Chapter 8

Soft J-Closed, Soft J*-Closed and Soft J**-Closed sets in Soft Topological spaces

§ 8.1. Introduction

In 1999, Molodtsov presented the concept of soft set theory as a mathematical tool for dealing with uncertainties. He built up the fundamental results of this new soft set theory and effectively applied the soft set theory into a few directions, for example smoothness of functions, operations research, Riemann integration, game theory, theory of probability etc. Shabir and Naz characterized soft topology by using soft sets and concentrated some basic notations of soft topological spaces such as soft open and closed sets, soft subspace, soft closure, soft neighbourhood of a point, soft separation axioms. After then numerous authors presented some of basic concepts and properties of soft topological spaces.In the year 1970,Levine introduced g-closed and g-open sets in topological spaces.Kannan(2012) characterized soft g-closed and soft g-open sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. In 2016,Annalakshmi has introduced regular*-open sets in topological spaces. In this Chapter, the theory of J-closed sets is extended to soft topology. soft regular*-open sets, soft regular*-closed sets,soft η*-open sets, soft η*-closed sets,soft J-closed sets,soft J*-closed sets and soft J**-closed sets are introduced and related properties are discussed.

§ 8.2. Soft Regular*-Open Sets

Definition 8.2.1. A soft subset (F,E) of a soft topological space (Y,ζ,E) is said to be a **soft regular*-open (soft r*-open) set** if (F,E) = Int(Cl^*(F,E)).Here Cl^*(F,E) = ∩{(O,E): (O,E) is soft g-closed and (F,E) $\tilde{\subseteq}$(O,E)}.

Notation 8.2.2. The set of all soft regular*-open sets (soft r*-open set) in (Y,ζ,E) is denoted by SR*O(Y,ζ,E).

Example 8.2.3. Let Y={a,b},E ={e_1,e_2}.Define $(F,E)_1$ = {(e_1,∅),(e_2, ∅)},$(F,E)_2$ = {(e_1,∅),(e_2,{a})},$(F,E)_3$ = {(e_1,∅),(e_2,{b})},$(F,E)_4$ = {(e_1,∅),(e_2,{a,b})}, $(F,E)_5$ =

$\{(e_1,\{a\}),(e_2, \emptyset)\}$,$(F,E)_6$ = $\{(e_1,\{a\}),(e_2,\{a\})\}$,$(F,E)_7$ = $\{(e_1,\{a\}),(e_2,\{b\})\}$,$(F,E)_8$ = $\{(e_1,\{a\}),(e_2,\{a,b\})\}$,$(F,E)_9$ = $\{(e_1,\{b\}),(e_2, \emptyset)\}$,$(F,E)_{10}$ = $\{(e_1,\{b\}),(e_2,\{a\})\}$,$(F,E)_{11}$ = $\{(e_1,\{b\}),(e_2,\{b\})\}$,$(F,E)_{12}$ = $\{(e_1,\{b\}),(e_2,\{a,b\})\}$,$(F,E)_{13}$ = $\{(e_1,\{a,b\}),(e_2, \emptyset)\}$,$(F,E)_{14}$ = $\{(e_1,\{a,b\}),(e_2,\{a\})\}$,$(F,E)_{15}$ = $\{(e_1,\{a,b\}),(e_2,\{b\})\}$,$(F,E)_{16}$ = $\{(e_1,\{a,b\}),(e_2,\{a,b\})\}$.Then soft topology ζ is $\{(F,E)_1, (F,E)_2, (F,E)_3, (F,E)_4,(F,E)_{16}\}$.We obtain SR*O(Y,ζ,E)= $\{(F,E)_1, (F,E)_2, (F,E)_3, (F,E)_4,(F,E)_{16}\}$.

Remark 8.2.4. In any soft topological space (Y,ζ,E), ∅ and Y are soft r*-open sets.

Remark 8.2.5. The union of two soft r*-open sets need not be soft r*-open as seen from the following Counter Example.

Counter Example 8.2.6. Let Y={a,b},E ={e_1,e_2}.Define $(F,E)_1$ = $\{(e_1,\emptyset),(e_2, \emptyset)\}$,$(F,E)_2$ = $\{(e_1,\emptyset),(e_2,\{a\})\}$,$(F,E)_3$ = $\{(e_1,\emptyset),(e_2,\{b\})\}$, $(F,E)_4$ = $\{(e_1,\emptyset),(e_2,\{a,b\})\}$,$(F,E)_5$ = $\{(e_1,\{a\}),(e_2, \emptyset)\}$,$(F,E)_6$ = $\{(e_1,\{a\}),(e_2,\{a\})\}$,$(F,E)_7$ = $\{(e_1,\{a\}),(e_2,\{b\})\}$, $(F,E)_8$ = $\{(e_1,\{a\}),(e_2,\{a,b\})\}$,$(F,E)_9$ = $\{(e_1,\{b\}),(e_2, \emptyset)\}$,$(F,E)_{10}$ = $\{(e_1,\{b\}),(e_2,\{a\})\}$, $(F,E)_{11}$ = $\{(e_1,\{b\}),(e_2,\{b\})\}$,$(F,E)_{12}$ = $\{(e_1,\{b\}),(e_2,\{a,b\})\}$, $(F,E)_{13}$ = $\{(e_1,\{a,b\}),(e_2, \emptyset)\}$,$(F,E)_{14}$ = $\{(e_1,\{a,b\}),(e_2,\{a\})\}$,$(F,E)_{15}$ = $\{(e_1,\{a,b\}),(e_2,\{b\})\}$, $(F,E)_{16}$=$\{(e_1,\{a,b\}),(e_2,\{a,b\})\}$.Then soft topology ζ is $\{(F,E)_1, (F,E)_4, (F,E)_5, (F,E)_8, (F,E)_{16}\}$.In this space(Y,ζ,E), the soft subsets $(F,E)_4$ = $\{(e_1,\emptyset),(e_2,\{a,b\})\}$ and $(F,E)_5$ = $\{(e_1,\{a\}),(e_2, \emptyset)\}$are soft r*-open sets, but $(F,E)_4 \cup (F,E)_5 = \{(e_1,\{a\}),(e_2,\{a,b\})\} = (F,E)_8$ is not soft r*-open.

Lemma 8.2.7. Intersection of any two soft r*-open sets is soft r*-open.

Proof Let (F,A) and (G,B) be soft r*-open sets then (F,A) = Int(Cl^*(F,A)) and (G,B) = Int(Cl*(G,B)). Consider, Int(Cl*((F,A)∩(G,B))) = Int(Cl*(((($(F,A)\cap(G,B))^c)^c$)) = Int((Cl*((($(F,A)^c\cup(G,B)^c))^c$)) [By Proposition 3 of (Shabir,2011)]=Int(Y\Int*((Y\(F,A))∪ (Y\(G,B))) = Int(Cl*(Y\((Y\(F,A))∪ (Y\(G,B))) = Int(Cl*(Y\(Y\(F,A)) ∪ Cl*(Y\(Y\(G,B))) = Int (Cl*(F,A) ∩Cl*(G,B)) = Int(Cl*(F,A) ∩ Int(Cl*(G,B)) = (F,A) ∩ (G,B).Hence (F,A)∩ (G,B) is a soft regular* -open set.

Theorem 8.2.8. SR*O(Y,ζ,E) forms a soft topology on Y if and only if it is closed under arbitrary union.

Proof Follows from **Remark 8.2.4.**,**Remark 8.2.5.** and **Lemma 8.2.7**.

Theorem 8.2.9. Every soft regular open set is soft r*-open.

Proof Let (F,E) be a soft regular - open set then (F,E) = Int(Cl(F,E)).Since (F,E) is soft regular-open, it is soft clopen.(i.e) (F,E) is soft closed and every soft closed set is soft generalized closed, (By Theorem 3.2 of (Kannan,2012)).Hence Cl(F,E) = Cl*(F,E)⇒Int (Cl(F,E))= Int(Cl*(F,E)).Hence (F,E) is soft r*-open.

Remark 8.2.10. Converse of the above Theorem 8.2.9. need not be true, as seen from the following Counter Example .

Counter Example 8.2.11. Consider the space as in **Example 8.2.3.**, in this space (Y,ζ,E), the soft subset $(F,E)_4 = \{(e_1,\emptyset),(e_2,\{a,b\})\}$ is soft r*-open but not soft regular-open.

Theorem 8.2.12. Every soft r*-open set is soft open.

Proof Let (F,E) is soft r*-open then (F,E) = Int(Cl*(F,E)).Now Int(F,E) = Int(Int(Cl*(F,E))) = Int(Cl*(F,E))= (F,E).(i.e) Int(F,E) = (F,E).Hence (F,E) is soft open.

Remark 8.2.13. The converse of the above Theorem 8.2.12. need not be true, as seen from the following Counter Example .

