7

Desingning Group Sampling Plan with Weighted Binomial Distribution for Truncated Life Tests Using Minimum Angle Method

7.1 Introduction

In this chapter author proposed a new approach of designing Group acceptance sampling plan using weighted binomial distribution for a truncated life test when the life time of the test items follows different distributions. The distributions considered in this chapter are Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution. Minimum angle method is applied to determine the design parameter group size g by satisfying both the risks at the specified quality levels simultaneously and at the same time minimizing the sum of risks. Tables of design parameters are provided and the results are explained with some examples. The comparisions are made among the distributions considered.

Radhakrishnan and Alagirisamy (2011), constructed Group acceptance sampling plan using weighted binomial distribution. In (2012c), Sudamani Ramasamy and Priya Anburajan presented the Group acceptance sampling plans using weighted binomial on truncated life test for Marshall – Olkin Extented distributions and (2012d), they presented the Group acceptance sampling plans using weighted binomial on truncated life test for inverse Rayleigh and log-logistic distributions.

7.2 Distributions

The following are the life time distributions used in this chapter.

7.2.1 Rayleigh distribution

The Rayleigh distribution and the cumulative distribution function (cdf) is given by

$$F(t,\lambda) = 1 - e^{-\frac{1}{2}\left(\frac{t}{\lambda}\right)^2}, \quad t>0, \lambda>0 \qquad (7.1)$$

where, λ is the scale parameter.

7.2.2 Generalized Exponential distribution

The cumulative distribution function (cdf) of the Generalized Exponential distribution is given by

$$F(t,\lambda) = \left(1 - e^{-\frac{t}{\lambda}}\right)^{\alpha}, \quad t>0, \lambda>0 \qquad (7.2)$$

where, λ is the scale parameter and α is the shape parameter and it is equal to 2

7.2.3 Weibull distribution

The cumulative distribution function (cdf) of the Weibull distribution is given by

$$F(t,\lambda) = 1 -- e^{-\left(\frac{t}{\lambda}\right)^{m}}, \quad t>0, \lambda>0 \qquad (7.3)$$

where, λ is the scale parameter and m is the shape parameter and it is equal to 2

7.2.4 Gamma distribution

The cumulative distribution function (cdf) of the Gamma distribution is given by

$$F(t,\lambda) = 1 - e^{-\frac{t}{\lambda}} \sum_{j=0}^{\gamma-1} \left(\frac{t}{\lambda}\right)^{j} / j!, \quad t>0, \lambda>0 \qquad (7.4)$$

where, λ is the scale parameter and $\gamma > 0$, is the shape parameter and it is equal to 2

If some other parameters are involved, then they are assumed to be known. The failure probability of an item by time t_0 is given by

$$p = F(t_0, \lambda), \quad t>0, \lambda>0 \qquad (7.5)$$

The quality of an item is usually represented by its true mean lifetime. Let us assume that the true mean λ can be represented by the scale parameter. In addition, it is convenient to specify the test time as a multiple of the specified life so that $a\lambda_0$ and the quality of an item as a ratio of the true mean to the specified life **(**λ/λ_0).

Then we can rewrite (7.5) as a function of '*a*' (termination time) and the ratio **(**λ/λ_0).

$$p = F(a\lambda_0, \lambda/\lambda_0), \qquad t > 0, \lambda > 0 \qquad (7.6)$$

when the underlying distribution is the Rayleigh distribution, the failure probability is

$$p = 1 - e^{-\frac{1}{2}\left(\frac{a}{\lambda/\lambda 0}\right)^2}, \quad t > 0, \lambda > 0 \qquad (7.7)$$

when the underlying distribution is the Generalized Exponential distribution, the failure probability is

$$p = \left(1 - e^{-\frac{a}{\lambda/\lambda 0}}\right)^{\alpha}, \quad t > 0, \lambda > 0 \qquad (7.8)$$

when the underlying distribution is the Weibull distribution, the failure probability is

$$p = 1 - e^{-\left(\frac{a}{\lambda/\lambda 0}\right)^m}, \quad t > 0, \lambda > 0 \qquad (7.9)$$

when the underlying distribution is the Gamma distribution, the failure probability is

$$P = 1 - e^{-a/(\lambda/\lambda o)} \sum_{j=0}^{\gamma-1} (a/(\lambda/\lambda o))^{j} \Big/ j!, \quad t > 0, \lambda > 0 \qquad (7.10)$$

7.3 Construction of Tables

The lot acceptance probability will be

$$L(p) = \left(\sum_{i=1}^{c} \binom{r-1}{i-1} p^{i-1} (1-p)^{r-1} \right)^{g} \qquad (7.11)$$

Where p is the probability that an item in a group fails before the termination time $t_0 = a\lambda_0$.

The minimum number of groups required can be determined by considering the consumer's risk when the true mean life equals the specified mean life (λ/λ_0) where p_0 is the failure probability at $\lambda = \lambda_0$. Here minimum testers r is obtained using the equations (7.1) and (7.4) along with lifetime distributions at worst case.

