

CHAPTER - VII

CHAPTER VII
FUZZY APPROXIMATELY IRRESOLUTE AND
CONTRA FUZZY IRRESOLUTE FUNCTIONS

Definition: 7.1

A map $f : X \rightarrow Y$ is said to be **fuzzy approximately irresolute (or Fap-irresolute)** if, $\text{scl}(\delta) \leq f^{-1}(\nu)$ whenever ν is a fuzzy semi-open subset of Y , δ is a Fsg-closed subset of X , and $\delta \leq f^{-1}(\nu)$.

Definition: 7.2

A map $f : X \rightarrow Y$ is said to be **fuzzy approximately semi-closed (or Fap-semi-closed)** if, $f(\beta) \leq \text{sint}(\alpha)$ whenever α is a Fsg-open subset of Y , β is a fuzzy semi-closed subset of X , and $f(\beta) \leq \alpha$.

Definition: 7.3

Let $f : X \rightarrow Y$ be a map from a fts X to a fts Y .

(i) f is called **fuzzy pre-semi-closed** if for every fuzzy semi-closed set β of X , $f(\beta)$ is fuzzy semi-closed in Y .

(ii) f is called **fuzzy pre-semi-open** if for every fuzzy semi-open set β of X , $f(\beta)$ is fuzzy semi-open in Y .

Note: 7.4

Fuzzy irresolute maps are Fap-irresolute and fuzzy pre-semi-closed maps are Fap-semi-closed, but not conversely.

The proof follows from Definitions.

The following example shows the converse implications do not hold.

Example: 7.5

Let $X = \{a, b\}$ and $Y = \{x, y\}$.

Define α and β as :

$$\alpha(a) = 0.3, \alpha(b) = 0.4, \beta(x) = 0.7, \beta(y) = 0.8.$$

Let $\tau = \{0, \alpha, 1\}$ and $\sigma = \{0, \beta, 1\}$. Then the map $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by

$f(a) = x, f(b) = y$ is Fap-irresolute but not fuzzy irresolute.

Example: 7.6

Let $X = \{x, y, z\}$ and $Y = \{a, b, c\}$.

Define α, β and λ as:

$$\alpha(x) = 0, \alpha(y) = 0.3, \alpha(z) = 0.2;$$

$$\beta(a) = 0, \beta(b) = 0.3, \beta(c) = 0.2;$$

$$\lambda(a) = 0.9, \lambda(b) = 0.6, \lambda(c) = 0.7.$$

Let $\tau = \{0, \alpha, 1\}$ and, $\sigma = \{0, \beta, \lambda, 1\}$. Then the map $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by

$f(x) = a, f(y) = b$ and $f(z) = c$ is Fap-semi-closed but not fuzzy pre-semi-closed.

Theorem: 7.7

(i) $f : X \rightarrow Y$ is Fap-irresolute if, $f^{-1}(\nu)$ is fuzzy semi-closed in X for every $\nu \in \text{FSO}(Y)$.

(ii) $f : X \rightarrow Y$ is Fap-semi-closed if, $f(\beta) \in \text{FSO}(Y)$ for every fuzzy semi-closed subset β of X .

Proof:

(i) Let $\delta \leq f^{-1}(\nu)$ where $\nu \in \text{FSO}(Y)$ and δ is a Fsg-closed subset of X .

Therefore $\text{scl}(\delta) \leq \text{scl}(f^{-1}(\nu)) = f^{-1}(\nu)$ since $f^{-1}(\nu)$ is fuzzy semi-closed.

Thus f is Fap-irresolute.

(ii) Let $f(\beta) \leq \alpha$, where β is a fuzzy semi-closed subset of X and α is a Fsg-open subset of Y . Therefore $\text{sint}(f(\beta)) \leq \text{sint}(\alpha)$. Then $f(\beta) \leq \text{sint}(\alpha)$ since $f(\beta)$ is Fs-open.

Thus f is Fap-semi-closed.

Result: 7.8

A fuzzy subset β is fuzzy semi-closed iff $\text{scl}(\beta) = \beta$.

Result: 7.9

A fuzzy subset α is fuzzy semi-open iff $\text{scl}(\alpha) = \alpha$.