Counter Example 8.2.14. Consider the space as in **Counter Example 8.2.6.**,In this space(Y,ζ,E),Then the soft subset $(F,E)_8 = \{(e_1,\{a\}),(e_2,\{a,b\})\}$ is soft open but not soft r*-open.

Theorem 8.2.15. In any soft topological space (Y,ζ,E), SRO(Y,ζ,E) $\tilde{\subseteq}$ SR*O(Y,ζ,E) $\tilde{\subseteq}$ ζ. That is, the class of soft regular*-open sets is placed between the class of soft regular - open sets and the class of soft open sets.

Proof Follows from **Theorem 8.2.9.** and **Theorem 8.2.12.**

Theorem 8.2.16. Every soft r*-open sets are soft pre-open.

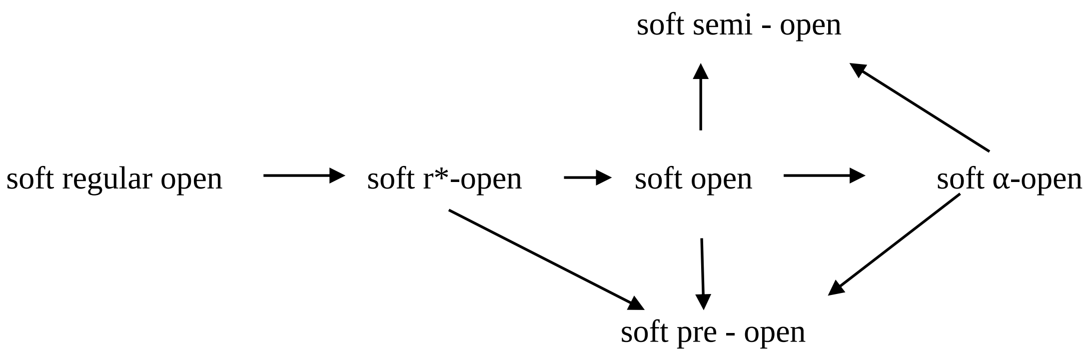
Proof Let (F,E) be a soft r*-open set, then (F,E) = Int(Cl*(F,E)) ---(1).Since Cl*(F,E) $\tilde{\subseteq}$ Cl(F,E).Then Int(Cl*(F,E)) $\tilde{\subseteq}$ Int(Cl(F,E)) ---(2).From (1) and (2) ⇒ (F,E) $\tilde{\subseteq}$ Int(Cl(F,E)). Hence (F,E) is soft pre-open.

Remark 8.2.17. The converse of the above Theorem 8.2.16. need not be true, as seen from the following Counter Example .

Counter Example 8.2.18. Consider the space as in **Counter Example 8.2.6**., in this space (Y,ζ,E), the soft subset $(F,E)_2 = \{(e_1,\emptyset),(e_2,\{a\})\}$is soft pre-open but not soft r*-open.

From the above discussions we have the following implication diagram.

Diagram 8.2.19.



Definition 8.2.20. Let (Y,ζ,E) be a soft topological space. Let (A,E) be a soft subset. A point x in Y is called a **soft r* -interior point** of (A,E) if (A,E) contains a soft r*-open set containing x.

Definition 8.2.21. Let (Y,ζ,E) be a soft topological space over Y then the **soft r*-interior** of the soft set (F,E) over Y is denoted by Sr*Int(F,E) and is defined as the union of all soft r*-open sets contained in (F,E).

Remark 8.2.22. If (A,E) is any subset of soft topological space, Sr*Int(A,E) need not be soft r* - open set, as seen from the following Counter Example .

Counter Example 8.2.23. Consider the space as in **Counter Example 8.2.6.**,in this space (Y,ζ,E), SR*O(Y,ζ,E) = $\{(F,E)_1, (F,E)_4, (F,E)_5,(F,E)_{16}\}$.Consider the soft subset $(F,E)_8 = \{(e_1,\{a\}),(e_2,\{a,b\})\}$.Sr*Int$(F,E)_8$= $\{(e_1,\{a\}),(e_2,\{a,b\})\}$ is not soft r*-open.

Theorem 8.2.24. In any soft topological space (Y,ζ,E), if (A,E) and (B,E) are subsets of soft topological space, then the following holds.

(i) **Sr*Int$(\phi, E) = (\phi, E)$**

(ii) **Sr*Int$(Y, E) = (Y, E)$**

(iii) **Sr*Int(A, E) ⊆̃ (A, E)**

(iv) **(A,E)⊆̃ (B,E)⇒ Sr*Int(A,E) ⊆̃ Sr*Int(B,E)**

(v) **Int(Sr*Int(A,E)) ⊆̃ Int(A,E)**

(vi) **srint(A,E) ⊆̃ Sr*Int(A,E) ⊆̃ Int(A,E) ⊆̃ (A,E)**

(vii) **Sr*Int((A,E)∪ (B,E)) ⊇̃ Sr*Int(A,E) ∪ Sr*Int(B,E)**

(viii) **Sr*Int((A, E) ∩(B,E)) = Sr*Int(A,E) ∩Sr*Int(B,E)**

(ix) **(A,E) is soft regular* - open iff (A,E) = Sr*Int(A,E).**

Proof (i),(ii) follows from **Definition 8.2.21.**

(iii) From the **Definition 8.2.21.**, Sr*Int(A,E) = ∪ {(B,E) / (B,E) ⊆̃ (A,E) and (B,E) is soft regular* - open}.Hence Sr*Int(A,E) ⊆̃ (A,E),since each (B,E) ⊆̃ (A,E).

(iv) Let (A,E) ⊆̃ (B,E).Let x ∈ Sr*Int(A,E) = ∪ {(C,E) / (C,E) ⊆̃ **(**A,E) and (C,E) is soft regular*-open}⇒ x∈(C,E) ⊆̃ (A,E) ⊆̃ (B,E)⇒x∈ Sr*Int(B,E) = ∪{(C,E) / (C,E) ⊆̃ (B,E) and (C,E) is soft regular* - open}.Hence Sr*Int(A,E) ⊆̃ Sr*Int(B,E).

(v) follows from **Definition 8.2.21.** and **(iv).**

(vi) follows from **Diagram 8.2.19.**

(vii) follows from **(iv)**.

(viii) We know, (A,E)∩(B,E) ⊆̃ (A,E),(B,E).By Condition **(iv),**Sr*Int((A, E) ∩ (B,E)) ⊆̃ Sr*Int(A,E), Sr*Int(B,E)⇒ Sr*Int((A,E)∩(B,E)) ⊆̃ Sr*Int(A,E) ∩ Sr*Int(B,E).To prove the converse, Let (C,E)∈ Sr*Int(A,E) ∩ Sr*Int(B,E).(C,E) ⊆̃ (A,E) where (C,E) is soft regular* - open,(C,E) ⊆̃ (B,E) where (C,E) is soft regular* - open,(C,E) ⊆̃ (A,E) ∩ (B,E) which is soft regular*-open by **Lemma 8.2.7.**Therefore (C,E) ∈ Sr*Int((A,E)∩(B,E)).

(ix) If (A,E) is a soft r*-open set over Y then (A,E) is itself a soft r*- open set over Y which contains (A,E).So Sr*Int(A,E) is the largest soft r*-open set contained in (A,E) and (A,E) = Sr*Int(A,E).Conversely,Suppose that (A,E) = Sr*Int(A,E).Since Sr*Int(A,E) is the largest soft r*-open set,so (A,E) is a soft r*-open set over Y.

Remark 8.2.25. The inclusions in the above Theorem 8.2.24 (vi) may be strict or equality holds in the following Example.

Example 8.2.26 Consider the space as in **Counter Example 8.2.6**., in this space (Y,ζ,E), SR*O(Y,ζ,E) = {$(F,E)_1$, $(F,E)_4$, $(F,E)_5$, $(F,E)_{16}$} = SRO(Y,ζ,E).Here take a soft set $(F,E)_2$ = {(e_1,∅),(e_2,{a})} and hence srint$(F,E)_2$ = Sr*Int$(F,E)_2$ = Int$(F,E)_2$ $\tilde{\subset}$ $(F,E)_2$.Cosider the soft set $(F,E)_5$ = {(e_1,{a}),(e_2, ∅)}.In this case srint$(F,E)_5$ = Sr*Int(F,E) $_5$ = Int$(F,E)_5$ = $(F,E)_5$.

Remark 8.2.27. The converse part in the above Theorem 8.2.24 (vii) does not hold which is seen in the following Counter Example.

Counter Example 8.2.28. Consider the space as in **Counter Example 8.2.6**., in this space (Y,ζ,E),SR*O(Y,ζ,E) = {$(F,E)_1$, $(F,E)_4$, $(F,E)_5$, $(F,E)_{16}$}.Then $(F,E)_2$ = {(e_1,∅),(e_2,{a})}, $(F,E)_{15}$ = {(e_1,{a,b}),(e_2,{b})} and $(F,E)_2$∪$(F,E)_{15}$ = {(e_1,{a,b}),(e_2,{a,b})}.Therefore Sr*Int((A,E) ∪ (B,E)) $\tilde{\not\subset}$ Sr*Int(A,E) ∪ Sr*Int(B,E).