The time termination ratio **t**/λ_0 values are fixed as 0.7, 0.8, 1.0, 1.2, 1.5 and 1.8, and the mean ratio λ/λ_0 values are fixed as 4,6,8,10 and 12. The failure probability p is obtained such that it satisfies the following inequality at worst case $\lambda = \lambda_0$, $L(p) \leq \beta$ where β is taken as 0.10. The parameter value n is obtained using minimum angle method for predetermined values of acceptance numbers and satisfying the conditions $L(p_1) \geq 0.95$ and $L(p_2) \leq 0.10$ for Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution are presented in Table 7.1, Table 7.2 Table 7.3 and Table 7.4 respectively. The value of θ and $\tan\theta$ are also provided in each table. The parameters can be selected such that the angle is minimum. The parameter n = rg and θ can be obtained from the selected table corresponding to λ/λ_0 a, r and g along with producer's risk and consumer's risk.

Table – 7.1 The number of groups and probability of acceptance for Minimum angle method Group sampling plan using weighted binomial distribution when the life time of the items follows Rayleigh distribution (r=6, c=2) Section 7.1.1

a	λ/λ_0	g	$L(p_1)$	$L(p_2)$	$\tan\theta$	θ
0.7	4	7	0.984427	0.083618	0.224354	12.64513
	4	8	0.982222	0.05866	0.218826	12.3426
	6	7	0.996828	0.083618	0.23052	12.98106
	6	8	0.99638	0.05866	0.2245	12.6529
	8	7	0.998986	0.083618	0.233212	13.12742
	8	8	0.998842	0.05866	0.227057	12.79251
	10	7	0.999583	0.083618	0.23456	13.20064
	10	8	0.999523	0.05866	0.228352	12.86308
	12	7	0.999798	0.083618	0.23532	13.24192
	12	8	0.99977	0.05866	0.229087	12.90305
0.9	4	3	0.982283	0.098303	0.348456	19.21121
	4	4	0.976447	0.045369	0.33083	18.30574
	6	3	0.996333	0.098303	0.35838	19.71667
	6	4	0.995114	0.045369	0.338866	18.71976
	8	3	0.998822	0.098303	0.362808	19.94116
	8	4	0.998429	0.045369	0.342806	18.92203
	10	3	0.999514	0.098303	0.365044	20.05429
	10	4	0.999352	0.045369	0.34485	19.02676
	12	3	0.999765	0.098303	0.36631	20.1183
	12	4	0.999686	0.045369	0.346022	19.08673
1.2	4	2	0.964886	0.029371	0.50159	26.63789
	4	3	0.947794	0.005034	0.497735	26.46115
	6	2	0.992478	0.029371	0.512348	27.12825
	6	3	0.988739	0.005034	0.50162	26.63927
	8	2	0.997554	0.029371	0.51856	27.40943
	8	3	0.996333	0.005034	0.506467	26.86072
	10	2	0.998986	0.029371	0.521933	27.56152
	10	3	0.998479	0.005034	0.509413	26.99487
	12	2	0.999507	0.029371	0.523906	27.6503
	12	3	0.999261	0.005034	0.511211	27.07662
1.5	4	1	0.959847	0.041119	0.661186	33.47212
	6	1	0.991103	0.041119	0.678517	34.15757

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	6	2	0.982286	0.001691	0.657337	33.31838
	8	1	0.997068	0.041119	0.68824	34.53732
	8	2	0.994145	0.001691	0.662925	33.5414
	10	1	0.998776	0.041119	0.693526	34.74232
	10	2	0.997554	0.001691	0.666919	33.70009
	12	1	0.999404	0.041119	0.696625	34.86204
	12	2	0.998808	0.001691	0.669496	33.80213

Table – 7.1 The number of groups and probability of acceptance for Minimum angle method Group sampling plan using weighted binomial distribution when the life time of the items follows Rayleigh distribution (r=9, c=2) Section 7.1.2

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	3	0.98186	0.093392	0.22747	12.815
	4	4	0.975887	0.042372	0.216493	12.21563
	6	3	0.996244	0.093392	0.233164	13.12484
	6	4	0.994995	0.042372	0.220982	12.4611
	8	3	0.998793	0.093392	0.235779	13.26685
	8	4	0.998391	0.042372	0.223295	12.58739
	10	3	0.999502	0.093392	0.237111	13.3391
	10	4	0.999336	0.042372	0.22451	12.65368
	12	3	0.999759	0.093392	0.237868	13.38016
	12	4	0.999679	0.042372	0.225211	12.69186
0.9	4	2	0.968603	0.03826	0.331091	18.31925
	4	3	0.953276	0.007484	0.325683	18.03954
	6	2	0.99331	0.03826	0.336984	18.62297
	6	3	0.989982	0.007484	0.327569	18.13721
	8	2	0.997828	0.03826	0.340481	18.80274
	8	3	0.996744	0.007484	0.330262	18.27642
	10	2	0.9991	0.03826	0.342389	18.90065
	10	3	0.99865	0.007484	0.331913	18.36169
	12	2	0.999563	0.03826	0.343507	18.95796
	12	3	0.999345	0.007484	0.332924	18.41385
1.2	4	1	0.954577	0.029732	0.507377	26.90221
	6	1	0.989859	0.029732	0.513939	27.20038
	6	2	0.979822	0.000884	0.504063	26.75099
	8	1	0.996649	0.029732	0.519239	27.44008
	8	2	0.99331	0.000884	0.505892	26.8345
	10	1	0.9986	0.029732	0.522335	27.57965
	10	2	0.997201	0.000884	0.507944	26.92803
	12	1	0.998615	0.000811	0.522406	27.49456
	12	2	0.998635	0.000884	0.509406	26.99456
	12	2	0.998635	0.000884	0.509406	26.99456
1.5	6	1	0.976575	0.002177	0.661517	33.48531
	6	2	0.953699	0.000474	0.675878	34.05389

