Empowering Entrepreneurs through Mathematics Education for Sustainable SME Development in India

By

Apaarana M

(23PMA019)

Supervisor

Dr. C Antony Crispin Sweety

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore – 641043

In Partial Fulfillment of the Requirement of the Degree of Master of Science in Mathematics

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Signature of the Director | Signature of the Supervisor

DECLARATION

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I declare that the thesis " **Empowering Entrepreneurs through Mathematics Education for Sustainable SME Development in India** " submitted by me for the degree of **Master of Science (M.Sc.)** is the record of work carried out during the period from December 2024 to April 2025 under the guidance of **Dr. C. Antony Crispin Sweety, M.Sc., B.Ed., M.Phil., Ph.D.,** Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for women, Coimbatore, and has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this institute or any other University or other similar institution of Higher Learning.

M. Apaakana
25/04/2025
Signature of the Candidate

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CONTENT

ABSTRACT

Small and Medium Enterprises (SMEs) are a vital component of India's economic landscape, contributing significantly to GDP, employment, and regional development. However, many SMEs face challenges in strategic decision-making, innovation, and sustainability due to limited resources and lack of analytical capabilities. This thesis explores the intersection of mathematics education and entrepreneurial development as a pathway to strengthen SME performance in India. It emphasizes educational strategies that integrate mathematical thinking, problem-solving, and real-world applications to nurture future mathematical entrepreneurs. The research introduces an innovative framework that combines entrepreneurship education with advanced mathematical tools, including Fermatean Neutrosophic logic. It applies the Fermatean Neutrosophic TOPSIS method - a robust Multi-Criteria Decision-Making (MCDM) model - to evaluate sustainable strategic practices in SMEs under uncertainty and imprecision. By using this approach, the thesis demonstrates how mathematical models can guide more informed, data-driven decisions that align with sustainability goals. Through pedagogical recommendations, and applied decision models, this work aims to empower students and entrepreneurs with the analytical skills necessary for the growth and sustainability of Indian SMEs. The integration of education, sustainability, and neutrosophic mathematics opens new avenues for innovative, impactful entrepreneurship in the Indian SME sector.

CHAPTER 1

CHAPTER 1

INTRODUCTION

SME stands for Small and Medium Enterprise. In the Indian context, SMEs refer to business entities that operate on a smaller scale in terms of investment, turnover, and number of employees, compared to large corporations. These enterprises form the backbone of India's economy by contributing to employment, innovation, exports, and inclusive development.

These enterprises play a key role in inclusive growth by promoting entrepreneurship, especially in semi-urban and rural regions. The Indian government supports SMEs through various initiatives, including financial schemes, infrastructure development, skill training, and digital empowerment, aiming to boost their competitiveness and sustainability in both domestic and global markets.

SMEs play a critical role in the country’s economic and social development. They account for approximately 30% of India’s Gross Domestic Product (GDP), making them a key contributor to national economic growth. SMEs employ over 110 million people, making them the second-largest source of employment after the agricultural sector and a crucial pillar for job creation and livelihood support.

Furthermore, they contribute about 45% of India’s total exports, showcasing their importance in global trade. Beyond economic contributions, SMEs help foster regional balance by encouraging industrial development in rural and backward areas, thereby reducing the urban-rural divide. They promote entrepreneurship, especially among youth and women, and drive innovation in niche markets.

Over time, SMEs have been widely studied for their potential in economic development, especially in developing countries. However, due to their limited resources and heightened exposure to risk, effective decision-making becomes crucial for their survival and growth. To support strategic and operational decisions—such as choosing suppliers, investing in new technology, or entering new markets—SMEs often turn to structured decision-making tools. To assist in such complex decision-making, especially when multiple conflicting criteria are involved, **Multi-Criteria Decision-Making (MCDM)** methods are widely used.

There are many Multi-Criteria Decision-Making (MCDM) methods, developed to handle different types of problems, data formats, levels of uncertainty, and decision-maker preferences.

Here's a some categorized list

1. AHP - Analytic Hierarchy process
2. ANP – Analytic Network Process
3. TOPSIS - Technique for Order Preference by Similarity to Ideal Solution
4. MOORA - Multi-Objective Optimization by Ratio Analysis
5. BWM - Best-Worst Method
6. SWARA - Step-wise Weight Assessment Ratio Analysis
7. CODAS - Combinative Distance-Based Assessment

One of the most popular MCDM tools is the **TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)** method. TOPSIS helps decision-makers evaluate and rank alternatives based on their closeness to an ideal solution (best case) and distance from a negative-ideal solution (worst case).

ABBREVIATION

- SME - Small and Medium Enterprises
- MCDM - Multi-Criteria Decision–Making
- AHP - Analytic Hierarchy process
- ANP - Analytic Network Process
- TOPSIS - Technique for Order Preference by Similarity to Ideal Solution
- MOORA - Multi-Objective Optimization by Ratio Analysis
- BWM - Best-Worst Method
- SWARA - Step-wise Weight Assessment Ratio Analysis
- CODAS - Combinative Distance-Based Assessment

1.1 LITERATURE REVIEW

Schlange (2006) emphasizes that traditional entrepreneurship primarily focused on maximizing profit and neglected social and environmental concerns.

Lans et al. (2014) define sustainable entrepreneurship as identifying sustainability as a business opportunity, leading to sustainable products and processes.

Muñoz & Cohen (2018) argue that sustainable entrepreneurship seeks to restore balance between nature, society, and the economy.

Terán-Yépez et al. (2020) highlight the shift in entrepreneurship from wealth generation to sustainable development, incorporating social and environmental dimensions.

Natividade et al. (2021) expand the concept to include TBL (Triple Bottom Line) values and long-term environmental protection.

Kostakis & Tsagarakis (2022) discuss the role of sustainability in influencing market behaviours and encouraging adoption of sustainable practices.

Bajdor et al. (2021) and Di Vaio et al. (2022) acknowledge the growing importance of sustainability practices but note a lack of clarity in criteria.

Masurel (2007) emphasizes SMEs' role in sustainability due to their flexibility and local impact.

Walker et al. (2008) point out that SMEs are not merely scaled-down versions of large enterprises and thus require unique sustainability strategies.

Omrane et al. (2020), Bocken et al. (2019), Iqbal et al. (2020), and others provided examples of environmental practices implemented in SMEs (e.g., renewable energy use, waste segregation, eco-product manufacturing).

Mendes et al. (2022) study the determinants of sustainable entrepreneurship in SMEs but focus on broad characteristics rather than specific practices.

Decision-making is the process of choosing the best option among several alternatives to achieve a specific goal or solve a problem. Fuzzy and Neutrosophic methods refine decision-making under uncertainty.