§ 8.3. Soft Regular*-Closed Sets

Definition 8.3.1 The complement of a soft regular* - open set is called a **soft regular* - closed set (soft r* - closed).**

Lemma 8.3.2. If a soft set is soft r* - closed, then (A,E) = Cl(Int*(A,E)).

Proof Let (A,E) be soft r* - closed. Then $(A,E)^c$ is soft r* - open. Then By **Definition 8.2.1.**, $(A,E)^c$ = Int(Cl*($(A,E)^c$)). This ⇒ (A,E) = $((A,E)^c)^c$= (Int(Cl*($(A,E)^c$)))^c. Hence (A,E) = Cl(Int*(A,E)).

Result 8.3.3. A soft set is said to be soft r*-closed if (A,E) = Cl(Int*(A,E)).Here Int*(F,E) = ∪{(O,E): (O,E) is soft g-open and (O,E) $\tilde{\subset}$ (F,E)}.

Notation 8.3.4. The set of all soft r*-closed sets in (Y,ζ,E) is denoted by SR*C(Y,ζ,E) (or) SR*C(Y).

Remark 8.3.5. In any space (Y,ζ,E), ϕ and Y are soft r*-closed sets.

Theorem 8.3.6. Union of any two soft r* - closed sets is soft r* - closed.

Proof Let (A,E) and (B,E) be soft regular* - closed sets then (A,E) = Cl(Int*(A,E))and (B,E) = Cl(Int*(B,E)).Now consider, Cl(Int*((A,E)∪ (B,E))) = Cl(Int*(Y\(Y\((A,E)∪ (B,E))))) = Cl(Int*(Y\((Y\(A,E))∩ (Y\(B,E)))) = Cl(Int*(Y\(Y\(A,E))) ∪ Int*(Y\(Y\(B,E)))) = Cl(Int*(A,E) ∪Int*(B,E)) = Cl(Int*(A,E))∪

Cl(Int*(B,E)).Hence Cl(Int*((A,E)∪ (B,E)) = (A, E) ∪(B,E).Therefore (A, E) ∪ (B, E) is soft regular*-closed.

Remark 8.3.7. The intersection of two soft r*-closed sets need not be soft r* - closed, as seen from the following Example.

Counter Example 8.3.8. Consider **Counter Example 8.2.6.**, SR*C(Y,ζ,E) ={$(F,E)_1$, $(F,E)_{12}$, $(F,E)_{13}$,$(F,E)_{16}$}.For $(F,E)_{12}$ and $(F,E)_{13}$,their intersection is $(F,E)_{12}$∩ $(F,E)_{13}$= $(F,E)_9$ is not a soft r*-closed set.

Theorem 8.3.9. Every soft r* - closed set is soft closed.

Proof Let (A,E) be soft r*-closed, then$(A,E)^c$ is soft r*-open.By **Theorem 8.2.12.**, $(A,E)^c$ is soft open⇒(A,E) is soft closed.

Remark 8.3.10. The converse of the above Theorem 8.3.9. need not be true, as seen from the following Counter Example.

Counter Example 8.3.11. Consider **Counter Example 8.2.6.**,soft r*-closed sets are {$(F,E)_1$, $(F,E)_{12}$, $(F,E)_{13}$,$(F,E)_{16}$}and soft closed sets are {$(F,E)_1$, $(F,E)_9$, $(F,E)_{12}$, $(F,E)_{13}$,$(F,E)_{16}$}.Here $(F,E)_9$ is soft closed but it is not soft r*-closed.

Theorem 8.3.12. Every soft regular closed set is soft r* -closed.

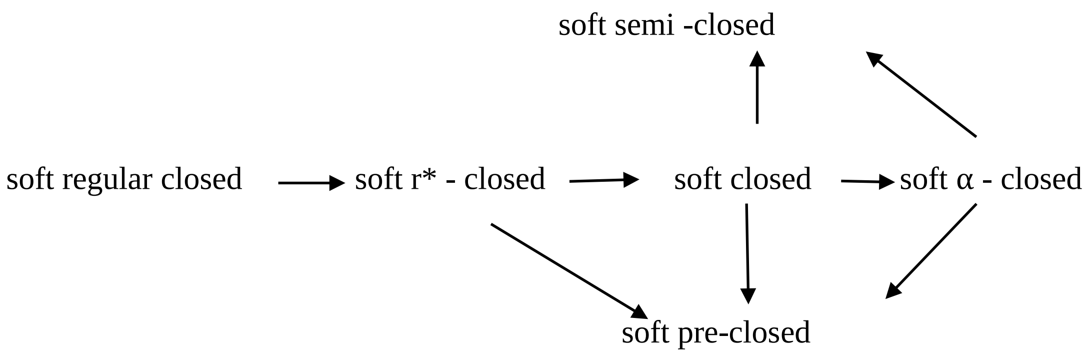
Proof Let (A,E) be a soft regular -closed set, then $(A,E)^c$ is soft regular - open. By **Theorem 8.2.9.**, $(A,E)^c$is soft r* - open⇒(A,E) is soft r* - closed.

Remark 8.3.13. The converse of the above Theorem 8.3.12. need not be true, as seen from the following Example.

Counter Example 8.3.14. In **Example 8.2.3.**, SR*C(Y,ζ,E) is {$(F,E)_1$, $(F,E)_{13}$, $(F,E)_{14}$, $(F,E)_{15}$,$(F,E)_{16}$} and soft regular closed sets are {$(F,E)_1$, $(F,E)_{14}$, $(F,E)_{15}$,$(F,E)_{16}$}.Here $(F,E)_{13}$ is soft r*-closed but it is not soft regular closed.

From the above discussions we have the following implication diagram.

Diagram 8.3.15.



Definition 8.3.16. The **soft r* -closure** of (A,E) is defined as the intersection of all soft r* -closed sets of (Y,ζ,E) containing (A,E). It is denoted by Sr*Cl(A,E).

Definition 8.3.17. Let (A,E) be a subset of soft topological space. An element x ∈ Y is called **soft r*-adherent point** of (A,E) if every soft r*-open set in (Y,ζ,E) containing x intersect (A,E).

Definition 8.3.18. Let (A,E) be a subset of soft topological space. An element x ∈ Y is called **soft r*-limit point** of (A,E) if every soft r* -open set in (Y,ζ,E) containing x intersects (A,E) in a point different from x.

Definition 8.3.19. The set of all soft regular* - limit points of (A,E) is called the **soft r* - Derived set** of (A,E). It is denoted by SD_{r*}(A,E).

Remark 8.3.20. If (A,E) is any subset of a soft topological space, Sr*Cl(A,E) need not be a soft r* -closed set, as seen from the following Counter Example.

Counter Example 8.3.21. Consider **Counter Example 8.2.6.,**SR*C(Y,ζ,E) ={$(F,E)_1$, $(F,E)_{12}$, $(F,E)_{13}$,$(F,E)_{16}$} but Sr*Cl$(F,E)_9$ = $(F,E)_9$ is not a soft r*-closed set.

Theorem 8.3.22. In any soft topological space (Y,ζ,E), the following results hold:

- **(i)** **Sr*Cl(ϕ, E) = (ϕ, E)**
- **(ii)** **Sr*Cl(Y,E) = (Y,E)**
- **(iii)** **(A,E) $\tilde{\subseteq}$ Sr*Cl(A,E)**
- **(iv)** **(A,E) $\tilde{\subseteq}$ (B,E) ⇒ Sr*Cl(A,E) $\tilde{\subseteq}$ Sr*Cl(B,E), if (A,E) and (B,E) are soft subsets of (Y,ζ,E)**

(v) **(A,E) $\tilde{\subseteq}$ Cl(A,E) $\tilde{\subseteq}$ Sr*Cl(A,E) $\tilde{\subseteq}$srcl(A,E)**

(vi) **Sr*Cl((A,E)∪ (B,E)) = Sr*Cl(A,E) ∪ Sr*Cl(B,E), if (A,E) and (B,E) are soft subsets of (Y,ζ,E)**

(vii) **Sr*Cl((A,E) ∩(B,E) $\tilde{\subseteq}$ Sr*Cl(A,E) ∩ Sr*Cl(B,E), if (A,E) and (B,E) are soft subsets of (Y,ζ,E)**

(viii) **Cl(A,E) $\tilde{\subseteq}$ Cl(Sr*Cl(A,E)).**

Proof The proof of **(i),(ii),(iii)** and **(iv),(vii)** is as given in **Theorem 8.2.24.**

(v) follows from **Diagram 8.3.15.**

(vi) obvious.

(viii) By the definition of soft r*-closure, (A,E) $\tilde{\subseteq}$ Sr*Cl(A,E) ⟹ Cl(A,E) $\tilde{\subseteq}$Cl(Sr*Cl(A,E)).

