semi-pre open set. So (i) implies that $\text{gspcl}(A^c) = (\text{gspint}(A))^c$ is an intuitionistic fuzzy generalized semi-pre open set. Thus $\text{gspcl}(A)$ is an intuitionistic fuzzy generalized semi-pre closed set in (X, τ) .

(ii) \Rightarrow (iii) Let A be an intuitionistic fuzzy generalized semi-pre open set. Then we have $\text{gspcl}(\text{gspcl}(A))^c = \text{gspcl}(\text{gspint}(A^c))$. Therefore $(\text{gspcl}(\text{gspcl}(A))^c)^c = (\text{gspcl}(\text{gspint}(A^c)))^c$. Since A is an intuitionistic fuzzy generalized semi-pre open set, A^c is an intuitionistic fuzzy generalized semi-pre closed set. So by (ii) $\text{gspint}(A^c)$ is an intuitionistic fuzzy generalized semi-pre closed set. That is $\text{gspcl}(\text{gspint}(A^c)) = \text{gspint}(A^c)$. Hence $(\text{gspcl}(\text{gspint}(A^c)))^c = (\text{gspint}(A^c))^c = \text{gspcl}(A)$.

(iii) \Rightarrow (iv) Let A and B be any two intuitionistic fuzzy generalized semi-pre open sets in (X, τ) such that $\text{gspcl}(A) = B^c$. (iii) implies $\text{gspcl}(A) = (\text{gspcl}(\text{gspcl}(A))^c)^c = (\text{gspcl}(B^c)^c)^c = (\text{gspcl}(B))^c$.

(iv) \Rightarrow (i) Let A be any intuitionistic fuzzy generalized semi-pre open set in (X, τ) . Put $B = (\text{gspcl}(A))^c$. Then $\text{gspcl}(A) = B^c$. Hence by (iv), $\text{gspcl}(A) = (\text{gspcl}(B))^c$. Since $\text{gspcl}(B)$ is an intuitionistic fuzzy generalized semi-pre closed set as the space is an intuitionistic fuzzy semi-pre $T_{1/2}$ space, it follows that $\text{gspcl}(A)$ is an intuitionistic fuzzy generalized semi-pre open set. This implies that (X, τ) is an intuitionistic fuzzy generalized semi-pre extremally disconnected space. \square

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