Biswas et al. (2016) [8] applied TOPSIS in a neutrosophic setting, and Peng & Ma (2019) [50] enhanced CODAS with a new score function, improving multi-criteria analysis. This involves assessing options based on criteria such as risks, benefits, and potential outcomes.

In Multi-Criteria Decision Making (MCDM), various methods are employed to evaluate the alternatives based on multiple criteria [26], including TOPSIS (Technique for order of preference by Similarity to Ideal solution) [8,71,80] for ranking alternatives based on proximity to ideal solutions.

Fermatean Neutrosophic Set (FNS) is a flexible framework and generalized theory introduced by Antony Crispin Sweety et al. (2021) [3] that includes fuzzy, intuitionistic fuzzy, Pythagorean fuzzy, spherical fuzzy, Fermatean fuzzy sets, and neutrosophic set theory.

1.2 OUTLINE OF THE THESIS

Chapter 1 provides a foundational understanding of Neutrosophic Sets, highlighting their advancement over classical, fuzzy, and intuitionistic fuzzy sets in handling indeterminate, inconsistent, and incomplete information. The chapter introduces key concepts such as truth, indeterminacy, and falsity membership functions, and discusses their relevance in real-world applications involving uncertainty.

Chapter 2 shifts focus to educational strategies for nurturing mathematical entrepreneurs in Small and Medium Enterprises (SMEs). It emphasizes the importance of mathematical thinking in entrepreneurial success and outlines pedagogical approaches for fostering innovation, critical reasoning, and problem-solving skills in potential entrepreneurs. The role of SMEs in driving economic development is also highlighted, connecting educational practices to practical entrepreneurial needs.

Chapter 3 introduces the Fermatean Neutrosophic TOPSIS method as a robust Multi-Criteria Decision-Making (MCDM) tool. This chapter applies the method to rank various sustainable practices. The model offers enhanced flexibility and accuracy in decision-making, suitable for SME environments.

1.3 BASIC CONCEPTS

This chapter presents an introduction to our study, highlighting the necessity of flexible sets that play a significant role in the later sections of the thesis.

Definition 1.3.1

Let X be a nonempty set. A fuzzy set A drawn from X is defined as

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\}$$

Where $\mu: X \rightarrow [0,1]$ is the membership function of the fuzzy set A.

Definition 1.3.2

Let X be a universe. An intuitionistic fuzzy set A on X can be defined as follows:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

Where $\mu: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for any $x \in X$Where, $\mu_A(x)$ and $\nu_A(x)$ is the degree of membership and degree of non-membership of the element x respectively.

Definition 1.3.3

Let U be a universe set. A Neutrosophic Set (NS) A in U is characterized by a truth membership functionT_A, an indeterminacy membership function I_A and a falsity membership function $F_A$where T_A , I_A and F_A are real standard elements of [0,1]. It can be written as

$$A = \{\langle X, (T_A(x)) + (I_A(x)) + (F_A(x))\rangle : x \in E, T_A, I_A, F_A \in]^-0, 1^+[\}$$

There is no restriction on the sum of $(T_A(x)), (I_A(x))$and$(F_A(x))$ and so

$$0^- \leq (T_A(x)) + (I_A(x)) + (F_A(x)) \leq 3^+$$

Definition 1.3.4

Let X is a space of points (objects) with generic elements in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$ if the functions $T_A(x), I_A(x), F_A(x)$ are singletons subintervals/subsets in the real standard [0,1], i.e. $T_A(x)$: X → [0, 1], $I_A(x)$: X → [0,1], $F_A(x)$: X → [0,1]. Then a simplification of the neutrosophic set A is denoted by

$$\mathrm{A} = \{< \mathrm{x}, T_A(x), I_A(x), F_A(x) >, x \in X\}.$$

Definition 1.3.5

Let X is a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $\mathrm{I}_A(x)$ and a falsity-membership function $F_A(x)$, for each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0,1]$. Therefore, a SVNS A can be written as $A_{\mathrm{SVNS}} = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$.

For two SVNS, $A_{\mathrm{SVNS}} = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$ and $B_{\mathrm{SVNS}} = \{< x: T_B(x), I_B(x), F_B(x) >, x \in X\}$, the following expressions are defined in as follows: $A_{\mathrm{NS}} \subseteq B_{\mathrm{NS}}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$. $A_{\mathrm{NS}} = B_{\mathrm{NS}}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$. $A^c =< x, F_A(x), 1 - I_A(x), T_A(x) >$

For convenience, a SVNS A is denoted by $A =< T_A(x), I_A(x), F_A(x) >$ for any x$\in X$; for two SVNSs A and B. Then,

(1) $A \cup B = < \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(T_A(x), T_B(x)) >$

(2) $A \cap B = < \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(T_A(x), T_B(x)) >$

Definition 1.3.6

A single-valued neutrosophic set (SVNS) A in X is a neutrosophic set which is of the form

$$A = \{\langle x: (T_A(x)) + (I_A(x)) + (F_A(x))\rangle, x \in X\}$$

that is characterized by the degree of membership (namely $(T_A(x))$, the degree of indeterminacy (namely $(I_A(x))$ and the degree of non-membership (namely $(F_A(x))$, where

$T_A(x), I_A(x), F_A(x) \in [0,1]$ such that $0 \le (T_A(x)) + (I_A(x)) + (F_A(x)) \le 3$, for all $x \in X$, respectively. For X, $SVNS(X)$ denotes the collection of all single valued neutrosophic sets of X.

Definition 1.3.7

A spherical fuzzy set $\tilde{A}$ of the universe of discourse U is given by

$$\tilde{A} = \{< x, (T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)) > | x \in U\}$$

where $T_{\tilde{A}}(x): U \to [0,1], I_{\tilde{A}}(x): U \to [0,1], F_{\tilde{A}}(x): U \to [0,1]$ and

$0 \le {T_{\tilde{A}}}^2(x) + {I_{\tilde{A}}}^2(x) + {F_{\tilde{A}}}^2(x) \le 1 \forall x \in U$.

The numbers $T_{\tilde{A}}(x), I_{\tilde{A}}(x)$ and $F_{\tilde{A}}(x)$ are the degree of membership, non-membership and hesitancy of x to$\tilde{A}$, respectively.

Definition 1.3.8

Consider X be a set that is not empty, and I be the unit interval $[0,1]$. A Pythagorean fuzzy set A is an object that has the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

where the degree of membership function is $\mu_A: X \to [0,1]$ and the degree of non-membership function is $\nu_A: X \to [0,1]$ for each element $x \in X$to the set A, and $(\mu_A(x))^2 + (\nu_A(x))^2 \le 1$ for each $x \in X$.