Remark 8.3.23. The inclusions in the above Theorem 8.3.22. (v) may be strict or equality holds in the following Example.

Example 8.3.24. Consider **Counter Example 8.2.6.**Then soft topology ζ is {$(F,E)_1$, $(F,E)_4$, $(F,E)_5$, $(F,E)_8$,$(F,E)_{16}$} and $ζ^c$ = {$(F,E)_1$, $(F,E)_9$, $(F,E)_{12}$, $(F,E)_{13}$,$(F,E)_{16}$}.Here SR*C(Y,ζ,E) ={$(F,E)_1$, $(F,E)_{12}$, $(F,E)_{13}$,$(F,E)_{16}$}= SRC(Y,ζ,E).Here take a soft set $(F,E)_5$ = {(e_1,{a}),(e_2, ∅)} and hence $(F,E)_5$$\tilde{\subseteq}$ Cl$(F,E)_5$ = Sr*Cl$(F,E)_5$ = srcl$(F,E)_5$ but $(F,E)_5$ is not equal to Cl$(F,E)_5$, Sr*Cl$(F,E)_5$ and srcl$(F,E)_5$.Cosider the soft set $(F,E)_{12}$ = {(e_1,{b}),(e_2,{a,b})}.In this case $(F,E)_5$ = Cl$(F,E)_5$ = Sr*Cl$(F,E)_5$ = srcl$(F,E)_5$.

Remark 8.3.25. The converse part in the above Theorem 8.3.22. (vii) does not hold which is shown in the following Counter Example.

Counter Example 8.3.26. Consider **Counter Example 8.2.6.**Then soft topology ζ is {$(F,E)_1$, $(F,E)_4$, $(F,E)_5$, $(F,E)_8$,$(F,E)_{16}$}.Here SR*C(Y,ζ,E) ={$(F,E)_1$, $(F,E)_{12}$, $(F,E)_{13}$,$(F,E)_{16}$}.Then $(F,E)_2$ = {(e_1,∅),(e_2,{a})}, $(F,E)_3$ = {(e_1,∅),(e_2,{b})} and $(F,E)_2$∩$(F,E)_3$ = {(e_1, ∅),(e_2, ∅)}.Therefore Sr*Cl(A,E) ∩ Sr*Cl(B,E) $\tilde{\not\subseteq}$ Sr*Cl((A,E) ∩ (B,E)).

§ 8.4. Soft η*-Open Sets

Definition 8.4.1. A subset (D,E) of a soft topological space (Y,ζ,E) is called a **soft η*-open set** if it is a union of soft regular*-open sets (soft r*-open sets).We denote the set of all soft η* -open sets in (Y,ζ,E) by Sη*O(Y,ζ,E).

Example 8.4.2. Consider **Counter Example 8.2.6.**Then for the soft topology ζ {$(F,E)_1$, $(F,E)_4$, $(F,E)_5$, $(F,E)_8$,$(F,E)_{16}$} the corresponding family of soft r*-open sets is {$(F,E)_1$, $(F,E)_4$, $(F,E)_5$,$(F,E)_{16}$} and the corresponding family of soft η*-open sets is **{$(F,E)_1$, $(F,E)_4$, $(F,E)_5$, $(F,E)_8$,$(F,E)_{16}$}.**

Theorem 8.4.3. Every soft δ-open set is a soft η*-open set but not conversely.

Proof Let (D,E) be a soft δ-open set.Then (D,E) = ∪ (B,E) where (B,E) is soft regular open.Since every soft regular open is soft r*-open by **Theorem 8.2.9.** (D,E) is a union of soft r*-open sets. Therefore (D,E) is soft η*-open. Hence every soft δ-open is soft η*-open.

Counter Example 8.4.4. Let Y={a,b},E ={e_1,e_2}.Define $(F,E)_1$ = {(e_1,∅),(e_2, ∅)},$(F,E)_2$ = {(e_1,∅),(e_2,{a})},$(F,E)_3$ = {(e_1,∅),(e_2,{b})}, $(F,E)_4$ = {(e_1,∅),(e_2,{a,b})},$(F,E)_5$ = {(e_1,{a}),(e_2, ∅)},$(F,E)_6$ = {(e_1,{a}),(e_2,{a})},$(F,E)_7$ = {(e_1,{a}),(e_2,{b})}, $(F,E)_8$ = {(e_1,{a}),(e_2,{a,b})},$(F,E)_9$ = {(e_1,{b}),(e_2, ∅)},$(F,E)_{10}$ = {(e_1,{b}),(e_2,{a})}, $(F,E)_{11}$ = {(e_1,{b}),(e_2,{b})},$(F,E)_{12}$ = {(e_1,{b}),(e_2,{a,b})}, $(F,E)_{13}$ = {(e_1,{a,b}),(e_2, ∅)},$(F,E)_{14}$ = {(e_1,{a,b}),(e_2,{a})},$(F,E)_{15}$ = {(e_1,{a,b}),(e_2,{b})}, $(F,E)_{16}$={(e_1,{a,b}),(e_2,{a,b})}.Then soft topology ζ is {$(F,E)_1$, $(F,E)_3$, $(F,E)_8$, $(F,E)_{16}$}**.**In this space(Y,ζ,E), SRO(Y,ζ,E) = {$(F,E)_1$, $(F,E)_{16}$} and SδO(Y,ζ,E) ={$(F,E)_1$,$(F,E)_{16}$} and SR*O(Y,ζ,E) = {$(F,E)_1$, $(F,E)_3$,$(F,E)_{16}$} and Sη*O(Y,ζ,E)={$(F,E)_1$, $(F,E)_3$,$(F,E)_{16}$}.Here $(F,E)_3$ is soft η*-open but it is not soft δ-open.

Theorem 8.4.5. Every soft η*-open set is a soft open set but not conversely.

Proof Let (D,E) be a soft η*-open set. Then (D,E) = ∪ (B,E) where (B,E) is soft r*-open. Since every soft r*-open is soft open by **Theorem 8.2.12.** (D,E) is a union of soft open sets. Therefore (D,E) is soft open. Hence every soft η*-open is soft open.

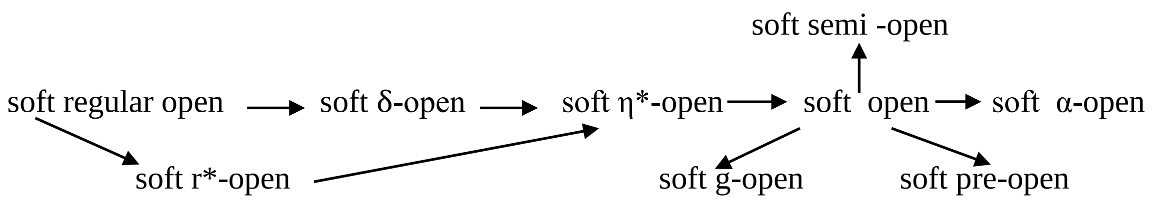
Counter Example 8.4.6. In previous **Counter Example 8.4.4.**,$(F,E)_8$ = {(e_1,{a}),(e_2,{a,b})} is soft open but it is not soft η*-open.

Theorem 8.4.7. Every soft r*-open set is a soft η*-open set but not conversely.

Proof Follows from the definition of soft η*-open set.

Counter Example 8.4.8. In previous **Counter Example 8.2.6.**, $(F,E)_8$ = {(e_1,{a}),(e_2,{a,b})} is soft η*-open but it is not soft r*-open.

From the above theorems we get the following implication diagram.



Theorem 8.4.9.

(a) **In any soft topological space (Y,ζ,E),Y and ϕ are soft η* -open sets.**

(b) **Any arbitrary union of soft η* -open sets are soft η* -open sets.**

(c) **The finite intersection of soft η* -open sets are soft η* -open sets.**

Result 8.4.10. Sη*O(Y,ζ,E) forms a soft topology which is finer than the set of all soft open sets ζ and coarser than the set of all soft *δ*-open sets ζ_δ.

Definition 8.4.11. A subset (D,E) of a soft topological space (Y,ζ,E) is called **soft η*-Interior of (D,E)** is the union of all of soft η*-open sets of Y contained in(D,E).We denote the symbol by **Sη*int(D,E)**.

Result 8.4.12.(i) Sη*int(D,E)is η*-open.

(ii) Sη*int(D,E) is the maximum soft η*-open set contained in (D,E).

Theorem 8.4.13. (D,E) is soft η*-open iff Sη*int(D,E) = (D,E) .

Proof (⟹) Let (D,E) be a soft η*-open set.Sη*int(D,E) is the union of all soft η* -open sets of Y contained in (D,E).That is Sη*int(D,E) is the maximum soft η* -open set contained in (D,E).Since (D,E) itself is a soft η*-open set.We get Sη*int(D,E)= (D,E).

(⟸) Let (D,E) = Sη*int(D,E).Since Sη*int (D,E) is the maximum soft η*-open set contained in (D,E).we get (D,E) is soft η*-open.