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	8	1	0.992072	0.002177	0.664639	33.60957
	8	2	0.984207	0.000474	0.668483	33.76206
	10	1	0.996649	0.002177	0.667852	33.73707
	10	2	0.99331	0.000474	0.668637	33.76815
	12	1	0.998356	0.002177	0.670126	33.82706
	12	2	0.996715	0.000474	0.669769	33.81293
1.8	6	1	0.954577	0.00078	0.794238	38.45794
	8	1	0.984176	0.000786	0.789664	38.29688
	8	2	0.968603	0.000618	0.802296	38.73994
	10	1	0.993221	0.000786	0.79146	38.36019
	10	2	0.986487	0.000618	0.796799	38.5478
	12	1	0.996649	0.000486	0.793636	38.43678
	12	2	0.99331	0.000618	0.796241	38.52825

Table – 7.2 The number of groups and probability of acceptance for Minimum angle method Group sampling plan using weighted binomial distribution when the life time of the Items follows Generalized Exponential distribution (r=6, c=2) Section 7.2.1

a	λ/λ_0	g	$L(p_1)$	$L(p_2)$	$\tan\theta$	θ
0.7	4	3	0.961612	0.098949	0.342107	18.88619
	6	3	0.990843	0.098949	0.352844	19.43507
	6	4	0.98781	0.045767	0.33406	18.47243
	6	5	0.98479	0.02117	0.32658	18.0861
	8	3	0.996839	0.098949	0.358769	19.73641
	8	4	0.995788	0.045767	0.339082	18.73087
	8	5	0.994737	0.021168	0.33088	18.30837
	10	3	0.998639	0.098949	0.362046	19.90259
	10	4	0.998185	0.045767	0.342002	18.88078
	10	5	0.997732	0.021168	0.333546	18.44591
	12	4	0.999096	0.045767	0.343779	18.9719
	12	3	0.999322	0.098949	0.363999	20.00145
0.9	6	3	0.97748	0.017624	0.436555	23.58393
	6	4	0.970087	0.004586	0.434003	23.46101
	6	2	0.98493	0.067723	0.456855	24.55352
	6	3	0.97748	0.017624	0.436555	23.58393
	6	4	0.970087	0.004586	0.434003	23.46101
	8	3	0.991981	0.017624	0.442015	23.84616
	8	4	0.989323	0.004586	0.437356	23.62247
	8	5	0.986672	0.001194	0.437027	23.60665
	10	2	0.997657	0.067723	0.469261	25.13882
	10	3	0.996488	0.017624	0.445804	24.02751
	10	4	0.99532	0.004586	0.440463	23.77173
	12	2	0.998821	0.067723	0.472117	25.27281
	12	3	0.998231	0.017624	0.448281	24.14577
	12	4	0.997642	0.004586	0.442661	23.87711
1.2	8	2	0.98493	0.008378	0.579538	30.09391
	8	1	0.992436	0.091534	0.628202	32.1371
	10	1	0.996607	0.091534	0.635832	32.44952
	10	2	0.993226	0.008378	0.584329	30.29899
	12	1	0.998264	0.091534	0.640646	32.64551
	12	2	0.996532	0.008378	0.587858	30.44947

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
1.5	6	1	0.957652	0.02783	0.68638	34.46491
	8	1	0.983579	0.02783	0.696615	34.86167
	8	2	0.967428	0.000774	0.688758	34.55742
	10	1	0.992436	0.02783	0.704727	35.1734
	10	2	0.98493	0.000774	0.690729	34.63397
	12	1	0.996064	0.02783	0.710402	35.39006
	12	2	0.992144	0.000774	0.693824	34.75383

Table - 7.2: The number of groups and probability of acceptance for Minimum angle method Group sampling plan using weighted binomial distribution when the life time of the Items follows Generalized Exponential distribution (r=9, c=2) Section 7.2.2

