Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD

Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC

Coimbatore-641 043, Tamil Nadu, India

Continuous Internal Assessment Test II – October 2024

Semester V

Class: III UG **Time: 2 Hrs**

Branch: Mathematics/ Special Education and Mathematics **Max. Marks: 60**

21BMAC15 / 21BSMC15 – Complex Analysis I

Course Outcomes:

CO1: Operate complex derivatives of a function.

CO2: Determine analyticity of a function using C-R equations.

CO3: Construct analytic functions.

CO4: Analyze bilinear transformations.

CO5: Apply Cauchy's theorem and Cauchy's integral formula to evaluate integrals.

Part A **6x1=6**

Choose the Correct Answer

1. The harmonic conjugate of $u = x^2 - y^2$ is _____. CO3K2

a. 2xy b. $\frac{xy}{2}$ c. $-2xy$ d. $-\frac{xy}{2}$

2. The value of m such that $2x-x^2 + my^2$ may be harmonic is _____ CO3K2

a. 1 b. 2 c. 3 d. 4

3. The transformation $w = \frac{1}{z}$ is _____. CO4K1

a. translation b. homothetic transformation

c. rotation d. inversion

4. The fixed points of the transformation $w = az$ are _____. CO4K2

a. $0, \infty$ b. $0, -\infty$ c. a, ∞ d. $a, -\infty$

5. An integral along a simple closed curve is called _____. CO5K1

a. Contour integral b. Jordan curves

c. Multiple point d. Simple closed curve

6. The value of the integral $\int_C \frac{dz}{2z-3}$ where C is |z| = 1. C05K2

a. 0 b. 3 c. $2\pi i$ d. $-2\pi i$

Part B **3x6=18**

Answer ALL questions

7**.** a. Find the analytic function $f(z)$ if $u(x,y) = \frac{x}{x^2+y^2}$ by substitution method. C03K4

(or)

7. b. Find the analytic function f(z) = u+iv, given that $u-v = e^x (\cos y - \sin y)$. C03K3

8. a. Find the image of the circle | z+2i | = 2 under the transformation $w = \frac{1}{z}$. C04K3

(or)

8. b. Prove that the bilinear transformation which transforms z_1, z_2, z_3 is w_1, w_2, w_3 CO4K2

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}.$$

9. a. State and prove Cauchy's fundamental theorem. CO5K4

(or)

9. b. Evaluate $\int_C \frac{1}{(z^2+4)^2} dz$, where C is | z-i | = 2. CO5K3

Part C **3x12=36**

Answer ALL questions

10. a. Find the analytical functions, $f(z) = u + iv$ given that $u - v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. CO3K5

(or)

10. b. Find the analytic functions $f(z) = u + iv$, given that CO3K5

$(a) - 2v(x, y) = e^x(\cos y - \sin y)$

$(b) 2u + v = e^x(\cos y - \sin y)$

$(c) u - 2v = e^x(\cos y - \sin y)$

11. a. Prove that the points which map the real axis onto the circle $|w| = 1$ are of the form $w = e^{i\lambda} \frac{z - z_1}{\overline{z - z_1}}$, where λ is real. CO4K5

(or)

11.b. Find the bilinear transformation which maps the points 1, i, -1 onto the points 0, 1, ∞. Show that the transformation maps the interior of the unit circle of the z plane onto the upper half of the w plane. CO4K3

12. a. If C is the circle |z| = 3, evaluate $\int_C \frac{e^z}{(z+1)^2(z+2)} dz$. CO5K4

(or)

12. b. State and prove Cauchy's integral formula for derivatives. CO5K5

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