

Avinashilingam Institute for Home Science and Higher Education for Women
 (Deemed to be University), Coimbatore-641 043
Bachelor's Degree Examination – November 2018
I Semester

Class : I UG

Major : B. Sc. Special Education and Mathematics

Time :3 hours

Max. Marks: 100

18BSMC01 – ANALYTICAL GEOMETRY AND CLASSICAL ALGEBRA

Part-A

10 x 1=10

Choose the correct answer

1. The radical axis is _____ to the line of centres.
 (a) perpendicular (b) parallel (c) tangent (d) normal
2. A system of circles in which every pair of circles has the same radical axis is called a _____ system.
 (a) radical (b) coaxial (c) orthogonal (d) Limiting
3. A circle will cut a parabola in 4 points and the algebraic sum of the ordinates of the 4 points is _____
 (a) 0 (b) 1 (c) constant (d) ∞
4. The line $y = mx, y = m_1x$ are conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $mm_1 =$ _____
 (a) $\frac{b^2}{a^2}$ (b) $\frac{a^2}{b^2}$ (c) $-\frac{b^2}{a^2}$ (d) $-\frac{a^2}{b^2}$
5. Find the remainder when $2x^6 + 3x^5 - 15x^2 + 2x - 4$ is divided by $x + 5$.
 (a) 20486 (b) 21486 (c) 22486 (d) 23486
6. An equation $f(x) = 0$ cannot have _____ positive roots than there are changes of sign in $f(x)$.
 (a) One (b) Two (c) Zero (d) more
7. The value of the Euler's function, $\phi(2) =$ _____
 (a) 0 (b) 1 (c) 2 (d) 3
8. The integral part of a number $x = \sqrt{2}$ is _____
 (a) 0 (b) 1 (c) 2 (d) 3
9. If two numbers are congruent with respect to the modulus m , each is called a _____ of the other to the modulus m .
 (a) residue (b) integral (c) congruence (d) coprime
10. Every integer which is a perfect cube is of the form _____
 (a) $17p$ (b) p (c) $7p$ (d) $9p$

Answer the following

Answer should not exceed 400 words or two pages

11. (a) Find the radical centre of the three circles

$$x^2 + y^2 - x + 3y - 3 = 0, x^2 + y^2 - 2x + 2y + 2 = 0 \text{ and}$$

$$x^2 + y^2 + 2x + 3y - 9 = 0.$$

(or)

- (b) Find the circles which cuts orthogonally each of the following circles:

$$x^2 + y^2 + 2x + 4y + 1 = 0$$

$$x^2 + y^2 - 4x + 3 = 0$$

$$x^2 + y^2 + 6y + 5 = 0$$

12. (a) Chords of a parabola are drawn through a fixed point. Show that the locus of the middle points is another parabola.

(or)

- (b) If a pair of conjugate diameters meet the hyperbola and its conjugate in P and D, then prove that
- $CP^2 - CD^2 = a^2 - b^2$
- .

13. (a) Change the equation
- $2x^4 - 3x^3 + 3x^2 - x + 2 = 0$
- into another the coefficient of whose highest term will be unity.

(or)

- (b) Find the quotient and remainder when
- $3x^3 + 8x^2 + 8x + 12$
- is divided by
- $x - 4$
- .

14. (a) Find the number and sum of all the divisors of 360.

(or)

- (b) Find the number of integers less than
- n
- and prime to it when
- $n = 729$
- and
- 720
- .

15. (a) Find a number having the remainders 5, 4, 3, 2 when divided by 6, 5, 4, 3 respectively.

(or)

- (b) If
- x, y, z
- be three consecutive integers, show that
- $(2x)^2 - 3x^2$
- is divided by 108.

Part C

5 x 12=60

Answer the following

Answer should not exceed 800 words or four pages

16. (a) Show that the circles
- $x^2 + y^2 - 6x + 9y + 13 = 0$
- and
- $x^2 + y^2 - 2x - 16y = 0$
- touch each other and find the co-ordinates of the point of contact.

(or)

- (b) Obtain the equation of a circle which passes through the point (1, 2) bisects the Circumference of the circle
- $x^2 + y^2 = 9$
- and cuts orthogonally the circle
- $x^2 + y^2 - 2x + 8y - 7 = 0$
- .

17. (a) Find the locus of the poles of all tangents to the parabola
- $y^2 = 4ax$
- with respect to the parabola
- $y^2 = 4bx$
- .

(or)

- (b) A tangent to the ellipse
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- whose centre is C meets the circle
- $x^2 + y^2 = a^2 + b^2$
- at Q and Q'. Prove that CQ and CQ' are conjugate diameters of the ellipse.

18. (a) Find the roots of the equation
- $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$
- .

(or)

- (b) Increase by 7 the roots of the equation
- $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$
- .

19. (a) Find the highest power of 3 dividing
- $1000!$
- .

(or)

- (b) Show that
- $n(n+1)(2n+1)$
- is divisible by 6.

20. (a) Find the remainder obtained in dividing
- 2^{45}
- by 47.

(or)

- (b) Show that
- $x^5 - x$
- is divisible by 30.