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	6	2	0.983523	0.038589	0.333038	18.41972
	6	3	0.975387	0.00758	0.325167	18.01283
	8	2	0.994223	0.038589	0.33709	18.62846
	8	3	0.991347	0.00758	0.327451	18.13106
	10	2	0.997494	0.038589	0.339689	18.76203
	10	3	0.996243	0.00758	0.329464	18.2352
	12	2	0.998746	0.038589	0.341334	18.84653
	12	3	0.99812	0.00758	0.330865	18.30756
0.9	6	1	0.979986	0.065067	0.457997	24.60763
	6	2	0.960373	0.004234	0.438252	23.66554
	8	1	0.992742	0.065067	0.464259	24.90349
	8	2	0.985536	0.004234	0.438887	23.69605
	10	1	0.996789	0.065067	0.46836	25.09652
	10	2	0.993588	0.004234	0.441077	23.80117
	12	1	0.998374	0.065067	0.471	25.22043
	12	2	0.99675	0.004234	0.442902	23.88864
1.2	8	1	0.979986	0.009343	0.583065	30.24496
	8	2	0.960373	0.000873	0.589354	30.51313
	10	1	0.99085	0.009343	0.586318	30.38385
	10	2	0.981783	0.000873	0.586205	30.37904
	12	1	0.995268	0.009343	0.589186	30.506
	12	2	0.990559	0.000873	0.586482	30.39085
1.5	8	1	0.957749	0.001076	0.695943	34.83573
	10	1	0.979986	0.001076	0.69443	34.77727
	12	1	0.989418	0.001076	0.69595	34.83597
1.8	10	1	0.963032	0.000108	0.782359	38.03818
	12	1	0.979986	0.000108	0.780075	37.95689

Table – 7.3 The number of groups and probability of acceptance for Minimum angle Group sampling plan using weighted binomial distribution when the life time of the Items follows Weibull distribution (r=6, c=2) Section 7.3.1

a	λ/λ_0	g	$L(p_1)$	$L(p_2)$	tanθ	θ
0.7	4	4	0.966196	0.016632	0.376186	20.61561
	4	5	0.957925	0.005973	0.375242	20.56822
	4	3	0.974538	0.046314	0.384835	21.04845
	6	4	0.992904	0.016632	0.382941	20.9539
	6	3	0.994673	0.046314	0.394212	21.51495
	6	5	0.99114	0.00597	0.37948	20.781
	8	3	0.998282	0.046314	0.398907	21.74739
	8	4	0.99771	0.016632	0.387071	21.15996
	8	5	0.997139	0.005973	0.383131	20.96339
	10	4	0.999054	0.016632	0.389329	21.27242
	10	3	0.99929	0.046314	0.401359	21.8685
	10	5	0.998818	0.005973	0.385242	21.06879
	12	3	0.999656	0.046314	0.402769	21.93804
	12	4	0.999542	0.016632	0.390653	21.33825
	12	5	0.999427	0.005973	0.386507	21.13185
0.8	4	2	0.971794	0.049941	0.470245	25.18504
	4	3	0.95799	0.011161	0.457841	24.60023
	6	2	0.994015	0.049941	0.482046	25.73619
	6	3	0.991036	0.011161	0.464433	24.91173
	6	4	0.988066	0.002494	0.461749	24.78508
	8	2	0.99806	0.049941	0.48808	26.01605
	8	3	0.997091	0.011161	0.469361	25.14353
	8	4	0.996124	0.002494	0.465724	24.97254
	8	5	0.995157	0.000557	0.46527	24.95115
	10	2	0.999197	0.049941	0.491257	26.16288
	10	3	0.998795	0.011161	0.472167	25.27512
	10	4	0.998394	0.002494	0.468248	25.09125
	10	5	0.997993	0.000557	0.467527	25.05737
	12	2	0.99961	0.049941	0.493091	26.24748
	12	3	0.999415	0.011161	0.473838	25.35339
	12	4	0.999221	0.002494	0.469811	25.16465
	12	5	0.999026	0.000557	0.468991	25.12617

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.9	4	2	0.956384	0.015909	0.537789	28.27085
	4	3	0.935295	0.002007	0.54193	28.45459
	4	4	0.914671	0.000253	0.553114	28.94758
	6	2	0.990555	0.015909	0.546755	28.66786
	6	3	0.985867	0.002007	0.541635	28.44153
	6	4	0.9812	0.000253	0.543244	28.51274
	8	2	0.996918	0.015909	0.553069	28.94561
	8	3	0.99538	0.002007	0.546185	28.64269
	8	4	0.993845	0.000253	0.546065	28.63739
	10	2	0.99872	0.015909	0.556643	29.10219
	10	3	0.99808	0.002007	0.549231	28.77696
	10	4	0.997441	0.000253	0.548617	28.74994
	12	2	0.999378	0.015909	0.55877	29.19514
	12	3	0.999067	0.002007	0.551153	28.86148
	12	4	0.998756	0.000253	0.550357	28.82648
1.2	6	2	0.971794	0.000163	0.744997	36.68599
	6	1	0.985796	0.012769	0.743928	36.64659
	8	1	0.995267	0.012769	0.754021	37.01706
	8	2	0.990555	0.000163	0.74801	36.79686
	10	1	0.998014	0.012769	0.759989	37.23445
	10	2	0.996032	0.000163	0.751882	36.93884
	12	1	0.999029	0.012769	0.763614	37.36586
	12	2	0.99806	0.000163	0.754709	37.04219
	12	3	0.997091	0.000208	0.755321	37.0645
1.5	6	1	0.967541	0.000565	0.862497	40.77766
	8	1	0.988869	0.000565	0.870233	41.03087
	8	2	0.977863	0.000319	0.879526	41.33246
	10	1	0.995267	0.000565	0.876999	41.25072
	10	2	0.990555	0.000319	0.88067	41.3694
	12	1	0.99767	0.000565	0.88165	41.401
	12	2	0.995346	0.000319	0.883208	41.45121

