CHAPTER 1

CHAPTER 1

STRONGLY AND WEAKLY GENERALIZED CLOSED SETS AND CONTINUOUS MAPS IN TOPOLOGICAL SPACES

In this chapter strongly-g-closed set and due to Sundaram and Pushpalatha [63] in topological spaces and wg-closed sets and wg-continuous maps due to Sundaram and Nagaveni [64] in topological spaces are discussed. Properties and characterization of these concepts are analyzed.

SECTION: 1.1
PRELIMINARIES

Definition: 1.1.1

Let (X,τ) be a topological space. A subset A of X is said to be **semi-closed** if $int(cl(A)) \subseteq A$.

Definition: 1.1.2

Let (X,τ) be a topological space. A subset A of X is said to be **pre-closed** if $cl(int(A)) \subseteq A$.

Definition: 1.1.3

Let (X,τ) be a topological space. A subset A of X is said to be **semi pre-closed** (or) **β-closed** if $int(cl(int(A))) \subseteq A$.

Definition: 1.1.4

Let (X,τ) be a topological space. A subset A of X is said to be **α-closed** if $cl(int(cl(A)) \subseteq A$.

Definition: 1.1.5

Let (X,τ) be a topological space. A subset A of X is said to be **b-closed** if $cl(int(A)) \subseteq int(cl(A)) \subseteq A$.

Definition: 1.1.6

Let (X,τ) be a topological space. A subset A of X is said to be **regular-closed set** if $A = cl(int(A))$.

Definition: 1.1.7

Let (X,τ) be a topological space. A subset A of X is said to be **g-closed** if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is open in X.

The complement of g-closed set is called **g-open** in X.

Definition: 1.1.8

Let (X,τ) be a topological space. A subset A of X is said to be **w-closed set** if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is semi-open in X.

Definition: 1.1.9

Let (X,τ) be a topological space. A subset A of X is said to be **Semi-generalized closed (sg-closed)** set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

Definition: 1.1.10

Let (X,τ) be a topological space. A subset A of X is said to be **generalized semi-closed (gs-closed)** set if scl (A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

Definition: 1.1.11

Let (X,τ) be a topological space. A subset A of X is said to be **generalized α-closed (gα-closed)** set if α-cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in X.

Definition: 1.1.12

Let (X,τ) be a topological space. A subset A of X is said to be **α-generalized closed (αg-closed)** set if α-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

Definition: 1.1.13

Let (X,τ) be a topological space. A subset A of X is said to be **generalized pre-closed (gp-closed)** set if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

Definition: 1.1.14

Let (X,τ) be a topological space. A subset A of X is said to be **generalized semi-pre-closed (gsp-closed)** set if spcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

Definition: 1.1.15

The semi-closure of a subset A of X is intersection of all semi-closed sets containing A and denoted by scl(A).

Definition: 1.1.16

The pre-closure of a subset A of X is intersection of all pre-closed sets containing A and denoted by pcl (A).

Definition: 1.1.17

The semi-pre-closure of a subset A of X is intersection of all semi-pre-closed sets containing A and denoted by spcl(A).

Definition: 1.1.18

The α-closure of a subset A of X is intersection of all α-closed sets containing A and denoted by αcl (A).

Definition: 1.1.19

A topological space X is said to be **$T_{1/2}$-space** if every g-closed set is closed.

Definition: 1.1.20

A function f: $X \to Y$ is said to be **pre-continuous** if $f^{-1}(V)$ is a pre-open in X for each open set V of Y.

Definition: 1.1.21

A function $f: X \to Y$ is said to be **β-continuous** if $f^{-1}(V)$ is a semi-pre-closed set of X for every closed set V of Y.

Definition: 1.1.22

A function f: X→ Y is said to be α-**continuous (or strongly semi-continuous)** if f^{-1} (V) is a α-open set in X for each open set V of Y.

Definition: 1.1.23

A function f: X→ Y is said to be **generalized semi-continuous (gs-continuous)** if f^{-1}(V) is a gs-open set in X for each open set V of Y.

Definition: 1.1.24

A function f: X→ Y is said to be **semi-generalized continuous (sg-continuous)** if f^{-1}(V) is a sg-open set in X for each open set V of Y.

Definition: 1.1.25

A function f: X→ Y is said to be **w-continuous if** f^{-1} (V) is a w-open in X for each open set V of Y.

Definition: 1.1.26

A function f: X→ Y is said to be **generalized semi-pre-continuous (gsp-continuous)** if f^{-1}(V) is a gsp-open set in X for each open set V of Y

Definition: 1.1.27

A map f: X→ Y is said to be **completely continuous** if f^{-1}(V) of regular closed in (X,τ) for every closed set V of (Y,σ).