Definition 1.3.9

Consider X be a set that is not empty (universe). $(\not{P}\not{N}\not{S})$is a Pythagorean neutrosophic set with T and F are dependent neutrosophic components A on X is an object of the form

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$$

Where $T_A(x), I_A(x), F_A(x) \in [0,1]$, $0 \le (T_A(x))^2 + (I_A(x))^2 + (F_A(x))^2 \le 2$, for every$x \in X$. The degree of membership is $T_A(x)$, the degree of indeterminacy is $I_A(x)$ and the degree of non- membership is$F_A(x)$. Here $T_A(x)$ and $F_A(x)$ are dependent components and $I_A(x)$ is an independent.

Definition 1.3.10

Let X be a non-empty set (universe). A Fermatean neutrosophic set (FNS) A_F on X is an object of the form:

$$A_F = \{< s, T_{A_F}(s), I_{A_F}(s), F_{A_F}(s)) > | s \in X\}$$

Where, $T_{A_F}(s), I_{A_F}(s), F_{A_F}(s) \in [0,1]$, $0 \leq T^3{}_{A_F}(s), F^3{}_{A_F}(s) \leq 1, F^3{}_{A_F}(s) \leq 1, I^3{}_{A_F}(s) \leq 1$ then $0 \leq T^3{}_{A_F}(s), I^3{}_{A_F}(s), F^3{}_{A_F}(s) \leq 2, \forall\, s \in X$

$T_{A_F}(s)$ is the degree of membership, $I_{A_F}(s)$ is the degree of indeterminacy and $F_{A_F}(s)$ is the degree of non-membership. Here $T_{A_F}(s)$ and $I_{A_F}(s)$are independent components and $F_{A_F}(s)$ is a dependent component on $T_{A_F}(s)$.

Definition 1.3.11

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a discrete confined set. A mapping $d: NS(X) \times NS(X) \rightarrow [0,1]$ is said to be a distance measure between two neutrosophic sets if it satisfies the following axioms:

i. $d(A, B) \geq 0$ for all $A, B \in NS(X)$.

ii. $d(A, B) = 0$ if and only if $A = B$ for all $A, B \in NS(X)$.

iii. $d(A, B) = d(B, A)$ for all $A, B \in NS(X)$.

iv. If $A \subseteq B \subseteq C$ for all $A, B, C \in NS(X)$, then $d(A, C) \geq d(A, B)$and $d(A, C) \geq d(B, C)$

Definition 1.3.15

The Hamming distance between two single-valued neutrosophic sets A and B is defined as

$$d_H(A, B) = \frac{1}{3}\sum_{j=1}^{n}\left(\left|T_A(x_j)^2 - T_B(x_j)^2\right| + \left|I_A(x_j)^2 - I_B(x_j)^2\right| + \left|F_A(x_j)^2 - F_B(x_j)^2\right|\right)$$

Definition 1.3.16

The Euclidean distance between two single-valued neutrosophic sets A and B is defined as

$$d_E(A,B)=\left\{\frac{1}{3}\sum_{j=1}^{n}\left(\left(T_A(x_j)-T_B(x_j)\right)^2+\left(I_A(x_j)-I_B(x_j)\right)^2+\left(F_A(x_j)-F_B(x_j)\right)^2\right)\right\}^{\frac{1}{2}}$$

Definition 1.3.17

A pythagorean fuzzy number P in fixed set X can be defined as given in Equations (1) - (3). Let X be fixed and $T_P(x)$: X → [0, 1] represents the degree of membership of the element x ∈ X. $F_P(x)$: X → [0, 1] represents the degree of non membership of the element x ∈X to P.

$P \approx \{x, T_P(x), F_P(x); x \in X\}$

$T_P(x) : X \to [0, 1]$ and $F_P(x) : X \to [0, 1]$

$0 \leq T_P(x)^2 + F_P(x)^2 \leq 1.$

The Hesitancy degree can be determined by:

$\pi P(x) = \sqrt{1 + T_P(x)^2 - F_P(x)^2}.$

Definition 1.3.18

The Normalized Euclidean distance between two single-valued neutrosophic sets *A* and *B* is defined as

$$d_{n-E}(A,B)=\left\{\frac{1}{3}\sum_{j=1}^{n}\left(\left(T_A(x_j)-T_B(x_j)\right)^2+\left(I_A(x_j)-I_B(x_j)\right)^2+\left(F_A(x_j)-F_B(x_j)\right)^2\right)\right\}^{\frac{1}{2}}$$

Definition 1.3.19

The distance measures between two single-valued neutrosophic sets *A* and *B* are defined as

$$D_1(A,B)=\frac{1}{3n}\sum_{j=1}^{n}\left(|T_A(x_j)^2-T_B(x_j)^2|+|I_A(x_j)^2-I_B(x_j)^2|+|F_A(x_j)^2-F_B(x_j)^2|\right)$$

And

$$D_2(A,B)=\frac{1}{3n}\sum_{j=1}^{n}\left|\left(T_A(x_j)^2-T_B(x_j)^2\right)-\left(I_A(x_j)^2-I_B(x_j)^2\right)-\left(F_A(x_j)^2-F_B(x_j)^2\right)\right|$$

Definition 1.3.20

A mapping $S: NS(X) \times NS(X) \rightarrow [0,1]$ is said to be a similarity measure between two neutrosophic sets if it satisfies the properties of axioms:

i. $S(A,B) \geq 0$ for all $A, B \in NS(X)$.

ii. $S(A,B) = 0$ if and only if $A = B$ for all $A, B \in NS(X)$.

iii. $S(A,B) = S(B,A)$ for all $A, B \in NS(X)$.

iv. If $A \subseteq B \subseteq C$ for all $A, B, C \in NS(X)$, then $S(A,C) \geq S(A,B)$ and $S(A,C) \geq S(B,C)$.

Definition 1.3.21

Let $A_F = \{(x, T_{A_F}(x), I_{A_F}(x), F_{A_F}(x)) \mid x \in X\}$,$B_F = \{(x, T_{A_F}(x), I_{A_F}(x), F_{A_F}(x)) \mid x \in X\}$ be any two Fermatean neutrosophic sets (FNSs). For comparing any two FNSs, a comparison method is developed as follows:

If Score (A_F) < Score (B_F) then $A_F < B_F$

If Score (A_F) > Score (B_F) then $A_F > B_F$If Score (A_F) = Score (B_F) then check Accuracy (A_F) in the next step

If Accuracy (A_F) > Accuracy (B_F) Then $A_F > B_F$

If Accuracy (A_F) < Accuracy (B_F) Then $A_F < B_F$

If Accuracy (A_F) = Accuracy (B_F) Then $A_F = B_F$

CHAPTER 2

Chapter 2

2.1 EDUCATIONAL STRATEGIES FOR NURTURING MATHEMATICAL ENTREPRENEURS IN SMEs

The traditional mathematics approach to teaching math often focuses heavily on rote learning, formulas, and standardized exams, rather than real-world applications.