Table – 7.3: The number of groups and probability of acceptance for Minimum angle method Group sampling plan using weighted binomial distribution when the life time of the Items follows Weibull distribution (r=9, c=2) Section 7.3.2

a	λ/λ_0	g	$L(p_1)$	$L(p_2)$	$\tan\theta$	θ
0.7	4	2	0.955377	0.01445	0.379639	20.78873
	6	2	0.990329	0.01445	0.383096	20.96161
	6	3	0.985528	0.001737	0.380014	20.80751
	6	4	0.980751	0.000209	0.381274	20.87053
	8	2	0.996843	0.01445	0.386553	21.13414
	8	3	0.995268	0.001737	0.382219	20.91781
	10	2	0.998688	0.01445	0.388611	21.23666
	10	3	0.998033	0.001737	0.383908	21.00217
	10	4	0.997378	0.000209	0.383571	20.98538
	12	2	0.999363	0.01445	0.389859	21.29875
	12	3	0.999044	0.001737	0.385013	21.05738
0.8	6	3	0.975894	0.000116	0.466384	25.00359
	6	2	0.983864	0.002385	0.463675	24.87595
	8	2	0.994679	0.002385	0.466351	25.00203
	8	3	0.99203	0.000116	0.46653	25.01047
	10	2	0.99778	0.002385	0.468486	25.10241
	10	3	0.996671	0.000116	0.46794	25.07678
	12	3	0.998378	0.000116	0.469088	25.13073
	12	2	0.998918	0.002385	0.469902	25.16892
0.9	6	2	0.974807	0.000284	0.546825	28.67093
	8	2	0.991596	0.000284	0.547321	28.6928
	10	2	0.996474	0.000284	0.549167	28.77412
1.2	12	2	0.998278	0.000284	0.550637	28.83882
	8	1	0.987323	0.000266	0.750537	36.8896
	10	1	0.994595	0.000266	0.753045	36.98141
	12	1	0.997336	0.000266	0.755335	37.06503
	8	1	0.970914	0.000105	0.885821	41.5352
1.5	10	1	0.987323	0.000105	0.883554	41.46233
	12	1	0.993675	0.000105	0.884694	41.49898
	8	1	0.954057	0.000109	0.965483	43.99391
	10	1	0.974961	0.000109	0.952813	43.6158
	12	1	0.987323	0.000109	0.950639	43.55042

Table – 7.4 The number of groups and probability of acceptance for Minimum angle Group sampling plan using weighted binomial distribution when the life time of the Items follows Gamma distribution (r=6, c=2) Section 7.4.1

a	λ/λ_0	g	$L(p_1)$	$L(p_2)$	$\tan\theta$	θ
0.7	4	12	0.978498	0.098578	0.161568	9.177861
	4	13	0.976727	0.08127	0.158765	9.021253
	6	12	0.995309	0.098578	0.166724	9.465506
	6	13	0.99492	0.08127	0.16364	9.29331
	8	12	0.998447	0.098578	0.169128	9.599484
	8	13	0.998317	0.08127	0.16596	9.422912
	10	12	0.999347	0.098578	0.170373	9.668799
	10	13	0.999293	0.08127	0.167171	9.490413
	12	12	0.99968	0.098578	0.171089	9.708667
	12	13	0.999653	0.08127	0.167869	9.529355
0.8	4	9	0.973635	0.079884	0.194332	10.99735
	4	10	0.970748	0.060327	0.190774	10.80076
	6	9	0.994153	0.079884	0.200238	11.32302
	6	10	0.993505	0.060327	0.19618	11.09933
	8	9	0.99805	0.079884	0.203154	11.48358
	8	10	0.997833	0.060327	0.198963	11.25278
	10	9	0.999177	0.079884	0.204694	11.56829
	10	10	0.999085	0.060327	0.200449	11.33468
	12	9	0.999595	0.079884	0.205588	11.61748
	12	10	0.99955	0.060327	0.201317	11.38249
0.9	4	6	0.972975	0.098997	0.235361	13.24413
	4	7	0.968542	0.067333	0.228249	12.85743
	6	6	0.993916	0.098997	0.242851	13.65007
	6	7	0.992906	0.067333	0.234808	13.21411
	8	6	0.997956	0.098997	0.246557	13.85043
	8	7	0.997616	0.067333	0.238255	13.40116
	10	6	0.999134	0.098997	0.248521	13.95645
	10	7	0.998989	0.067333	0.240113	13.50184
	12	6	0.999573	0.098997	0.249665	14.01816
	12	7	0.999502	0.067333	0.241203	13.56089