Definition: 1.1.28

A map f: X→ Y is said to be **perfectly continuous** if the inverse image of every open set in Y in both open and closed in X.

Definition: 1.1.29

A map f: X→ Y is said to be α **-irresolute** if f^{-1}(V) of α-closed in (X,τ) for every α-closed set V of (Y,σ).

SECTION: 1.2

STRONGLY AND WEAKLY GENERALIZED CLOSED SETS

Definition: 1.2.1

Let (X,τ) be a topological space. A subset A of X is said to be **strongly-g-closed** if cl(A)$\subseteq$G whenever A$\subseteq$G and is G is g-open in X.

Preposition: 1.2.2

Union of two strongly-g-closed sets is strongly-g-closed.

Preposition: 1.2.3

If A is strongly-g-closed and g-open then A is closed.

Preposition: 1.2.4

A sub set A of X is strongly g-closed in X if and only if cl(A) − A contains no non empty g-closed set in X.

Preposition: 1.2.5

If A is strongly g-closed and $A \subseteq B \subseteq cl(A)$ then B is strongly g-closed.

Preposition: 1.2.6

Every strongly g-closed set in X is a g-closed set in X.

Preposition: 1.2.7

For each $x \in X$, $\{x\}$ is g-closed in X or $\{x\}^C$ is strongly g-closed in X.

Preposition: 1.2.8

A subset A of X is strongly g-open in X if and only if $F \subseteq int(A)$ whenever $F \subseteq A$ and F is g-closed in X.

Definition: 1.2.9

A subset A of a topological space X is said to be **weakly generalized closed (wg-closed)**set in X if G contains cl(int(A)) whenever G contains A and G is open in X. The complement of wg-closed is **wg-open**.

Theorem: 1.2.10

If a subset A of a topological space X is g-closed then it is wg-closed in X.

Remark: 1.2.11

The converse of the above theorem [1.2.10] need not be true.

Example: 1.2.12

Let X = {a, b, c} and τ = {X, ϕ, {a}, {a, b}}, then {b} is wg-closed but not g-closed.

Theorem: 1.2.13

If a subset A of a topological space is both open and wg-closed then it is closed.

Corollary: 1.2.14

If A is both open and wg-closed in X, then it is both regular open and regular closed in X.

Corollary: 1.2.15

If A is both open and wg-closed then it is rg-closed.

Theorem: 1.2.16

If a subset A of a topological space X is both wg-closed and semi-open then it is g-closed

Corollary: 1.2.17

If subset A of a topological space X is both wg-closed and open then it is g-closed.

Theorem: 1.2.18

A set A is wg-closed if and only if cl (int (A)) − A is closed.

Theorem: 1.2.19

Suppose that $B \subseteq A \subseteq X$, B is wg-closed set relative to A and A is a wg-closed subset of X, then B is a wg-closed set relative to X.

Corollary: 1.2.20

Let A be wg-closed and suppose that F is closed then $A \cap F$ is a wg-closed set.

Theorem: 1.2.21

Let A be wg-closed and $A \subseteq B \subseteq cl(int(A))$ then B is wg-closed.

Theorem: 1.2.22

Let $A \subseteq Y \subseteq X$ and suppose that A is wg-closed in X then A is wg-closed relative to Y.

Theorem: 1.2.23

If subset A of a topological space X is wg-closed then it is gsp-closed.

Remark: 1.2.24

The converse of the above theorem [1.2.23] need not be true.

Example: 1.2.25

Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a , b}}, then {a} is gsp-closed but not wg-closed.

Theorem: 1.2.26

If a subset A of a topological space X is no where dense then it is wg-closed.

Remark: 1.2.27

The converse of the above theorem [1.2.26] need not be true.

Example: 1.2.28

Let X = {a, b, c} and τ = {X, ϕ , {a}, {b , c}}, then {a} is wg-closed but not no where dense.

Remark: 1.2.29

If A and B are wg-closed sets then their union need not be wg-closed.

Example: 1.2.30

Let X = {a, b, c} and τ = {X, ϕ, {a}, {a , b}}, then the subsets {a} and {b} are wg-closed but their union {a ,b} is not wg-closed.

Remark: 1.2.31

If the subset A and B are wg-closed then their intersection need not be wg-closed.

Example: 1.2.32

Let X = {a, b, c} and τ = {X, ϕ, {a}}. Then the subsets {a, b} and {a, c} are wg-closed but their intersection {a} is not wg-closed.

Theorem: 1.2.33

If a subset A of a topological space X is pre-closed then it is wg-closed.

Example: 1.2.34

Let X = {a, b, c} and τ = {X, ϕ, {a}}. Then the subsets {a, b} is wg-closed but not pre-closed.

Theorem: 1.2.35

If a subset A of a topological space X is α g-closed then it is wg-closed.

Remark: 1.2.36

The converse of the above theorem [1.2.35] need not be true.