<u>Properties of Soft η*-Open Sets</u>

Theorem 8.4.14. In any soft topological space (Y,ζ,E), if (C,E) and (D,E) are subsets of Y then we get the following :

- **a.** **Sη*int(ϕ,E) = (ϕ,E)**
- **b.** **Sη*int(Y,E) = (Y,E)**
- **c.** **Sη*int (D,E) $\tilde{\subseteq}$ (D,E)**
- **d.** **(C,E) $\tilde{\subseteq}$ (D,E)⇒Sη*int(C,E) $\tilde{\subseteq}$Sη*int(D,E)**
- **e.** **Sη*int(Sη*int(D,E)) $\tilde{\subseteq}$Sη*int(D,E)**
- **f.** **Sη*int($\cup_{i\in\varepsilon}$\{ D_i, E\}) = $\cup_{i\in\varepsilon}$ Sη*int(\{D_i, E\})**
- **g.** **Sη*int((C,E)∩(D,E)) = Sη*int(C,E) ∩Sη*int(D,E).**

Proof Obvious.

§ 8.5. Soft η*-Closed Sets

Definition 8.5.1. The complement of a soft η*-open set is called a **soft η*-closed set**. We denote soft η*-closed sets in (Y,ζ,E) by Sη*C(Y,ζ,E).

Theorem 8.5.2. Every soft η*-closed set is soft closed but not conversely.

Proof Let (D,E) be a soft η*-closed set in (Y,ζ,E),then $(D,E)^c$ is soft η*-open.By **Theorem 8.4.5.**, $(D,E)^c$ is soft open implies that (D,E) is soft closed. Hence every soft η*-closed set is soft closed.

Counter Example 8.5.3. In **Counter Example 8.4.4.** Sη*C(Y,ζ,E)={$(F,E)_1$,$(F,E)_{14}$, ,$(F,E)_{16}$}.Here $(F,E)_9$ is soft closed but it is not soft η*-closed.

Theorem 8.5.4. Every soft r*-closed set is soft η*-closed but not conversely.

Proof Follows from the definition of soft η*-closed set.

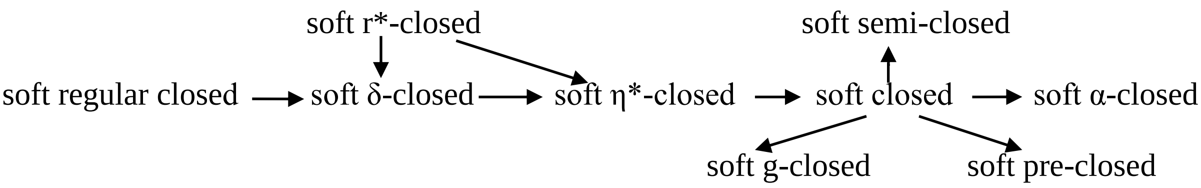
Counter Example 8.5.5. In previous **Counter Example 8.2.6.**, $(F,E)_9$ = {(e_1,{b}),(e_2, ∅) }is soft η*-closed but it is not soft r*-closed.

Theorem 8.5.6. Every soft δ-closed is soft η*-closed but not conversely.

Proof Let (D,E) be a soft δ-closed set in (Y,ζ,E),then $(D,E)^c$ is soft δ-open. By **Theorem 8.4.3.**, $(D,E)^c$ is soft η*-open implies that (D,E) is soft η*-closed. Hence every soft δ-closed is soft η*-closed.

Counter Example 8.5.7. In **Counter Example 8.4.4**, SδC(Y,ζ,E) ={$(F,E)_1$,$(F,E)_{16}$} and and Sη*C(Y,ζ,E)={$(F,E)_1$, $(F,E)_{14}$,$(F,E)_{16}$}.Here $(F,E)_{14}$ is soft η*-closed but it is not soft δ-closed.

From the above theorems we get the following diagrammatic implications.



Definition 8.5.8. The intersection of all soft η*-closed sets of Y containing (D,E) is called as the **soft η*-closure** of (D,E) and denoted by **Sη*Cl(D,E)**.

Note 8.5.9. Sη*Cl(D,E) is soft η*-closed.

Definition 8.5.10. Let (D,E) $\tilde{\subseteq}$ (Y,E). An element (y,E)∈(Y,E) is called **soft η*-adherent point** of (D,E) if every soft η*-open set in Y containing (y,E) intersects (D,E).

Definition 8.5.11. Let (D,E) be a subset of the topological space (Y,ζ,E).A point y∈Y is called **a soft η*-cluster point** of D if for every soft η*- open set (V,E) containing (y,E) intersects (D,E) in a point different from (y,E).

Definition 8.5.12. We denote the set of all soft η*-cluster points of (D,E) by **Sη*-D(D,E)**.

<u>Properties of Soft η*-Closed Sets</u>

Theorem 8.5.13. In any soft topological space (Y,ζ,E), if (C,E) and (D,E) are subsets of Y then we get the following :

a. **Sη*Cl(∅,E) = (∅,E)**

b. **Sη*Cl(Y,E) = (Y,E)**

c. **(D,E) $\tilde{\subseteq}$ Sη*Cl(D,E)**

d. **(C,E) $\tilde{\subseteq}$(D,E) ⇒ Sη*Cl(C,E) $\tilde{\subseteq}$ Sη*Cl(D,E),if (C,E) and (D,E) are subsets of (Y,ζ,E)**

e. **Sη* Cl((C,E)∪(D,E)) = Sη* Cl (C,E) ∪ Sη* Cl (D,E)**

f. **Sη*Cl($\cap_{i\in\varepsilon}$ {D_i, E}) = $\cap_{i\in\varepsilon}$ Sη*Cl ({D_i, E})**

g. **Cl(D, E)$\tilde{\subseteq}$ Cl(Sη*Cl(D, E))**

h. Sη* Cl(Sη*Cl(C,E)) = Sη*Cl(C,E).

Proof Obvious.

§ 8.6. Soft J-Closed Sets

Definition 8.6.1. A soft set (D,E) of a soft topological space (Y,ζ,E) is said to be a **soft J-closed** set if Cl(D,E)⊆̃ (M,E) whenever (D,E) ⊆̃ (M,E) and (M,E) is soft η*-open in Y.

Notation 8.6.2. The set of all soft J-closed sets in (Y,ζ, E) is denoted by SJC(Y,ζ,E).

Lemma 8.6.3. Every soft closed is soft J-closed but not conversely.

Proof Let(D,E) be a soft closed set and (M,E) is a soft η*-open set containing (D,E).Since (D, E) is soft closed, Cl(D, E) = (D, E)⊆̃ (M,E).Hence every soft closed set is soft J-closed.

Counter Example 8.6.4. Let Y = {p,q,r},E ={e_1,e_2}.Then ζ= {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$,$(F,E)_{16}$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1,{q,r}),(e_2,{p,q})}, $(F,E)_3$ = {(e_1,{p,q}),(e_2,{p,q,r})}, $(F,E)_4$ = {(e_1,{p,q}),(e_2,{p,r})}, $(F,E)_5$ = {(e_1,{q}),(e_2,{p,q})}, $(F,E)_{16}$ = {(e_1,{p,q,r}),(e_2,{p,q,r})}.Here (D,E) = {(e_1,∅),(e_2,{q,r})} is soft J-closed but it is not soft closed.

Theorem 8.6.5. If (D,E) is soft J-closed in Y and (D,E) ⊆̃ (B,E) ⊆̃ Cl(D,E),then (B,E) is soft J-closed.

Proof Let (M,E) is soft η* -open in Y such that (B,E) ⊆̃ (M,E).Since (D,E) ⊆̃ (B,E),we have (D,E) ⊆̃ (M,E).It is given that (D,E) is soft J-closed in Y and hence Cl(D,E) ⊆̃ (M,E).Since (B,E)⊆̃ Cl(D,E),Cl(B,E) ⊆̃ Cl(Cl(D,E)) implies Cl(B,E)⊆̃Cl(D,E)⊆̃(M,E).Therefore Cl(B,E)⊆̃ (M,E).Hence (B,E) is soft J-closed.

Theorem 8.6.6. If (D,E) and (H,E) are soft J-closed sets then their union is also soft J-closed.

Proof Let (D,E) ∪ (H,E) ⊆̃ (M,E) such that (M,E) is soft η*-open in Y. This implies (D,E) ⊆̃ (M,E) and (H,E) ⊆̃ (M,E).Since (D,E) and (H,E) are soft J-closed, we have Cl(D,E) ⊆̃ (M,E) and Cl(H,E) ⊆̃ (M,E).Hence Cl((D,E) ∪ (H,E)) = Cl(D,E) ∪ Cl(H,E) ⊆̃ (M,E) by Theorem 1 (6) of (Shabir,2011).Therefore (D,E) ∪ (H,E) is soft J-closed.