Theorem: 7.10

Let $f : X \rightarrow Y$ be a map from a fuzzy topological space X in a fuzzy topological space Y .

(i) If the fuzzy semi-open and fuzzy semi-closed sets of X coincide, then f is Fap-irresolute iff, $f^{-1}(\nu)$ is fuzzy semi-closed in X for every $\nu \in \text{FSO}(Y)$.

(ii) If the fuzzy semi-open and fuzzy semi-closed sets of Y coincide, then f is Fap-semi-closed iff, $f(\beta) \in \text{FSO}(Y)$ for every fuzzy semi-closed subset β of X .

Proof:

(i) Assume f is Fap-irresolute. Let α be an arbitrary fuzzy subset of X such that $\alpha \leq \eta$ where $\eta \in \text{FSO}(X)$. Then by hypothesis $\text{scl}(\alpha) \leq \text{scl}(\eta) = \eta$. Therefore all fuzzy subsets of X are Fsg-closed (and hence all are Fsg-open). So for any $\nu \in \text{FSO}(Y)$,

$f^{-1}(\nu)$ is Fsg-closed in X. Since f is Fap-irresolute $\text{scl}(f^{-1}(\nu)) \leq f^{-1}(\nu)$. Therefore $\text{scl}(f^{-1}(\nu)) = f^{-1}(\nu)$, i.e., $f^{-1}(\nu)$ is fuzzy semi-closed in X (Result 7.8).

The converse is clear by Theorem 7.7 (i).

(ii) Assume f is Fap-semi-closed. Reasoning as in (i), we obtain that all fuzzy subsets of Y are Fsg-open. Therefore for any fuzzy semi-closed fuzzy subset β of X, $f(\beta)$ is Fsg-open in Y. Since f is Fap-semi-closed $f(\beta) \leq \text{sint}(f(\beta))$. Therefore $f(\beta) = \text{sint}(f(\beta))$, i.e., $f(\beta)$ is fuzzy semi-open (Result 7.9). The converse is clear by Theorem 7.7 (ii).

As immediate consequence of Theorem 7.10, we have the following corollary.

Corollary: 7.11

Let $f : X \rightarrow Y$ be a map from a fuzzy topological space X in a fuzzy topological space Y.

(i) If the fuzzy semi-open and fuzzy semi-closed sets of X coincide, then f is Fap-irresolute iff, f is fuzzy irresolute.

(ii) If the fuzzy semi-open and fuzzy semi-closed sets of Y coincide, then f is Fap-semi-closed iff, f is fuzzy pre-semi-closed.

Definition: 7.12

A map $f : X \rightarrow Y$ is called **contra-fuzzy irresolute** if $f^{-1}(\nu)$ is fuzzy semi-closed in X for each $\nu \in \text{FSO}(Y)$, and **contra-fuzzy pre-semi-closed** if $f(\beta) \in \text{FSO}(Y)$ for each fuzzy semi-closed set β of X.

Remark: 7.13

The concepts of contra-fuzzy irresoluteness and fuzzy irresoluteness are independent.

Example: 7.14

A fuzzy irresolute map need not be contra-fuzzy irresolute.

Let $X = \{x, y, z\}$ and $Y = \{a, b\}$.

Define α and β as:

$$\alpha(x) = 0.3, \alpha(y) = 0.5, \alpha(z) = 0.4;$$

$$\beta(a) = 0.4, \beta(b) = 0.6.$$

Let $\tau = \{0, \alpha, 1\}$ and $\sigma = \{0, \beta, 1\}$. Then the map $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(x) = a$, $f(y) = f(z) = b$ is fuzzy irresolute but not contra-fuzzy irresolute.

Theorem: 7.15

Let $f : X \rightarrow Y$ be a map. Then the following conditions are equivalent:

- (i) f is contra-fuzzy irresolute,
- (ii) The inverse image of each fuzzy semi-closed set in Y is fuzzy semi-open in X .

Remark: 7.16

By Theorem 7.7, we have that every contra-fuzzy irresolute map is Fap-irresolute and every contra-fuzzy semi-closed is Fap-semi-closed, the converse implication do not hold, as the map defined in Example 6.4 is Fap-irresolute but not contra-fuzzy irresolute and the map defined in Example 7.6 is Fap-semi-closed but not contra fuzzy semi-closed.