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
1.2	6	6	0.982345	0.008636	0.328486	18.18462
	6	7	0.979433	0.003912	0.327876	18.15305
	8	6	0.993916	0.008636	0.332075	18.37003
	8	7	0.992906	0.003912	0.330828	18.30566
	10	6	0.997385	0.008636	0.334487	18.49441
	10	7	0.99695	0.003912	0.333042	18.41993
	12	6	0.998699	0.008636	0.336033	18.57404
	12	7	0.998483	0.003912	0.33451	18.4956
1.5	6	2	0.98673	0.071868	0.454359	24.4351
	6	3	0.980161	0.019267	0.432592	23.39294
	8	2	0.995331	0.071868	0.462007	24.79729
	8	3	0.993005	0.019267	0.438153	23.66079
	10	2	0.997968	0.071868	0.46646	25.00719
	10	3	0.996953	0.019267	0.441848	23.83813
	12	2	0.998981	0.071868	0.469181	25.13508
	12	3	0.998472	0.019267	0.444221	23.95181
1.8	6	2	0.974835	0.020877	0.52437	27.67117
	6	3	0.962491	0.003017	0.521355	27.53551
	8	2	0.990909	0.020877	0.531266	27.98021
	8	3	0.986395	0.003017	0.524056	27.65706
	10	2	0.995985	0.020877	0.536127	28.19697
	10	3	0.993984	0.003017	0.527547	27.81375
	12	2	0.997968	0.020877	0.539333	28.33945
	12	3	0.996953	0.003017	0.530192	27.93217

Table – 7.4 The number of groups and probability of acceptance for Minimum angle Group sampling plan using weighted binomial distribution when the life time of the items follows Gamma distribution (r=9, c=2) Section 7.4.2

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	6	0.970777	0.067962	0.157471	8.948927
	4	7	0.96599	0.043415	0.154098	8.760263
	6	6	0.993518	0.067962	0.161531	9.175801
	6	7	0.992442	0.043415	0.157536	8.952595
	8	6	0.997842	0.067962	0.16367	9.295162
	8	7	0.997482	0.043415	0.15952	9.063474
	10	6	0.99909	0.067962	0.164818	9.359215
	10	7	0.998939	0.043415	0.16061	9.12434
	12	6	0.999553	0.067962	0.165489	9.396674
	12	7	0.999479	0.043415	0.161253	9.16028
0.8	4	4	0.968326	0.078551	0.195201	11.04529
	4	5	0.960565	0.041585	0.188997	10.70251
	6	4	0.992842	0.078551	0.200233	11.32275
	6	5	0.99106	0.041585	0.192813	10.91344
	8	4	0.997596	0.078551	0.20296	11.47289
	8	5	0.996996	0.041585	0.195234	11.04715
	10	4	0.998982	0.000001	0.188366	10.66755
	10	5	0.998727	0.041585	0.196599	11.12246
	12	4	0.999498	0.078551	0.205312	11.6023
	12	5	0.999373	0.041585	0.197415	11.16747
0.9	4	3	0.963811	0.077069	0.231973	13.06007
	4	4	0.952041	0.032797	0.223771	12.61334
	6	3	0.991657	0.077069	0.237628	13.36717
	6	4	0.988891	0.032797	0.227312	12.80642
	8	3	0.997173	0.077069	0.240891	13.54399
	8	4	0.996232	0.000001	0.222483	12.54306
	10	3	0.998796	0.077069	0.242699	13.64188
	10	4	0.998396	0.032797	0.231673	13.04375
	12	3	0.999405	0.077069	0.243774	13.70003
	12	4	0.999207	0.032797	0.232657	13.09726
1.2	6	2	0.984035	0.035552	0.337222	18.63525
	6	3	0.976149	0.006703	0.329931	18.2593
	8	2	0.99443	0.035552	0.341219	18.84059

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	8	3	0.991657	0.006703	0.332185	18.37573
	10	2	0.997591	0.035552	0.343774	18.97162
	10	3	0.996388	0.006703	0.334171	18.47812
	12	2	0.998797	0.035552	0.345389	19.05433
	12	3	0.998196	0.006703	0.335548	18.5491
1.5	6	1	0.982322	0.068828	0.455039	24.4674
	6	2	0.964956	0.004737	0.432897	23.40763
	8	1	0.993656	0.068828	0.461325	24.76508
	8	2	0.987352	0.00001	0.432116	23.36996
	10	1	0.997211	0.068828	0.465313	24.95317
	10	2	0.99443	0.004737	0.436488	23.58068
	12	1	0.998593	0.068828	0.467842	25.07219
	12	2	0.997188	0.004737	0.438292	23.66747
1.8	6	1	0.967068	0.021658	0.529111	27.88381
	6	2	0.95522	0.000469	0.535144	28.15321
	8	1	0.987788	0.021658	0.533412	28.07599
	8	2	0.975726	0.000469	0.52842	27.85288
	10	1	0.994533	0.021658	0.537358	28.25169
	10	2	0.989097	0.000469	0.528796	27.86969
	12	1	0.997211	0.021658	0.540183	28.37716
	12	2	0.99443	0.000469	0.530179	27.9316