Theorem 8.6.7. A soft set (D,E) is soft J-closed iff Cl(D,E) \ (D,E) contains only null soft η* -closed set.

Proof Let (D,E) be a soft J-closed set. Consider (F, E) be soft η* -closed and (F,E) ⊆̃ Cl(D, E) \ (D, E).Since (F,E) is soft η* - closed, we have its relative complement $(F,E)^c$ is soft η*-open. Since (F, E) ⊆̃ Cl(D, E) \ (D, E), we have (F, E) ⊆̃ Cl(D, E) -----(1) and (F,E) ⊆̃ $(D,E)^c$ Hence (D, E) ⊆̃ $(F,E)^c$. Consequently Cl(D, E) ⊆̃ $(F,E)^c$ as (D, E) is soft J-closed in Y. Therefore (F,E) ⊆̃ Cl $(D,E)^c$ -----(2). From (1) and (2) we get (F, E) =∅.Therefore Cl(D,E) \ (D,E) contains only null soft η*-closed set. Conversely, Cl(D,E) \ (D,E) = ∅.Then Cl(D,E) = (D,E).Therefore (D,E) is soft closed. Hence (D,E) is soft J- closed by **Lemma 8.6.3.**

Corollary 8.6.8. Let (D,E) be a soft J-closed set. Then (D,E) is soft closed iff Cl(D,E) \ (D,E) is soft η* -closed.

Proof Let (D,E) is soft closed, then Cl(D,E) = (D,E) which implies Cl(D,E) \ (D,E) = ∅ is soft η*-closed. Conversely, suppose that Cl(D,E) \ (D,E) is soft η*-closed. Since (D,E) is soft J-closed, by **Theorem 8.6.7.,**we get Cl(D,E) \ (D,E) = ∅ which implies Cl(D,E) = (D,E).Hence a soft set (D,E) is soft closed.

Theorem 8.6.9. Every soft g-closed set is soft J-closed but not conversely.

Proof Let (D,E) be a soft g-closed set and (M,E) is a soft η*-open set containing (D,E). Since (D, E) is soft g-closed,Cl(D,E) ⊆̃ (M,E).Since every soft η*-open is soft open by **Theorem 8.4.5.**We get (D,E) is soft J-closed.

Counter Example 8.6.10. Let Y = {p,q,r},E ={e_1,e_2}.Then ζ= {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$,$(F,E)_{16}$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1,{q,r}),(e_2,{p,q})}, $(F,E)_3$ = {(e_1,{p,q}),(e_2,{p,q,r})}, $(F,E)_4$ = {(e_1,{p,q}),(e_2,{p,r})}, $(F,E)_5$ = {(e_1,{q}),(e_2,{p,q})}, $(F,E)_{16}$ = {(e_1,{p,q,r}),(e_2,{p,q,r})}.Here (D,E) = {(e_1,∅),(e_2,{q,r})} is soft J-closed but it is not soft g-closed.

Proposition 8.6.11. Let (G,E) be a soft subset of a soft topological space (Y,ζ,E).If (G,E) is soft η* -open and soft J-closed,then (G,E) is a soft closed set.

Proof Let (G,E) be soft η* -open and soft J-closed. Then Cl(G,E) ⊆̃ (M,E) whenever (G,E) ⊆̃ (M,E) and (M,E) is soft η*-open. Therefore Cl(G,E) ⊆̃ (G,E).Always (G,E) ⊆̃Cl(G,E).Hence (G,E) is a soft closed set.

Theorem 8.6.12. Every soft ĝ-closed set is soft J-closed but not conversely.

Proof Let (D,E) be a soft ĝ-closed set and (M,E) is soft η*-open containing (D,E).Since every soft η*-open is soft semi open and (D, E) is soft ĝ-closed,Cl(D,E) $\tilde{\subseteq}$ (M,E).Hence (D,E) is soft J-closed.

Counter Example 8.6.13. Let Y = {p,q},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$,$(F,E)_{16}$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1, ∅),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q}),(e_2, ∅)}, $(F,E)_4$ = {(e_1,{q}),(e_2,{p})}, $(F,E)_{16}$ = {(e_1,{p,q}),(e_2,{p,q})}.Here (D,E) = {(e_1,∅),(e_2,{q})} is soft J-closed but it is not soft ĝ-closed.

Theorem 8.6.14. Every soft g*-closed set is soft J-closed but not conversely.

Proof Let (D,E) be a soft g*-closed set and (M,E) is soft η*-open containing (D,E).Since (D, E) is soft g*-closed,Cl(D,E) $\tilde{\subseteq}$ (M,E). Since every soft η*-open is soft g-open.Therefore (D,E) is soft J-closed.

Remark 8.6.15. The converse part of the above Theorem is not true.

Counter Example 8.6.16. Let Y = {p,q},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$,$(F,E)_{16}$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1, ∅),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q}),(e_2, ∅)}, $(F,E)_4$ = {(e_1,{q}),(e_2,{p})}, $(F,E)_{16}$ = {(e_1,{p,q}),(e_2,{p,q})}.Here (H,E) = {(e_1,∅),(e_2,{q})} is soft J-closed but it is not soft g*-closed.

Theorem 8.6.17. (i) Every soft J-closed is soft gpr-closed.

(ii) Every soft J-closed is soft rwg-closed.

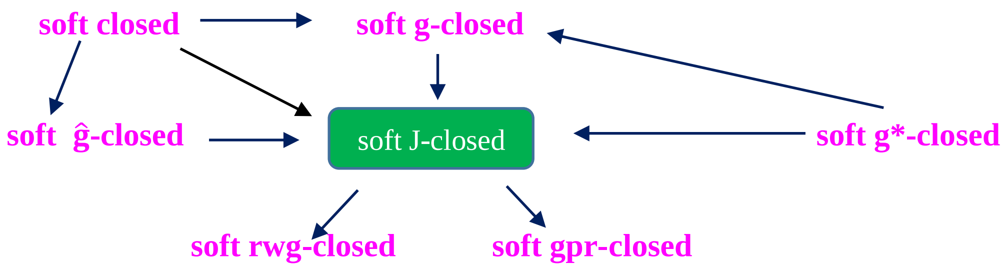
Proof (i) Let (D,E) be a soft J-closed set and (M,E) is soft regular-open containing (D,E).Since every soft regular-open is soft η*-open and (D,E) is soft J-closed, Cl(D,E) $\tilde{\subseteq}$ (M,E).As pCl(D,E) $\tilde{\subseteq}$ Cl(D,E) implies (D,E) is soft gpr-closed.

(ii) Let (D,E) be a soft J-closed set and (M,E) is soft regular-open containing (D,E).Since every soft regular-open is soft η*-open and (D,E) is soft J-closed, Cl(D,E) $\tilde{\subseteq}$(M,E).As Int(D,E) $\tilde{\subseteq}$ (D,E) implies Cl(Int((D,E)) $\tilde{\subseteq}$ Cl(D,E)$\tilde{\subseteq}$ (M,E).Hence (D,E) is soft rwg-closed.

Remark 8.6.18. The converse parts of the above Theorem are not true.

Counter Example 8.6.19. Let Y = {p,q},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$,$(F,E)_{16}$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1, ∅),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q}),(e_2, ∅)}, $(F,E)_4$ = {(e_1,{q}),(e_2,{p})}, $(F,E)_{16}$ = {(e_1,{p,q}),(e_2,{p,q})}.Here $(F,E)_4$ = {(e_1,{q}),(e_2,{p})} is soft gpr-closed and soft rwg-closed but it is not soft J-closed.

From the above discussions to represent as a picture format.



§ 8.7. Soft J*-Closed Sets

Definition 8.7.1. A soft set (D,E) of a soft topological space (Y,ζ,E) is said to be **soft J*-closed** set if Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E) whenever (D,E) $\tilde{\subseteq}$ (M,E) and (M,E) is soft open in Y.

Notation 8.7.2. The set of all soft J*-closed sets in (Y,ζ, E) is denoted by SJ*C(Y,ζ,E).

Lemma 8.7.3. Every soft η*-closed is soft J*-closed but not conversely.

Proof Let(D,E) be a soft η*-closed set and (M,E) is a soft open set containing (D,E).Since (D, E) is soft η*-closed, Sη*Cl(D, E) = (D, E) $\tilde{\subseteq}$ (M,E).Hence every soft η*-closed set is soft J*-closed.

Counter Example 8.7.4. Let Y = {p,q,r},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$,$(F,E)_{16}$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1,{p}),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q}),(e_2,{q})}, $(F,E)_4$ = {(e_1,{p,q}),(e_2,{p,q})}, $(F,E)_{16}$ = {(e_1,{p,q,r}),(e_2,{p,q,r})}.Here (D,E) = {(e_1,{p,q}),(e_2,{p,q,r})} is soft J*-closed but it is not soft η*-closed.

Theorem 8.7.5. If (D,E) is soft J*-closed in Y and (D,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ Sη*Cl(D,E),then (B,E) is soft J*-closed.