Example: 1.2.37

Let X = {a, b, c, d} and τ = {X, ϕ, {c}, {a , b}, {a, b, c}, {a, b, d}}, then the subset {a} is wg-closed but not α g-closed.

Corollary: 1.2.38

If a subset A of a topological space X is α-closed then it is wg-closed but not conversely.

Corollary: 1.2.39

If a subset A of a topological space X is gα-closed then it is wg-closed but not conversely.

Theorem: 1.2.40

If a subset A of a topological space w-closed then it is wg-closed.

Remark: 1.2.41

The converse of the above theorem [1.2.40] need not be true.

Example: 1.2.42

Let X = {a, b, c} and τ = {X, ϕ, {a}},then the subsets {a, b} is wg-closed but not w-closed.

Remark: 1.2.43

The following examples show that rg-closed sets and wg-closed sets are independent.

Example: 1.2.44

Let X = {a, b, c, d} and τ = {X, ϕ, {d}, {a , c}, {a, c, d }}, then the subset {c} is wg-closed but not rg-closed.

Example: 1.2.45

Let X = {a, b, c, d} and τ = {X, ϕ, {a}, {a , d}}, then the subset {a} is rg-closed but not wg-closed.

Remark: 1.2.46

The following examples show that wg-closed sets and semi-closed sets are independent.

Example: 1.2.47

Let X = {a, b, c} and τ = {X, ϕ, {a}}, then the subset {a, b} is wg-closed but not semi-closed.

Example: 1.2.48

Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}, then the subset {b} is semi-closed but not wg-closed.

Remark: 1.2.49

The following examples show that β-closed and wg-closed sets are independent.

Example: 1.2.50

Let X = {a, b, c} and τ = {X, ϕ, {a}}, then the subset {a , b} is wg-closed but not β-closed.

Example: 1.2.51

Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}, then the subset {a} is β-closed but not wg-closed.

Remark: 1.2.52

The following examples show that sg-closed set and wg-closed set are independent.

Example: 1.2.53

Let X = {a, b, c} and τ = {X, ϕ, {a}}, then the subset {a , b} is wg-closed but not sg-closed.

Example: 1.2.54

Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}, then the subset {b} is sg-closed but not wg-closed.

Remark: 1.2.55

The following examples show that gs-closed set and wg-closed set are independent.

Example: 1.2.56

Let X = {a, b, c, d} and τ = {X, ϕ, {c}, {a, b, c}, {a, b},{a, b, d}}, then the subset {a} is wg-closed but not gs-closed.

Example: 1.2.57

Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}, then the subset {b} is gs-closed but not wg-closed.

Remark: 1.2.58

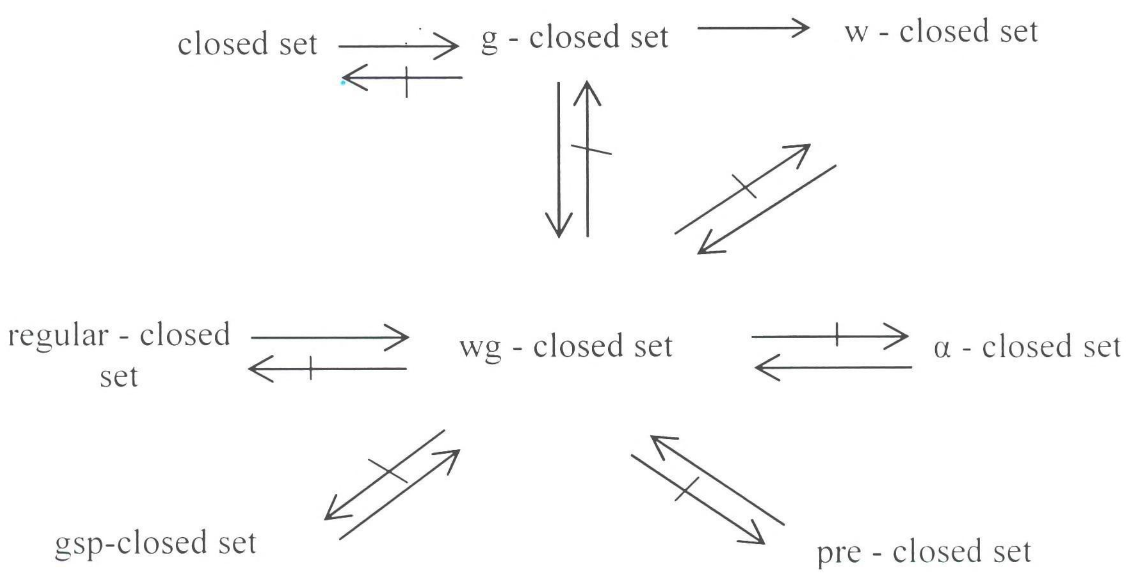
The following examples show that generalized A-set and wg-closed set are independent.