7.4 Example

Assume that an experimenter wants to establish that the lifetime of the electrical devices produced in the factory ensures that the true unknown mean life is at least 1000 hours when the ratio of the time mutlipilier is a = 0.7 and specified life is $\lambda/\lambda_0 = 6$ and acceptance numbers c = 2. Following are the results obtained when the lifetime of the test items follows the Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution respectively. A Group acceptance sampling plan using weighted binomial distribution is developed to test if the true unknown mean is greater than 1,000 hours based on a testing time of 700 hours and using testers equipped with 6 items each. Based on producer's risk, consumer's risk values and the test termination time multiplier, the number of groups of Group acceptance sampling plan using weighted binomial distribution can be found using equations (7.1) to (7.4).

7.4.1 Rayleigh distribution

Suppose that the lifetime of a product follows the Rayleigh distribution. From Table 7.1, the minimum number of groups required for r = 6 items are g = 8 and one can observe that the minimum angle is $\theta = 12.6529^0$ and also α = 0.0036, β= 0.0586 which is very much less than the specified risk. Thus, we will draw a random sample of size n = 48 (*n = gr*) items and allocate 6 items to each of 8 groups to put on test for 700 hrs. The lot will be accepted if not more than 2 failure occurs before 700 hrs in each of 8 groups and also it satisfies the condition of producer's risk and consumer's risk α ≤ 0.05, β ≤ 0.10. Thus the required Group sampling plan using weighted binomial distribution has parameters as (6,8). For the same conditions when the time of experiment is 1500 hours, the probability of acceptance is 0.982286, the producer's risk is 0.0178 and consumer's risk 0.0017. The sample size is 12 and number of groups is 2 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life to specified life is 12, there is no change in the sample size but there is increase in the probability of acceptance. The probability of acceptance is 0.9997 which is almost equal to 1 and consumer's risk is 0.0586 which shows that there is a reduction in consumer's risk.

7.4.2 Generalized Exponential distribution

Suppose that the lifetime of a product follows the Generalized Exponential distribution. From Table 7.2, the minimum number of groups required for r = 6 items and g = 5 and one can observe that the minimum angle is $\theta = 18.0861^0$ and $\alpha = 0.0152$, $\beta = 0.0211$ which is very much less than the specified risk. Thus, we will draw a random sample of size n = 30 (*n = gr*) items and allocate 6 items to each of 5 groups to put on test for 700 hrs. The lot will be accepted if not more than 2 failure occurs before 700 hrs in each of 5 groups and it satisfies the condition of producer's risk and consumer's risk $\alpha \leq 0.05$, $\beta \leq 0.10$. Thus, the required Group sampling plan using weighted binomial distribution has parameters as (6,5). For the same conditions when the time of experiment is 1500 hours, the probability of acceptance is 0.9576, the producer's risk is 0.0423 and consumer's risk 0.0278. The sample size is 6 and number of groups is 1 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life to specified life is 12, there is slight change in the sample size but there is increase in the probability of acceptance. The probability of acceptance is 0.99909 which is almost equal to 1 and consumer's risk is 0.045767 which shows that there is a reduction in consumer's risk.

7.4.3 Weibull distribution

Suppose that the lifetime of a product follows the Weibull distribution. From Table 7.3, the minimum number of groups required for r = 6 items are g = 5 and one can observe that the minimum angle is $\theta = 20.78097^0$ and also $\alpha = 0.0088$, $\beta = 0.0059$ which is very much less than the specified risk. Thus, we will draw a random sample of size n = 30 (*n = gr*) items and allocate 6 items to each of 5 groups to put on test for 700 hrs. The lot will be accepted if not more than 2 failure occurs before 700 hrs in each of 5 groups and it satisfies the condition of producer's risk and consumer's risk $\alpha \leq 0.05$, $\beta \leq 0.10$. Thus, the required Group sampling plan using weighted binomial distribution has parameters as (6, 5). For the same conditions when the time of experiment is 1500 hours, the probability of acceptance is 0.9675, the producer's risk is 0.0325 and consumer's risk 0.0005.

The sample size is 6 and number of groups is 1 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life to specified life is 12, there is no change in the sample size but there is increase in the probability of acceptance. The probability of acceptance is 0.999 which is almost equal to 1 and consumer's risk is 0.0059 which shows that there is a reduction in consumer's risk.