Proof Let (M,E) is soft open in Y such that (B,E) $\tilde{\subseteq}$ (M,E).Since (D,E) $\tilde{\subseteq}$ (B,E),we have (D,E) $\tilde{\subseteq}$ (M,E).It is given that (D,E) is soft J*-closed in Y and hence Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E).Since (B,E) $\tilde{\subseteq}$ Sη*Cl(D,E), Sη*Cl(B,E) $\tilde{\subseteq}$ Sη*Cl(Sη*Cl(D,E)) implies Sη*Cl(B,E)$\tilde{\subseteq}$

Sη*Cl(D,E)⊆̃(M,E) by **Theorem 8.5.13.h.**Therefore Sη*Cl(B,E)⊆̃ (M,E).Hence (B,E) is soft J*-closed.

Theorem 8.7.6. If (D,E) and (H,E) are soft J*-closed sets then their union is also soft J*-closed.

Proof Let (D,E) ∪ (H,E) ⊆̃ (M,E) such that (M,E) is soft open in Y.This implies (D,E) ⊆̃ (M,E) and (H,E) ⊆̃ (M,E).Since (D,E) and (H,E) are soft J*-closed,we have Sη*Cl(D,E) ⊆̃ (M,E) and Sη*Cl(H,E) ⊆̃ (M,E).Hence Sη*Cl((D,E) ∪ (H,E)) = Sη*Cl(D,E) ∪ Sη*Cl(H,E) ⊆̃ (M,E) by **Theorem 8.5.13.e.**.Therefore (D,E) ∪ (H,E) is soft J*-closed.

Theorem 8.7.7. A soft set (D,E) is soft J*-closed iff Sη*Cl(D,E) \ (D,E) contains only null soft closed set.

Proof Let (D,E) be a soft J*-closed set. Consider (F,E) be soft closed and (F,E) ⊆̃ Sη*Cl(D, E) \ (D, E).Since (F,E) is soft closed, we have its relative complement $(F,E)^c$ is soft open. Since (F, E) ⊆̃Sη*Cl(D, E) \ (D, E), we have (F, E) ⊆̃ Sη*Cl(D, E) -----(1) and (F,E) ⊆̃ $(D,E)^c$ Hence (D, E) ⊆̃ $(F,E)^c$. Consequently Sη*Cl(D, E) ⊆̃ $(F,E)^c$ as (D, E) is soft J*-closed in Y. Therefore (F,E) ⊆̃ $Sη^*Cl(D,E)^c$ -----(2). From (1) and (2) we get (F,E) =∅.Therefore Sη*Cl(D,E) \ (D,E) contains only null soft closed set. Conversely, Sη*Cl(D,E) \ (D,E) = ∅.Then Sη*Cl(D,E) = (D,E).Therefore (D,E) is soft η*-closed. Hence (D,E) is soft J*-closed by **Lemma 8.7.3.**

Corollary 8.7.8. Let (D,E) be a soft J*-closed set. Then (D,E) is soft η*-closed iff Sη*Cl(D,E) \ (D,E) is soft closed.

Proof Let (D,E) is soft η*-closed, then Sη*Cl(D,E) = (D,E) which implies Sη*Cl(D,E) \ (D,E) = ∅ is soft closed. Conversely, suppose that Sη*Cl(D,E) \ (D,E) is soft closed. Since (D,E) is soft J*-closed, by **Theorem 8.7.7.,**we get Sη*Cl(D,E) \ (D,E) = ∅ which implies Sη*Cl(D,E) = (D,E).Hence a soft set (D,E) is soft η*-closed.

Theorem 8.7.9. Every soft J*-closed is soft g-closed but the converse is not true.

Proof Let (D,E) be a soft J*-closed set and (M,E) is soft open containing (D,E).Since (D, E) is soft J*-closed, Sη*Cl(D,E) ⊆̃ (M,E).As Cl(D,E) ⊆̃ Sη*Cl(D,E) by **Theorem 8.5.2**.Hence (D,E) is soft g-closed.

Counter Example 8.7.10. Let Y = {p,q,r},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1,{p}),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q}),(e_2,{q})}, $(F,E)_4$ = {(e_1,{p,q}),(e_2,{p,q})}, $(F,E)_5$ = {(e_1,{p,q,r}),(e_2,{p,q,r})}.Here (D,E) = {(e_1, ∅),(e_2,{p})} is soft g-closed but it is not soft J*-closed.

Proposition 8.7.11. Let (G,E) be a soft subset of soft topological space (Y,ζ,E).If (G,E) is soft open and soft J*-closed,then (G,E) is a soft η*-closed set.

Proof Let (G,E) be soft open and soft J*-closed.Then Sη*Cl(G,E) ⊆̃ (M,E) whenever (G,E) ⊆̃ (M,E) and (M,E) is soft open.Therefore Sη*Cl(G,E) ⊆̃ (G,E).Always (G,E) ⊆̃ Sη*Cl(G,E).Hence (G,E) is a soft η*-closed set.

Theorem 8.7.12. (i) Every soft J*-closed is soft gpr-closed.

(ii) Every soft J*-closed is soft rwg-closed.

(iii) Every soft J*-closed is soft J-closed.

Proof (i) Let (D,E) be a soft J*-closed set and (M,E) is soft regular-open containing (D,E).Since every soft regular-open is soft open and (D,E) is soft J*-closed, Sη*Cl(D,E) ⊆̃ (M,E).As pCl(D,E) ⊆̃ Sη*Cl(D,E) implies (D,E) is soft gpr-closed.

(ii) Let (D,E) be a soft J*-closed set and (M,E) is soft regular-open containing (D,E).Since every soft regular-open is soft open and (D,E) is soft J*-closed, Sη*Cl(D,E) ⊆̃ (M,E).As Int(D,E) ⊆̃ (D,E) implies Sη*Cl(Int((D,E)) ⊆̃ Sη*Cl(D,E)⊆̃(M,E).Hence (D,E) is soft rwg-closed.

(iii) Let (D,E) be a soft J*-closed set and (M,E) is soft η∗-open containing (D,E).Since every soft η*-open is soft open and (D,E) is soft J*-closed, Sη*Cl(D,E) ⊆̃ (M,E).As Cl(D,E) ⊆̃ Sη*Cl(D,E) implies Cl(D,E) ⊆̃ (M,E).Hence (D,E) is soft J-closed.

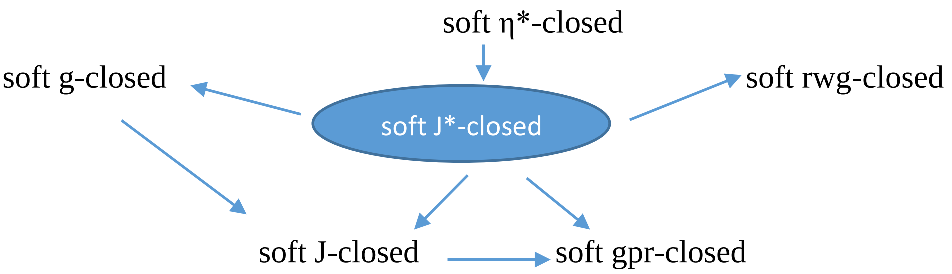
Remark 8.7.13. The converse parts of the above Theorem are not true.

Counter Example 8.7.14. Let Y = {p,q},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1, ∅),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q}),(e_2, ∅)},

$(F,E)_4 = \{(e_1,\{q\}),(e_2,\{p\})\}$, $(F,E)_5 = \{(e_1,\{p,q\}),(e_2,\{p,q\})\}$.Here $(F,E)_4 = \{(e_1,\{q\}),(e_2,\{p\})\}$ is soft gpr-closed and soft rwg-closed but it is not soft J*-closed.

Counter Example 8.7.15. Let Y = {p,q,r},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$}.Define $(F,E)_1 = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $(F,E)_2 = \{(e_1,\{p\}),(e_2,\{p\})\}$, $(F,E)_3 = \{(e_1,\{q\}),(e_2,\{q\})\}$, $(F,E)_4 = \{(e_1,\{p,q\}),(e_2,\{p,q\})\}$, $(F,E)_5 = \{(e_1,\{p,q,r\}),(e_2,\{p,q,r\})\}$.Here $(F,E)_4 = \{(e_1,\{p,q\}),(e_2,\{p.q\})\}$ is soft J-closed but it is not soft J*-closed.

From the above arguments to get the following diagram.



§ 8.8. Soft J**-Closed Sets

Definition 8.8.1. A soft set (D,E) of a soft topological space (Y,ζ,E) is said to be **soft J**-closed** set if Sη*Cl (D,E) ⊑̃ (M,E) whenever (D,E) ⊑̃ (M,E) and (M,E) is soft η*-open in Y.