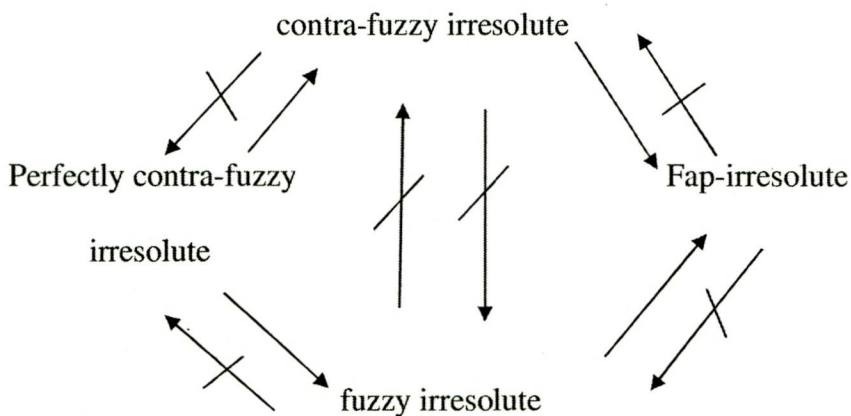
Definition: 7.17

A map $f : X \rightarrow Y$ is called **perfectly contra -fuzzy irresolute** if the inverse of every fuzzy semi-open set in Y is fuzzy semi-clopen in X .

Remark: 7.18

Every perfectly contra-fuzzy irresolute map is contra-fuzzy irresolute and fuzzy irresolute.

Clearly the following diagram holds and none of its implications is reversible:



Theorem: 7.19

If a map $f : X \rightarrow Y$ is fuzzy irresolute and Fap-semi-closed, then $f^{-1}(\alpha)$ is Fsg-closed (resp. Fsg-open) whenever α is Fsg-closed (resp. Fsg-open) subset of Y .

Proof:

Let α be a Fsg-closed subset of Y . Suppose that $f^{-1}(\alpha) \leq v$ where $v \in \text{FSO}(X)$. Taking complements we obtain $v^c \leq f^{-1}(\alpha^c)$ or $f(v^c) \leq \alpha^c$. Since f is an Fap-semi-closed $f(v^c) \leq \text{sint}(\alpha^c) = (\text{scl}(\alpha))^c$. It follows that $v^c \leq (f^{-1}(\text{scl}(\alpha)))^c$ and hence $f^{-1}(\text{scl}(\alpha)) \leq v$. Since f is fuzzy irresolute $f^{-1}(\text{scl}(\alpha))$ is fuzzy semi-closed. Thus we have $\text{scl}(f^{-1}(\alpha)) \leq \text{scl}(f^{-1}(\text{scl}(\alpha))) = f^{-1}(\text{scl}(\alpha)) \leq v$.

This implies that $f^{-1}(\alpha)$ is Fsg-closed in X .

A similar argument shows that inverse images of sg-open are sg-open.

The following theorem 7.20 shows that the fuzzy ap-irresolute pre-semi-closed image of a Fsg-closed set is Fsg-closed.

Theorem: 7.20

If a map $f : X \rightarrow Y$ is Fap-irresolute and fuzzy pre-semi-closed, then for every Fsg-closed δ of X , $f(\delta)$ is Fsg-closed set of Y .

Proof:

Let δ be a Fsg-closed subset of X . Let $f(\delta) \leq \nu$ where $\nu \in \text{FSO}(Y)$. Then $\delta \leq f^{-1}(\nu)$ holds. Since f is Fap-irresolute $\text{scl}(\delta) \leq f^{-1}(\nu)$ and hence $f(\text{scl}(\delta)) \leq \nu$.

Therefore, we have :

$\text{scl}(f(\delta)) \leq \text{scl}(f(\text{scl}(\delta))) = f(\text{scl}(\delta)) \leq \nu$. Hence $f(\delta)$ is Fsg-closed in Y .

