

CHAPTER - III

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MODIFIED FUZZY ANALYTIC HIERARCHY PROCESS METHOD

The Modified Fuzzy Analytic Hierarchy Process method is a new methodology based on Fuzzy Analytic Hierarchy Process. This methodology has many advantages. It enables multiple decision makers on evaluation and uses triangular fuzzy scale that includes both positive and negative fuzzy numbers, in order to evaluate hierarchy. It uses more understandable scale to compare factors. This methodology includes simple mathematical calculations, and it yields triangular fuzzy numbers of alternatives' weights. At the last step, obtained alternatives' weights are ranked by Centroid - Based Distance Method. And it follows the steps of the traditional AHP. This proposed methodology can handle the problems effectively and with efficiency.

METHODOLOGY

Step 1: Structure Hierarchy

To start with, decision makers determine goal, criteria and alternatives of the problem in a hierarchical form. An established hierarchy has to give the whole details of information on the structure so that there should not be lack of fact about the problem.

Step 2: Make Pairwise Comparisons for Factors

Decision makers are required to compare each factor in the hierarchy. Decision makers use the fuzzy scale shown in Table 4 to compare factors. They use experimental data, perception, background, knowledge, etc. to make comparisons. Because decision makers may have different viewpoints, they can use different linguistic variables in comparisons matrices. The weights (e) are allocated to decision makers on the basis of their knowledge, experience, etc. Suppose that m decision makers exist in the group and the k^{th} decision maker E_k is assigned a decision maker weight e_k , where $e_k \in [0,1]$, $e_1 + \dots + e_m = 1$.

Linguistic scale	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Equally important	(0, 1, 3)	(-3, -1, 0)
Weakly important	(1, 3, 5)	(-5, -3, -1)
Strongly more important	(3, 5, 7)	(-7, -5, -3)
Very strongly more important	(5, 7, 9)	(-9, -7, -5)
Absolutely more important	(7, 9, 9)	(-9, -9, -7)

Table 4. Triangular Fuzzy Conversion Scale

Step 3: Aggregate Individual TFNs to Group TFNs

The purpose of this step is to apply an acceptable operator to get a group preference from individual preferences. The aggregation of TFNs scores is performed by applying the fuzzy weighted triangular averaging operator, as defined by (13).

$$\widetilde{a}_{ij} = \widetilde{a}_{ij1} \otimes e_1 \oplus \widetilde{a}_{ij2} \otimes e_2 \oplus \dots \oplus \widetilde{a}_{ijm} \otimes e_m \quad (13)$$

where \widetilde{a}_{ij} is the aggregated fuzzy score for $A_i - A_j$ comparisons, $i = 1, \dots, n$; $\widetilde{a}_{ij1}, \widetilde{a}_{ij2}, \dots, \widetilde{a}_{ijm}$ are corresponding TFN scales assigned by decision makers E_1, E_2, \dots, E_m , respectively. \otimes and \oplus indicates fuzzy multiplication and fuzzy addition operators, respectively.

Step 4: Convert Negative Fuzzy TFNs to Positive TFNs.

Since the scores in the classical AHP are based on an exponential importance, we should calculate the corresponding exponential values of negative scores in our method. This conversion is obtained by (14).

$$\widetilde{a}_{ij}^* = e^{(\widetilde{a}_{ij}/4)} \quad (14)$$

where

$$\widetilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$$

Step 5: Calculate the Priority Weights of Factors

Consider a triangular fuzzy comparison matrix expressed by

$$\tilde{A} = (\tilde{a}_{ij}) = \begin{bmatrix} (1,1,1) & (l_{12}, m_{12}, u_{12}) & \dots & (l_{1n}, m_{1n}, u_{1n}) \\ (l_{21}, m_{21}, u_{21}) & (1,1,1) & \dots & (l_{2n}, m_{2n}, u_{2n}) \\ \dots & \dots & \dots & \dots \\ (l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) & \dots & (1,1,1) \end{bmatrix}$$

where $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ and $\tilde{a}_{ij}^{-1} = (1/u_{ij}, 1/m_{ij}, 1/l_{ij})$ for $i, j = 1, \dots, n$ and $i \neq j$. Since the aim is to bring out a simplified Fuzzy AHP, complicated normalization formula is not used. A normalized matrix \tilde{N} can be calculated as follows:

$$\tilde{N} = [\tilde{n}_{ij}]_{n \times n}$$

$$\tilde{n}_{ij} = \left(\frac{l_{ij}}{u_j^*}, \frac{m_{ij}}{u_j^*}, \frac{u_{ij}}{u_j^*} \right) \quad (15)$$

where

$$u_j^* = \max_i u_{ij}$$

The normalization method clarified above is to preserve the property that the ranges of normalized triangular fuzzy numbers belong to $[0, 1]$.

And the importance weights of the factors can be calculated as follows:

$$W_i = \frac{\sum_{j=1}^n \tilde{n}_{ij}}{\sum_{k=1}^n \sum_{j=1}^n \tilde{n}_{kj}} \quad k = 1, \dots, n \quad (16)$$

Step 6: Calculate Final Weights

In this step the rating of each alternative is multiplied by the weights of the sub-criteria and aggregated to get local ratings with respect to each criterion. The local ratings are then multiplied by the weights of the criteria and aggregated to get global ratings.

Step7: Compare the Weights using a Ranking Method

In the last step, we rank the obtained fuzzy numbers. In order to rank the fuzzy numbers, we use the Centroid - Based Distance Method [69].

Finally the alternative with highest rank is selected as the best alternative