Entrepreneurship is becoming increasingly important in today's economy for several reasons:

- **Job Creation**: Entrepreneurs drive employment by starting new businesses and creating new opportunities.
- **Innovation**: They are a major source of innovation, bringing new ideas, technologies, and products to the market.
- **Economic Growth**: SMEs and startups contribute significantly to GDP and economic development.
- **Self-Reliance**: Encouraging entrepreneurship promotes a mindset of independence and problem-solving.
- **Bridging the Education-Industry Gap**: Entrepreneurs apply knowledge-often including mathematics-to solve real-world problems, thus making education more meaningful and practical.

In the evolving landscape of global entrepreneurship, there is an increasing need to prepare students with the analytical and strategic skills necessary to launch and sustain small and medium-sized enterprises (SMEs). Entrepreneurship education, when integrated with mathematical thinking and quantitative tools, can empower students to make data-driven decisions, assess risks, optimize resources, and design sustainable business models. This approach referred to as mathematical entrepreneurship bridges the gap between theoretical knowledge and practical business applications.

Using quantitative metrics to evaluate SMEs eco-efficiency and sustainability offers clear, actionable benefits. These metrics provide a solid foundation for assessing performance, identifying inefficiencies, and guiding better resource management. By applying mathematical models, we not only streamline decision-making but also enhance the precision and impact of sustainability initiatives. It ensures that every decision is data-driven and strategically aligned with long-term environmental goals.

Indian SMEs are already showing how strategic sustainability practices can be embedded effectively. With a strong focus on planning, innovation, and long-term value creation, many of these enterprises are leveraging sustainability as a growth driver. Importantly, these capabilities-especially strategic thinking and innovation-can and should be nurtured early through education. This early development equips future entrepreneurs and managers with the mindset and tools needed to lead sustainable change from the ground up.

In traditional entrepreneurship education, the emphasis often lies on creativity, ideation, and soft skills such as leadership and communication. While these are undoubtedly important, there is a growing recognition that entrepreneurial success – particularly in the context of SMEs – also demands a strong foundation in analytical and quantitative skills. Financial forecasting, cost-benefit analysis, market demand estimation, risk management, and optimization of operations are just few areas where mathematical tools and models play a vital role. Integrating these concepts into entrepreneurship curricula can enable students to approach business development with precision and strategic clarity.

Moreover, the increasing emphasis on sustainability and responsible business practices calls for a new generation of entrepreneurs who are not only innovative but also conscious of environmental and social impacts. Mathematical models can support sustainable decision-making by enabling students to assess trade-offs, quantify environmental footprints, and simulate long-term outcomes. For example, teaching students how to calculate eco-efficiency or perform life cycle assessments can deepen their understanding of sustainable resource use and operational efficiency in SMEs.

Embedding mathematical entrepreneurship into educational institutions also has the potential to democratize access to entrepreneurship. By providing students with structured, data-driven frameworks, educators can lower the perceived risk of starting a business and foster confidence among those who may not see themselves as “natural” entrepreneurs. Through problem-based learning, case studies, simulations, and real-world project work, students can mathematical reasoning to develop viable business plan, evaluate market opportunities, and make informed strategic decisions.

Mathematical entrepreneurship must be effectively taught in educational institutions to cultivate entrepreneurial mindsets among students. Integrating mathematical modeling, sustainability principles, and strategic thinking equips future entrepreneurs with the necessary tools and confidence to establish and manage successful SMEs. It enhances students ability to think analytically and strategically, enabling them to tackle real-world challenges with innovative and sustainable business solutions.

2.2 EDUCATIONAL TACTICS

It is essential to prepare students not just academically, but also with practical life skills that foster independence, creativity, and leadership. One of the most effective ways to do this is by nurturing entrepreneurial thinking from a young age. The school environment offers the perfect platform to introduce students to spirit of innovation, problem-solving, and value creation through well-planned educational strategies. As such, integrating entrepreneurial thinking into math education is not just innovative - it's essential. The following are key educational tactics that can be used to encourage entrepreneurial skills through mathematics in school students.

KEY EDUCATIONAL TACTICS TO ENCOURAGE ENTREPRENEURIAL SKILLS THROUGH MATHEMATICS IN SCHOOL STUDENTS

Mathematics plays a crucial role in entrepreneurial success, particularly in financial planning, budgeting, pricing, and data analysis. By integrating mathematics learning with entrepreneurial concepts, educators can help students in middle and high school build a strong foundation for future ventures.

Some concepts are

1. Business-Based Math Projects
2. Budgeting and -Financial Planning Lesson
3. Real-Life Simulations and Role Plays
4. Math Integrated with Marketing
5. Using Technology and Spreadsheets
6. Problem-Solving Challenges

The following concepts highlight practical ways to integrate mathematics education into entrepreneurship. Each idea is briefly explained to show how it can inspire creativity, problem-solving, and real-world thinking in students.

BUSINESS-BASED MATH PROJECTS

Hand-on projects that simulate real businesses can help students apply mathematical concepts in practical ways. For example, a school store or class-run snack counter gives students the chance to use arithmetic for pricing, calculate profits, and understanding basic accounting. These activities make math tangible and teach responsibility and decision-making in a fun and interactive way.

BUDGETING AND FINANCIAL PLANNING LESSONS

Learning how to manage money is a core entrepreneurial skill. Teachers can guide students in creating simple budgets for mock businesses or personal savings goals. Lessons can conclude calculating income and expenses, distinguishing between needs and wants, and analyzing how small financial choices affect larger outcomes. These activities introduce foundational knowledge in financial literacy and smart money habits.

REAL-LIFE SIMULATIONS AND ROLE PLAYS

Role-playing as business owners or team members in a startup allows students to experience world while using math. Whether calculating wages, processing sales, or predicting expenses, students use math to solve realistic challenges. These simulations also improve communication, teamwork, and strategic thinking – key components of successful entrepreneurship.

MATH INTEGRATED WITH MARKETNG

Marketing strategies often depend on understanding numbers, trend, and customer behaviour. By using basic statistics and data interpretation, students can analyze survey results, track product performance, and identify preferences. Teaching how to create and interpret bar graph, pie charts, and percentage changes helps them make data-driven decisions in their mini-businesses.

USING TECHNOLOGY AND SPREADSHEETS

Introducing students to digital tools like Excel or Google Sheets prepares them for modern business environments. They can use spreadsheets to record sales, apply formulas, to calculate profits, and generate financial reports. These skills help students become comfortable with data management and provide a realistic look into how businesses track grow their finances.