Theorem: 7.21

Let $f : X \rightarrow Y$, $g : Y \rightarrow Z$ be two maps such that $g \circ f : X \rightarrow Z$. Then

- (i) $g \circ f$ is Fap-semi-closed, if f is fuzzy pre-semi-closed and g is Fap-semi-closed..
- (ii) $g \circ f$ is Fap-semi-closed, if f is Fap-semi-closed and g is fuzzy pre-semi-open and g^{-1} preserves Fsg-open sets.
- (iii) $g \circ f$ is Fap-irresolute, if f is Fap-irresolute and g is fuzzy irresolute.

Proof:

In order to prove the statement (i), suppose β is an arbitrary fuzzy semi-closed subset in X and α a Fsg-open subset of Z for which $g \circ f(\beta) \leq \alpha$. Then $f(\beta)$ is fuzzy semi-closed in Y because f is fuzzy pre-semi-closed. Since g is Fap-semi-closed, $g(f(\beta)) \leq \text{sint}(\alpha)$. This implies that $g \circ f$ is Fap-semi-closed.

In order to prove the statement (ii), suppose β is an arbitrary fuzzy semi-closed subset of X and α a Fsg-open subset of Z for which $g \circ f(\beta) \leq \alpha$. Hence $f(\beta) \leq g^{-1}(\alpha)$. Then $f(\beta) \leq \text{sint}(g^{-1}(\alpha))$ because $g^{-1}(\alpha)$ is Fsg-open and f is Fap-semi-closed. Thus , $(g \circ f)(\beta) = g(f(\beta)) \leq g(\text{sint}(g^{-1}(\alpha))) \leq \text{sint}(g(g^{-1}(\alpha))) \leq \text{sint}(\alpha)$.

This implies that $g \circ f$ is Fap-semi-closed.

In order to prove the statement (iii), suppose δ is an arbitrary Fsg- closed subset of X and $\nu \in \text{FSO}(Z)$ for which $\delta \leq (g \circ f)^{-1}(\nu)$. Then $g^{-1}(\nu) \in \text{FSO}(Y)$ because g is fuzzy irresolute. Since f is Fap-irresolute, $\text{scl}(\delta) \leq f^{-1}(g^{-1}(\nu)) = (g \circ f)^{-1}(\nu)$. This proved that $g \circ f$ is Fap-irresolute .

As a consequence of Theorem 7.21, we have the following corollary 7.22.

Corollary: 7.22

Let $f_i: X \rightarrow Y_i$ be a map for each $i \in \Omega$ and $f: X \rightarrow \prod Y_i$ the product map given by $f(x) = (f_i(x))$. If f is Fap-irresolute, then f_i is Fap-irresolute for each i .

Proof:

For each j let $P_j: \prod Y_i \rightarrow Y_j$ be the projection map. Then $f_j = P_j \circ f$ where P_j is fuzzy irresolute . By Theorem 7.21 (iii), f_j is Fap-irresolute.

Theorem: 7.23

Let X be a fuzzy topological space. Then the following statements are equivalent:

- (i) X is a Fuzzy-Semi- $T_{1/2}$ space,
- (ii) For every space Y and every map $f : X \rightarrow Y$, f is Fap-irresolute.

Proof:

(i) \Rightarrow (ii) :

Let δ be a Fsg-closed subset of X and suppose that $\delta \leq f^{-1}(\nu)$ where $\nu \in \text{FSO}(Y)$. Since X is a Fuzzy-semi- $T_{1/2}$ space, δ is fuzzy semi-closed (i.e., $\delta = \text{sCl}(\delta)$). Therefore $\text{scl}(\delta) \leq f^{-1}(\nu)$. Then f is Fap-irresolute.

(ii) \Rightarrow (i) :

Let β be a Fsg-closed subset of X and let Y be the fuzzy set X with the topology $\sigma = \{0, \beta, 1\}$. Finally let $f : X \rightarrow Y$ be the identity map. By assumption f is Fap-irresolute. Since β is Fsg-closed in X and fuzzy semi-open in (Y, σ) and $\beta \leq f^{-1}(\beta)$, it follows that $\text{scl}(\beta) \leq f^{-1}(\beta) = \beta$. Hence β is fuzzy semi-closed in X and therefore is Fuzzy-semi- $T_{1/2}$.