7.4.4 Gamma distribution

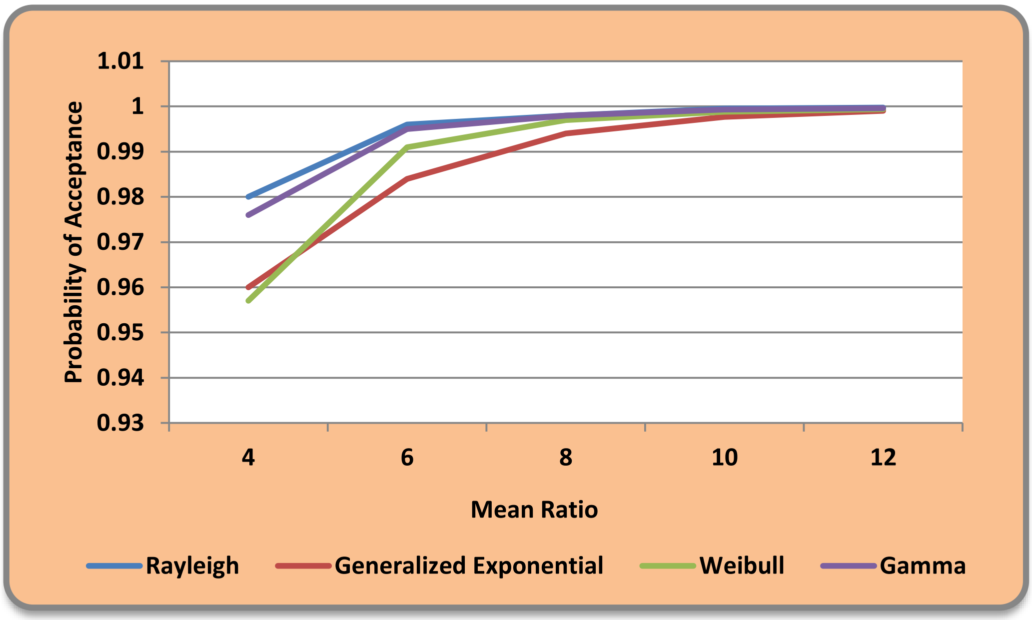
Suppose that the lifetime of a product follows the Gamma distribution. From Table 7.4, the minimum number of groups required for r = 6 items are g = 13 and one can observe that the minimum angle is $\theta = 9.2933^0$ and also $\alpha = 0.0051$, $\beta = 0.08127$ which is very much less than the specified risk. Thus, we will draw a random sample of size n = 78 ($n = gr$) items and allocate 6 items to each of 13 groups to put on test for 700 hrs. The lot will be accepted if not more than 2 failure occurs before 700 hrs in each of 13 groups and it satisfies the condition of producer's risk and consumer's risk $\alpha \le 0.05$, $\beta \le 0.10$. Thus, the required Group sampling plan using weighted binomial distribution has parameters as (6,13). For the same conditions when the time of experiment is 1500 hours, the probability of acceptance is 0.980161, the producer's risk is 0.0198 and consumer's risk 0.01926. The sample size is 18 and number of groups is 3 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life to specified life is 12, there is no change in the sample size but there is increase in the probability of acceptance. The probability of acceptance is 0.9996 which is almost equal to 1 and consumer's risk is 0.08127 which shows that there is a reduction in consumer's risk.

Comparison of results of Producer's risk, Consumer's risk and sample size for Group sampling plan using weighted binomial distribution when the life time of the items follows different distributions provided in table 7.5.

Table – 7.5 Comparison of results of Producer's risk, Consumer's risk and group size for Group sampling plan using weighted binomial distribution when the life time of the items follows different distributions (a=0.7, r=6,c=2)

S.No	λ/λ_0	Distribution	Producer's risk	Consumer's risk	g
1	6	Rayleigh	0.0036	0.0586	8
		Generalized Exponential	0.0152	0.0211	5
		Weibull	0.0088	0.0059	5
		Gamma	0.0051	0.0812	13
2	8	Rayleigh	0.0012	0.0586	8
		Generalized Exponential	0.0052	0.0211	5
		Weibull	0.0029	0.0059	5
		Gamma	0.0017	0.0812	13
3	10	Rayleigh	0.0005	0.0586	8
		Generalized Exponential	0.0023	0.0211	5
		Weibull	0.0012	0.0059	5
		Gamma	0.0007	0.0812	13
4	12	Rayleigh	0.0002	0.0586	8
		Generalized Exponential	0.0009	0.0457	4
		Weibull	0.0005	0.0059	5
		Gamma	0.0004	0.0812	13

Figure - 7.1 O C curve of Group sampling Plan using weighted binomial distribution when the life time of the items follows different distributions with (a=0.7, r=6, c=2)



From the Table 7.5 one can conclude that when the Generalized Exponential distribution and Weibull distribution is followed, the sample size is very much less than the sample size of all other distribution. And at the same time producer's risk and consumer's risk also less and the sum of the risks is also very much less for Generalized Exponential distribution and Weibull distribution. Figure 7.1 shows the OC curves of all four distributions. From the figure, one can observe that Probability of acceptance of Rayleigh distribution and Gamma distribution is more than any other distributions. It can be seen that by applying minimum angle method minimizes simultaneously the consumer's and producer's risk. This minimum angle method plan provides better discrimination of accepting good lots.