PROBLEM-SOLVING CHALLENGES

Entrepreneurship is rooted in solving problems creatively and efficiently. Assign students math-based challenges such as optimizing a product with limited resources, determining a pricing strategy to achieve a profit goal, or balancing a budget with constraints. These exercises encourage logical thinking, flexibility, and practical application of math concepts.

2.3 TEACHING STUDENTS TO START A BUSINESS IN SMES AND EXPLORING FUTURE SCOPE

Teaching students how to start a business within the SME (Small and Medium Enterprises) sector begins with instilling an entrepreneurial mindset and providing them with foundational knowledge of business operations. At the initial stage, educators can guide students through the basics of identifying market needs, generating business ideas, preparing business plans, and understanding financial management on a small scale.

Practical teaching methods such as business simulations, case studies, field visits to local SMEs, and mentorship from real entrepreneurs can help students visualize the real-world scenario of starting and managing a small business. They can be encouraged to begin with micro-projects such as home-based businesses, online reselling, handmade products, or simple service startups to gain firsthand experience.

As they grow in knowledge and confidence, the future scope expands significantly - students can scale their ventures, attract investors, adopt digital tools, and even tap into international markets. With increasing support from government schemes and startup incubators young entrepreneurs have a promising path to turn their small business ideas into successful, sustainable enterprises that contribute to economic growth and job creation.

Teaching entrepreneurship through SMEs not only equips students with business skills but also prepares them for a future of innovation, self-employment, and leadership.

CHAPTER 3

Chapter 3

A Fermatean Neutrosophic TOPSIS-Based Framework for Evaluating Strategic Decisions in SMEs

SMEs often operate with limited data, rapidly changing markets, and unclear outcomes. Traditional decision-making methods may fail to capture these ambiguous realities. As a result, it becomes essential to adopt advanced decision-making frameworks that can accommodate the indeterminate nature of real-world business scenarios. To address this complexity, a Multi-Criteria Decision-Making (MCDM) approach was employed, A wide range of Multi-Criteria Decision-Making (MCDM) methods have been developed to address various problem types, data structures, uncertainty levels, and decision-maker preferences. Among these, the Fermatean Neutrosophic Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method was selected. This method is well-suited for environments where information is incomplete, imprecise, or inconsistent, as it incorporates three membership degrees-truth, indeterminacy, and falsity-using Fermatean logic to better represent expert judgment. By applying this model, SMEs can enhance their decision-making capabilities, prioritize strategic options more effectively, and make robust choices even under conditions of high uncertainty.

The **Fermatean Neutrosophic Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)** method approach that effectively addresses uncertainty, vagueness, and imprecise information in decision environments. It extends the classical TOPSIS method by integrating Fermatean Neutrosophic Sets (FNS), which represent each evaluation with three membership degrees: truth (T), indeterminacy (I), and falsity (F), where the sum of the cubes of these values is less than or equal to 2. The core concept of this method is to identify the alternative that is closest to the positive-ideal solution (the best possible performance across all criteria) and farthest from the negative-ideal solution (the worst performance).

SMEs frequently rely on qualitative assessments and expert opinions rather than hard data, and the Fermatean Neutrosophic framework allows for more realistic representation of these evaluations. When combined with the TOPSIS method, which ranks alternatives based on their closeness to ideal solutions, it enables SMEs to make more informed and reliable decisions across multiple conflicting criteria. This is especially valuable in areas such as supplier selection, risk management, and strategic planning, where decisions must be made with limited resources and high uncertainty.

When integrated with the TOPSIS method, it enables SMEs to prioritize and rank alternatives more accurately, supporting better choices in areas like supplier selection, investment decisions, technology adoption, or strategic planning. This method is especially valuable in environments where precision is difficult to achieve, yet clear, rational decisions are essential for competitiveness and growth.

3.1 FERMATEAN NEUTROSOPHIC TOPSIS

Fermatean Neutrosophic TOPSIS is an advanced decision-making method that combines the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) with the Fermatean Neutrosophic Set (FNS) theory. It is used to rank alternatives in multi-criteria decision-making (MCDM) problems under uncertainty, imprecision, and incomplete information**.**

The MCDM Fermatean Neutrosophic TOPSIS approach is explained in the following section presents the step-by-step algorithm used to implement the Fermatean Neutrosophic TOPSIS method for ranking the alternatives under consideration.

ALGORITHM

STEP 1:

Construction of the aggregated single valued Neutrosophic decision matrix based on decision makers assessments

$$D = (d_{ij})_{1\le i\le n, 1\le j\le m} = (T_{ij}, I_{ij}, F_{ij})_{1\le i\le n, 1\le j\le m}$$

Where T_{ij} denote truth, I_{ij} indeterminacy and F_{ij} falsity membership score of preference i with respect to criterion j in single valued Neutrosophic.

W = $(\omega_1, \omega_2, \ldots, \omega_n)$ with a single valued Neutrosophic weight of criteria (so $\omega_i = (a_i, b_i, c_i)$).

STEP 2:

Aggregation of the weighted Neutrosophic decision matrix

$$D^{W} = D \otimes W = (d_{ij})_{1 \le i \le n, 1 \le j \le m} = (T_{ij}, I_{ij}, F_{ij})_{1 \le i \le n, 1 \le j \le m}$$

The Fermatean Neutrosophic Weighted Averaging (FNWA) operator is given below

$$r_{ij} =$$

$$\left(\sqrt[3]{1 - \prod_{k=1}^{d}(1 - T^3{}_{jk})^{w_k}}, \sqrt[3]{\prod_{k=1}^{d}(I^3{}_{jk})^{w_k}}, \sqrt[3]{\prod_{k=1}^{d}\left(1 - T^3{}_{jk}\right)^{w_k} - \prod_{k=1}^{d}\left(1 - T^3{}_{jk} - F^3{}_{jk}\right)^{w_k}}\right)$$

STEP 3:

Determination of the relative Neutrosophic positive ideal solution (RNPIS) and the relative negative ideal solution (RNIS) for SVNSs**.**

$$T_j^{w+} = \{(max_i\{T_{ij}^{w_i} | j \in B\}), (min_i\{T_{ij}^{w_i} \ | j \ \in C\})\}$$

$$Q_N^{+} = (d_1^{w+}, d_2^{w+}, \ldots, d_n^{w+})$$

$$T_j^{w+} = \{(max_i\{T_{ij}^{w_i} | j \in B\}), (min_i\{T_{ij}^{w_i} \ | j \ \in C\})\}$$

$$I_j^{w+} = \{(min_i\{I_{ij}^{w_i} | j \in B\}), (max_i\{I_{ij}^{w_i} \ | j \ \in C\})\}$$

$$F_j^{w+} = \{(min_i\{F_{ij}^{w_i} | j \in B\}), (max_i\{F_{ij}^{w_i} \ | j \ \in C\})\}$$

$$Q_N^{-} = (d_1^{w-}, d_2^{w-}, \ldots, d_n^{w-})$$

$$T_j^{w-} = \{(min_i\{d_{ij}^{wj} | j \in B\}), (max_i\{T_{ij}^{w_i} \ | j \ \in C\})\}$$

$$I_j^{w-} = \{(max_i\{I_{ij}^{w_i} | j \in B\}), (min_i\{I_{ij}^{w_i} \ | j \ \in C\})\}$$

$$F_j^{w-} = \{(max_i\{F_{ij}^{w_i} | j \in B\}), (min_i\{F_{ij}^{w_i} \ | j \ \in C\})\}$$

Where sets B and C are associated with the benefit and cost attribute sets, respectively.

STEP 4:

Determination of the distance measure of each alternative from the RNPIS and the RNNIS for SVNSS.

Euclidean distance between two FN numbers
$\beta_1 = G(T_{\beta_1}, I_{\beta_1}, F_{\beta_1})$ and $\beta_2 = G(T_{\beta_2}, I_{\beta_2}, F_{\beta_2})$ are shown as follows:

$$E_{+}(\beta_1, \beta_2) = \sqrt{\frac{1}{3}\left(\left(\left|T_{\beta_1}{}^3 - T_{\beta_2}{}^3\right|\right)^2 + \left(\left|I_{\beta_1}{}^3 - I_{\beta_2}{}^3\right|\right)^2 + \left(\left|F_{\beta_1}{}^3 - F_{\beta_2}{}^3\right|\right)^2\right)}$$

$$E_{-}(\beta_1, \beta_2) = \sqrt{\frac{1}{3}\left(\left(\left|T_{\beta_1}{}^3 - T_{\beta_2}{}^3\right|\right)^2 + \left(\left|I_{\beta_1}{}^3 - I_{\beta_2}{}^3\right|\right)^2 + \left(\left|F_{\beta_1}{}^3 - F_{\beta_2}{}^3\right|\right)^2\right)}$$

STEP 5:

Determination of the relative closeness coefficient to the Neutrosophic ideal solution for SVNSs.

$$C_i^* = \frac{NS_i^-}{NS_i^+ + NS_i^-} \; ; \; i = 1,2\ldots, m$$

A set of alternatives can now be ranked according to the descending order of the value of C_i^* .

ILLUSTRATION

A sustainable entrepreneurship committee wants to evaluate and rank four potential small and medium-sized enterprises (SMEs) based on their performance in sustainability-oriented dimensions. The SMEs are evaluated under 9 criteria using Fermatean Neutrosophic Sets. The alternatives are A1, A2, A3, A4 and the 9 evaluation criteria are C1, C2, C3, C4, C5, C6, C7, C8, C9.

The Alternatives are

A1: The use of renewable energy sources

A2: Reusing materials/raw materials

A3: The use of energy-saving devices

A4: Recycling of broken, defective, and used products

The Criteria are

C1: Energy saved

C2: Amount of materials reused

C3: Use of clean/renewable energy

C4: Products successfully recycled

C5: Positive impact on the environment

C6: Initial setup

C7: Operation and maintenance expenses

C8: Waste generation rate

C9: Time required to implement

Each alternative is rated using Fermatean Neutrosophic Numbers (T, I, F), where the cube sum $T^3 + I^3 + F^3 \leq 2$. Using the Fermatean Neutrosophic TOPSIS method

Rank the alternatives from most to least suitable for sustainable entrepreneurship investment.

3.1.2 A FRAMEWORK FOR FERMATEAN NEUTROSOPHIC TOPSIS

Let decision-maker take part in the procedure for evaluation. Each criterion has been weighted to reflect its significance within the overall assessment framework.

In the considered criteria, indeterminacy can manifest in various forms. That may include ambiguous definitions of criteria, challenges with trade-offs and waiting, uncertain future performance, and uncertain preferences.

Based on this assumption of ambiguity, the judgements made by the decision-maker compiled using the linguistics variables listed in Table 3.1.2.1.

Table 3.1.2.1. Terms used in linguistics and their associated Spherical Neutrosophic Number

Linguistic Terms	(T, I, F)
Extremely High (EH)	(0.9, 0.6, 0.4)
Very High (VH)	(0.8, 0.7, 0.5)
High (H)	(0.8, 0.6, 0.5)
Above Average (AA)	(0.7, 0.7, 0.6)
Average (A)	(0.7, 0.6, 0.6)
Low (L)	(0.6, 0.7, 0.7)
Very Low (VL)	(0.6, 0.6, 0.7)

The following tables 3.1.2.2 elucidates the decisions with respect to the decision makers.

Table 3.1.2.2. Decisions of DM

ALTERNATIVE	A1	A2	A3	A4
C1	EH	VH	A	L
C2	H	A	EH	VL
C3	AA	EH	VL	H
C4	EH	AA	H	L
C5	VH	VL	A	EH
C6	A	VH	EH	VL
C7	L	EH	H	A
C8	A	L	EH	H
C9	H	AA	VL	EH

STEP 1:

Construction of the aggregated single valued Neutrosophic decision matrix based on decision makers assessments

$$D = (d_{ij})_{1 \le i \le n, 1 \le j \le m} = (T_{ij}, I_{ij}, F_{ij})_{1 \le i \le n, 1 \le j \le m}$$

Table 3.1.2.3. Aggregate Single Valued Neutrosophic Decision Matrix

ALTERNATIVE	**A1**	**A2**	**A3**	**A4**
C1	(0.9, 0.6, 0.4)	(0.8, 0.7, 0.5)	(0.7, 0.6, 0.6)	(0.6, 0.7, 0.7)
C2	(0.8, 0.6, 0.5)	(0.7, 0.6, 0.6)	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C3	(0.7, 0.7, 0.6)	(0.9, 0.6, 0.4)	(0.6, 0.6, 0.7)	(0.8, 0.6, 0.5)
C4	(0.9, 0.6, 0.4)	(0.7, 0.7, 0.6)	(0.8, 0.6, 0.5)	(0.6, 0.7, 0.7)
C5	(0.8, 0.7, 0.5)	(0.6, 0.6, 0.7)	(0.7, 0.6, 0.6)	(0.9, 0.6, 0.4)
C6	(0.7, 0.6, 0.6)	(0.8, 0.7, 0.5)	(0.9, 0.6, 0.4)	(0.6, 0.6, 0.7)
C7	(0.6, 0.7, 0.7)	(0.9, 0.6, 0.4)	(0.8, 0.6, 0.5)	(0.7, 0.6, 0.6)
C8	(0.7, 0.6, 0.6)	(0.6, 0.7, 0.7)	(0.9, 0.6, 0.4)	(0.8, 0.6, 0.5)
C9	(0.8, 0.6, 0.5)	(0.7, 0.7, 0.6)	(0.6, 0.6, 0.7)	(0.9, 0.6, 0.4)