Notation 8.8.2. The set of all soft J-closed sets in (Y,ζ, E) is denoted by SJ**C(Y,ζ,E).**

Lemma 8.8.3. Every soft η* -closed is soft J-closed.**

Proof Let(D,E) be a soft η* -closed set and (M,E) is soft η* -open containing (D,E).Since (D, E) is soft η* -closed, Sη*Cl(D, E) = (D, E) ⊑̃ (M,E).Hence every soft η*-closed set is soft J**-closed.

Theorem 8.8.4. If (D,E) is soft J-closed in Y and (D,E) ⊑̃ (B,E) ⊑̃ Sη*Cl(D,E),then (B,E) is soft J**-closed.**

Proof Let (M,E) is soft η*-open in Y such that (B,E) ⊑̃ (M,E).Since (D,E) ⊑̃ (B,E),we have (D,E) ⊑̃ (M,E).It is given that (D,E) is soft J**-closed in Y and hence Sη*Cl(D,E) ⊑̃ (M,E).Since (B,E) ⊑̃ Sη*Cl(D,E), Sη*Cl(B,E) ⊑̃ Sη*Cl(Sη*Cl(D,E)) implies Sη*Cl(B,E)⊑̃ Sη*Cl(D,E)⊑̃(M,E) by **Theorem 8.5.13.h.**Therefore Sη*Cl(B,E)⊑̃ (M,E).Hence (B,E) is soft J**-closed.

Theorem 8.8.5. If (D,E) and (H,E) are soft J-closed sets then their union is also soft J**-closed.**

Proof Let (D,E) ∪ (H,E) $\tilde{\subseteq}$ (M,E) such that (M,E) is soft η*-open in Y.This implies (D,E) $\tilde{\subseteq}$ (M,E) and (H,E) $\tilde{\subseteq}$ (M,E).Since (D,E) and (H,E) are soft J**-closed,we have Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E) and Sη*Cl(H,E) $\tilde{\subseteq}$ (M,E).Hence Sη*Cl((D,E) ∪ (H,E)) = Sη*Cl(D,E) ∪ Sη*Cl(H,E) $\tilde{\subseteq}$ (M,E) by **Theorem 8.5.13.e**.Therefore (D,E) ∪ (H,E) is soft J**-closed.

Theorem 8.8.6. A soft set (D,E) is soft J-closed iff Sη*Cl(D,E) \ (D,E) contains only null soft η*-closed set.**

Proof Let (D,E) be a soft J**-closed set. Consider (F,E) be soft η*-closed and (F,E) $\tilde{\subseteq}$ Sη*Cl(D, E) \ (D, E).Since (F,E) is soft η*-closed, we have its relative complement $(F,E)^c$ is soft η*-open. Since (F, E) $\tilde{\subseteq}$ Sη*Cl(D, E) \ (D, E), we have (F, E) $\tilde{\subseteq}$ Sη*Cl(D, E) -----(1) and (F,E) $\tilde{\subseteq}$ $(D,E)^c$ Hence (D, E) $\tilde{\subseteq}$ $(F,E)^c$. Consequently Sη*Cl(D, E) $\tilde{\subseteq}$ $(F,E)^c$ as (D, E) is soft J**-closed in Y. Therefore (F,E) $\tilde{\subseteq}$ Sη*Cl$(D,E)^c$ -----(2). From (1) and (2) we get (F,E) =∅.Therefore Sη*Cl(D,E) \ (D,E) contains only null soft η*-closed set. Conversely, Sη*Cl(D,E) \ (D,E) = ∅.Then Sη*Cl(D,E) = (D,E).Therefore (D,E) is soft η*-closed. Hence (D,E) is soft J**-closed by **Lemma 8.8.3.**

Corollary 8.8.7. Let (D,E) be a soft J-closed set. Then (D,E) is soft η*-closed iff Sη*Cl(D,E) \ (D,E) is soft η*-closed.**

Proof Let (D,E) is soft η*-closed, then Sη*Cl(D,E) = (D,E) which implies Sη*Cl(D,E) \ (D,E) = ∅ is soft η*-closed. Conversely, suppose that Sη*Cl(D,E) \ (D,E) is soft η*-closed. Since (D,E) is soft J**-closed, by **Theorem 8.8.6.,**we get Sη*Cl(D,E) \ (D,E) = ∅ which implies Sη*Cl(D,E) = (D,E).Hence a soft set (D,E) is soft η*-closed.

Theorem 8.8.8. Every soft J-closed is soft g-closed.**

Proof Let (D,E) be a soft J**-closed set and (M,E) is soft η*-open containing (D,E).Since every soft η*-open is soft open and (D, E) is soft J**-closed, Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E).As Cl(D,E) $\tilde{\subseteq}$ Sη*Cl(D,E) by **Theorem 8.5.2.**Hence (D,E) is soft g-closed.

Proposition 8.8.9. Let (G,E) be a soft subset of soft topological space (Y,ζ,E).If (G,E) is soft η*-open and soft J-closed, then (G,E) is a soft η*-closed set.**

Proof Let (G,E) be soft η*-open and soft J**-closed.Then Sη*Cl(G,E) $\tilde{\subseteq}$ (M,E) whenever (G,E) $\tilde{\subseteq}$ (M,E) and (M,E) is soft η*-open. Therefore Sη*Cl(G,E) $\tilde{\subseteq}$(G,E).Always (G,E) $\tilde{\subseteq}$ Sη*Cl(G,E).Hence (G,E) is a soft η* -closed set.

Theorem 8.8.10.(i) Every soft J-closed is soft gpr-closed.**

(ii) Every soft J-closed is soft rwg-closed.**

(iii) Every soft J*-closed is soft J-closed.**

(iv) Every soft J-closed is soft J-closed.**

Proof (i) Let (D,E) be a soft J**-closed set and (M,E) is soft regular open containing (D,E).Since every soft regular open is soft η*-open and (D,E) is soft J**-closed, Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E).As pCl(D,E) $\tilde{\subseteq}$ Sη*Cl(D,E) implies (D,E) is soft gpr-closed.

(ii) Let (D,E) be a soft J**-closed set and (M,E) is soft regular open containing (D,E).Since every soft regular open is soft η* -open and (D,E) is soft J**-closed, Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E).As int(D,E) $\tilde{\subseteq}$(D,E) implies Sη*Cl(int((D,E)) $\tilde{\subseteq}$ Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E).Hence (D,E) is soft rwg-closed.

(iii) Let (D,E) be a soft J*-closed set and (M,E) is soft η* -open containing (D,E).Since every soft η* -open is soft open and (D,E) is soft J*-closed, Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E).Hence (D,E) is soft J**-closed.

(iv) Let (D,E) be a soft J**-closed set and (M,E) is soft η* -open containing (D,E).Since (D,E) is soft J**-closed, Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E).As Cl(D,E) $\tilde{\subseteq}$ Sη*Cl(D,E) implies Cl(D,E)$\tilde{\subseteq}$(M,E).Hence (D,E) is soft J-closed.

Remark 8.8.11. The converse parts of the above Theorem are not true.

Counter Example 8.8.12. Let Y = {p,q},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1, ∅),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q}),(e_2, ∅)},

$(F,E)_4$ = {(e_1,{q}),(e_2,{p})}, $(F,E)_5$ = {(e_1,{p,q}),(e_2,{p,q})}.Here $(F,E)_4$ = {(e_1,{q}),(e_2,{p})} is soft gpr-closed and soft rwg-closed but it is not soft J**-closed.

Counter Example 8.8.13. Let Y = {p,q,r},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$, $(F,E)_6$,$(F,E)_7$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1,{q}),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q,r}),(e_2,{p,q})}, $(F,E)_4$ = {(e_1,{p,q}),(e_2,{p,q,r})}, $(F,E)_5$ = {(e_1,{p,q}),(e_2,{p,r})}, $(F,E)_6$ = {(e_1,{q}),(e_2,{p,q})}, $(F,E)_7$ = {(e_1,{p,q,r}),(e_2,{p,q,r})}.Here (D,E) = {(e_1,{p,q}),(e_2,{q})} is soft J**-closed but it is not soft J*-closed.

Counter Example 8.8.14. Let Y = {p,q,r},E ={e_1,e_2}.Then ζ = {$(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$, $(F,E)_6$,$(F,E)_7$}.Define $(F,E)_1$ = {(e_1,∅),(e_2,∅)}, $(F,E)_2$ = {(e_1,{q}),(e_2,{p})}, $(F,E)_3$ = {(e_1,{q,r}),(e_2,{p,q})}, $(F,E)_4$ = {(e_1,{p,q}),(e_2,{p,q,r})}, $(F,E)_5$ = {(e_1,{p,q}),(e_2,{p,r})}, $(F,E)_6$ = {(e_1,{q}),(e_2,{p,q})}, $(F,E)_7$ = {(e_1,{p,q,r}),(e_2,{p,q,r})}.Here (D,E) = {(e_1,{r}),(e_2,{p})} is soft J-closed but it is not soft J**-closed.

The figure shows that soft J-closed sets is stronger as well as weaker than other soft sets.**