STEP 2:

Aggregation of the weighted Neutrosophic decision matrix

$$D^W = D \otimes W = (d_{ij})_{1 \le i \le n, 1 \le j \le m} = (T_{ij}, I_{ij}, F_{ij})_{1 \le i \le n, 1 \le j \le m}$$

Table 3.1.2.4. Weights

ALTERNATIVE	WEIGHTS
A1	(0.06,0.08.0.08)
A2	(0.16,0.11,0.06)
A3	(0.04,0.24,0.16)
A4	(0.06,0.08,0.08)
A5	(0.16,0.11,0.06)
A6	(0.04,0.24,0.16)
A7	(0.06,0.08,0.80)
A8	(0.16,0.11,0.06)
A9	(0.04,0.24,0.16)

The Fermatean Neutrosophic Weighted Averaging (FNWA) operator is given below

$$r_{ij} = \left(\sqrt[3]{1 - \prod_{k=1}^{d} (1 - T^3{}_{jk})^{w_k}}, \sqrt[3]{\prod_{k=1}^{d} (I^3{}_{jk})^{w_k}}, \sqrt[3]{\prod_{k=1}^{d} \left(1 - T^3{}_{jk}\right)^{w_k} - \prod_{k=1}^{d} \left(1 - T^3{}_{jk} - F^3{}_{jk}\right)^{w_k}} \right)$$

Table 3.1.2.5. Weighted Averaging

ALTERNATIVE	WEIGHT AVERAGING
A1	(0.75, 0.55, 0.57)
A2	(0.69, 0.58, 0.67)
A3	(0.79, 0.53, 0.52)
A4	(0.74, 0.53, 0.60)

STEP 3:

Determination of the relative Neutrosophic positive ideal solution (RNPIS) and the relative negative ideal solution (RNIS) for SVNSs**.**

$$T_j^{w+} = \{(max_i\{T_{ij}^{w_i}|j \in B\}), (min_i\{T_{ij}^{w_i} \ |j \ \in C\})\}$$

$$Q_N^+ \ = \ (d_1^{w+}, d_2^{w+}, \dots, d_n^{w+})$$

$$T_j^{w+} = \{(max_i\{T_{ij}^{w_i}|j \in B\}), (min_i\{T_{ij}^{w_i} \ |j \ \in C\})\}$$

$$I_j^{w+} = \{(min_i\{I_{ij}^{w_i}|j \in B\}), (max_i\{I_{ij}^{w_i} \ |j \ \in C\})\}$$

$$F_j^{w+} = \{(min_i\{F_{ij}^{w_i}|j \in B\}), (max_i\{F_{ij}^{w_i} \ |j \ \in C\})\}$$

$$Q_N^- \ = \ (d_1^{w-}, d_2^{w-}, \dots, d_n^{w-})$$

$$T_j^{w-} = \{(min_i\{d_{ij}^{w_j}|j \in B\}), (max_i\{T_{ij}^{w_i} \ |j \ \in C\})\}$$

$$I_j^{w-} = \{(max_i\{I_{ij}^{w_i}|j \in B\}), (min_i\{I_{ij}^{w_i} \ |j \ \in C\})\}$$

$$F_j^{w-} = \{(max_i\{F_{ij}^{w_i}|j \in B\}), (min_i\{F_{ij}^{w_i} \ |j \ \in C\})\}$$

Where sets B and C are associated with the benefit and cost attribute sets, respectively.

Table 3.1.2.6. RNIPS and RNNIS

ALTERNATIVE	RNIPS	RNNIS
C1	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C2	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C3	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C4	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C5	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C6	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C7	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C8	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)
C9	(0.9, 0.7, 0.4)	(0.6, 0.6, 0.7)

STEP 4:

Determination of the distance measure of each alternative from the RNPIS and the RNNIS for SVNSS.

Euclidean distance between two FN numbers

$\beta_1 = G(T_{\beta_1}, I_{\beta_1}, F_{\beta_1})$ and $\beta_2 = G(T_{\beta_2}, I_{\beta_2}, F_{\beta_2})$ are shown as follows:

$$D_{+}(\beta_1, \beta_2) = \sqrt{\frac{1}{3}\left(\left(\left|{T_{\beta_1}}^3 - {T_{\beta_2}}^3\right|\right)^2 + \left(\left|{I_{\beta_1}}^3 - {I_{\beta_2}}^3\right|\right)^2 + \left(\left|{F_{\beta_1}}^3 - {F_{\beta_2}}^3\right|\right)^2\right)}$$

$$D_{-}(\beta_1, \beta_2) = \sqrt{\frac{1}{3}\left(\left(\left|{T_{\beta_1}}^3 - {T_{\beta_2}}^3\right|\right)^2 + \left(\left|{I_{\beta_1}}^3 - {I_{\beta_2}}^3\right|\right)^2 + \left(\left|{F_{\beta_1}}^3 - {F_{\beta_2}}^3\right|\right)^2\right)}$$

TABLE 3.1.2.7. Distance Measure D+ AND D-

ALTERNATIVE	D+	D-
A1	0.61	0.64
A2	0.67	0.61
A3	0.65	0.68
A4	0.76	0.58

STEP 5:

Determination of the relative closeness coefficient to the Neutrosophic ideal solution for SVNSs.

$$C_i^* = \frac{NS_i^-}{NS_i^+ + NS_i^-} \; ; \; i = 1,2\ldots,m$$

A set of alternatives can now be ranked according to the descending order of the value of C_i^* .

Table 3.1.2.8. Relative Closeness Coefficient

ALTERNATIVE	CLOSENESS COEFFICIENT
A1	0.51
A2	0.48
A3	0.51
A4	0.43

Table 3.1.2.9 Ranking

ALTERNATIVE	CLOSENESS COEFFICIENT	RANK
A1	0.51	1
A2	0.48	3
A3	0.51	2
A4	0.43	4

From the table 3.1.2.9. the result obtained

A1 > A3 > A2 > A4

A1: The use of renewable energy sources achieved the highest closeness coefficient and was ranked first among the four SMEs.

The decision-making process involved evaluating multiple alternatives under conditions of significant uncertainty and imprecise expert judgments. The initial analysis was conducted using a traditional ranking method; however, due to the presence of high indeterminacy in the expert evaluations, it was necessary to adopt a more flexible and uncertainty-tolerant approach.

Table 3.1.2.10 Fuzzy Rank

ALTERNATIVE	TOTAL SCORE	RANK
A1	0.064343	4
A2	0.241287	1
A3	0.080429	3
A4	0.160858	2

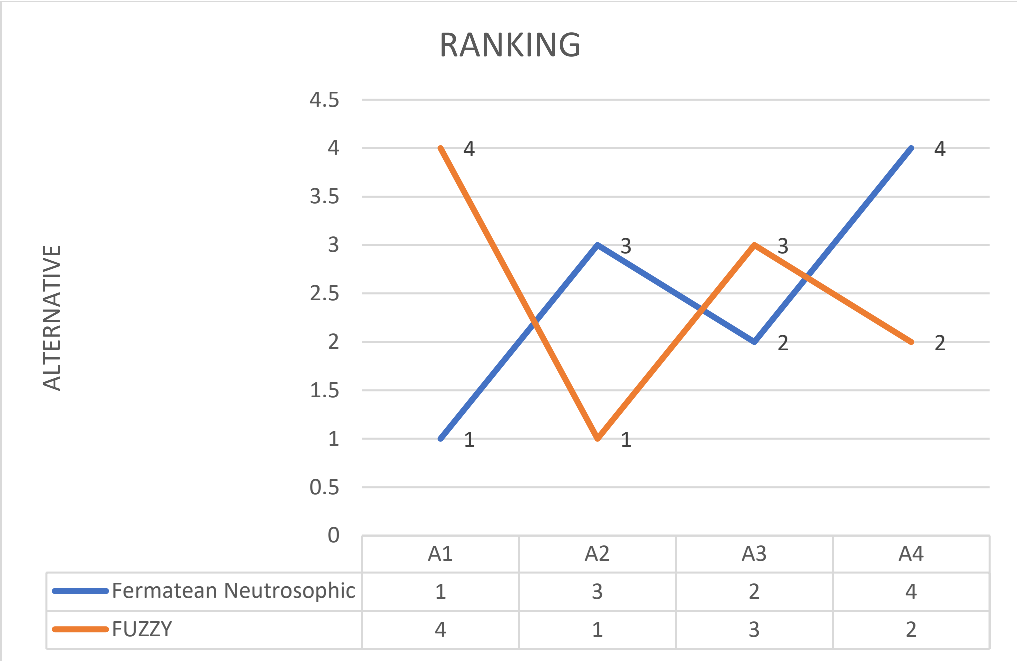
A paper related to SME decision-making using fuzzy sets was referred to as a baseline. The same set of evaluation values, criteria, and alternatives from the original fuzzy model were considered to maintain consistency and enable a meaningful comparison. However, instead of applying the traditional fuzzy set approach, this study utilized Fermatean Neutrosophic Sets to represent the data. The motivation behind this shift lies in the Fermatean model's enhanced ability to express higher degrees of indeterminacy, which is often encountered in SME decision-making due to limited data and expert hesitation.

By transforming the same fuzzy input values into Fermatean Neutrosophic terms, the analysis captured a more realistic level of uncertainty. Subsequently, the Fermatean Neutrosophic TOPSIS method was applied, leading to a revised ranking of alternatives.

The differences in rankings are presented in the graph below, where the coloured lines visually represent the shifts and highlight the impact of accounting for higher indeterminacy in the decision-making process.

The comparison between the two methods is illustrated in the graph below, where the **blue line** represents the rankings obtained using the Fermatean Neutrosophic approach, and the **red line** shows the rankings from the fuzzy sets approach. The variations between the two highlight how considering higher indeterminacy can influence decision outcomes in SMEs.

GRAPH



Comparison between Fuzzy and Fermatean Neutrosophic TOPSIS

The comparison between the two approaches reveals significant differences in the final rankings of the alternatives. In the existing paper, **A2: Reusing materials/raw materials** was ranked highest, whereas in the Fermatean Neutrosophic TOPSIS approach applied in this study, **A1: The use of renewable energy sources** emerged as the top-ranked alternative. This change highlights the influence of incorporating a higher degree of indeterminacy in the decision-making process. Fermatean Neutrosophic Sets provide a more flexible and realistic representation of expert judgment. Therefore, the Fermatean Neutrosophic TOPSIS method offers a more robust and reliable alternative for decision-making in uncertain environments, ultimately leading to more informed and context-sensitive outcomes.

SUMMARY AND CONCLUSION

The findings of this study underscore the critical importance of integrating mathematics education with entrepreneurial training from an early age. Mathematics develops essential skills such as logical reasoning, problem-solving, risk analysis, and strategic planning-all of which are vital for entrepreneurial success. In the context of SMEs, where resource optimization, financial management, and data-driven decision-making are crucial for survival and growth, a solid mathematical background provides a significant advantage. Early exposure to mathematical thinking not only enhances students ability to navigate uncertainty and complexity but also empowers them to design sustainable and scalable business models. By fostering mathematical competencies alongside entrepreneurial skills, students are better prepared to launch, manage, and sustain successful SMEs in an increasingly competitive and dynamic environment.

A sustainability model for SMEs was developed to explain and demonstrate to students the practical application of mathematical decision-making in entrepreneurship. The Fermatean Neutrosophic TOPSIS method was employed to evaluate and rank four key alternatives based on important sustainability criteria. This approach allows for handling uncertainty and expert subjectivity effectively, providing a realistic decision-making framework for future entrepreneurs. By evaluating four alternatives based on critical sustainability criteria, the method provided a clear and structured ranking of options. The results identified **A1: The use of renewable energy sources** as the top-ranked alternative, highlighting its superior alignment with sustainability goals and long-term business viability. **A3: The use of energy-saving devices** and **A2: Reusing materials/raw materials** followed, while **A4: Recycling of broken, defective, and used products** ranked lowest, showing room for improvement in several key areas such as scalability and innovation.

Encouraging students to prioritize high-impact alternatives - like investing in renewable energy - instills a deeper understanding of sustainable entrepreneurship and prepares them to create SMEs that are not only profitable but also socially and environmentally responsible. Through this approach, students may be empowered to make strategic and informed choices that support both their entrepreneurial ambitions and broader societal needs.

By engaging with these analytical and logical thinking approaches early in their education, students can better understand what drives successful, sustainable businesses and are more likely to develop the skills and confidence to launch their own ventures. The ability to apply structured reasoning and sustainability principles from a young age enhances their readiness to enter the entrepreneurial ecosystem, positioning them to contribute meaningfully to economic and environmental progress.

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