Example: 1.2.59

Let X = {a, b, c} and τ = {X, ϕ, {a}, {a, b}}, then the subset {a} is generalized A-set but not wg-closed set and the subset {b} is wg-closed but not generalized A-set.

Remark: 1.2.60

From the above discussion we obtain the following diagram:



Theorem: 1.2.61

If a subset A of a topological space X is g-open then it is wg-open.

Remark: 1.2.62

The converse of the above theorem [1.2.61] need not be true.

Example: 1.2.63

Let X = {a, b, c} and τ = {X, ϕ, {a}, {a, b}}, then the subset {a, c} is wg-open but not g-open.

Remark: 1.2.64

By the remark [1.2.29] and [1.2.31] both the union and the intersection of two wg-open sets need not be wg-open.

Theorem: 1.2.65

A subset A of a topological space X is wg-open if and only if $F \subseteq int(cl(A))$ whenever F is closed and $F \subseteq A$.

SECTION: 1.3
WEAKLY GENERALIZED CONTINUOUS MAPS

Definition: 1.3.1

A function f: X $\rightarrow$ Y is said to be **generalized continuous (g-continuous)** if $f^{-1}(V)$ is g-open in X for each open set V in Y.

Preposition: 1.3.2

If f: $(X, \tau) \rightarrow (Y, \sigma)$ is continuous then it is g-continuous.

Definition: 1.3.3

Let X and Y be topological space. A map f: $X \rightarrow Y$ is said to be **Weakly generalized continuous (wg-continuous)**. If the inverse image of every open set in Y is wg-open in X.

Theorem: 1.3.4

If a map f: $X \rightarrow Y$ from a topological space X into a topological space Y is continuous then it is wg-continuous.

Remark: 1.3.5

The converse of the above theorem [1.3.4] need not be true.

Example: 1.3.6

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{b, c\}\}$, and a map f: $X \rightarrow Y$ is defined by f (a) = b, f (b) = c, f(c) = a. Then f is wg-continuous but not continuous as the inverse image of the open set $\{b,c\}$ in Y is {a, b} is not open in X .

Theorem: 1.3.7

A map f: $X \rightarrow Y$ is wg-continuous if only if the inverse image of every closed set in Y is wg-closed in X.

Theorem: 1.3.8

Let X and Y be topological spaces. If a map f: $X \to Y$ is g-continuous then it is wg-continuous.

Remark: 1.3.9

The converse of the above theorem [1.3.8] need not be true.

Example: 1.3.10

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a, c\}\}$ and f be the identity map, then f is wg-continuous, but not g-continuous, as the inverse image of the open set {a, c} in Y is {a, c} in X is not g-open.

Theorem: 1.3.11

If a map f: $X \to Y$ is perfectly continuous and wg-continuous, then it is g-continuous.

Theorem: 1.3.12

If a map f: $X \to Y$ is completely continuous (resp.R-map) then it is wg-continuous.

Theorem: 1.3.13

If a map f: $X \to Y$ is α-continuous (or strongly semi-continuous) then it is wg-continuous.

Remark: 1.3.14

The converse of the above theorem [1.3.13] need not be true.

Example: 1.3.15

` Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, b}} and σ ={Y, ϕ, {a}, {a, b}}. Consider f: X$\rightarrow$Y is defined by f (a) = a − f (b), f(c) = b. Then f is wg-continuous but not α- continuous, since the pre-image of the open set {a} in Y is {a, b} is not α-open in X.

Theorem: 1.3.16

If a map f: X$\rightarrow$Y is w-continuous then it is wg-continuous.

Remark: 1.3.17

The converse of the above theorem [1.3.16] need not be true.

Example: 1.3.18

The function given example [1.3.10] is wg-continuous but not w-continuous as f^{-1}({a, c}) = {a, c} is not w-open in X.

Theorem: 1.3.19

If a map f: X$\rightarrow$Y is pre-continuous then it is wg-continuous.

Remark: 1.3.20

The converse of the above theorem [1.3.19] need not be true.

Example: 1.3.21

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, c}} and σ ={Y, ϕ, {a}, {b, c}}, and a map f: X$\rightarrow$Y is defined by f(a) = a = f(b), f(c) = c.Then f is wg-continuous but not pre-continuous as the inverse image of an the open set {b, c} in Y is {c} is not pre-open in X.

Remark: 1.3.22

The following examples show that wg-continuous function and sg-continuous function are independent.

Example: 1.3.23

Consider the topological spaces (X, τ) and (Y, σ) and the function given in example [1.3.21]. Then the function f is wg-continuous but not sg-continuous as the inverse image of the open set {b, c} in Y is {c} is not sg-open in X.

Example: 1.3.24

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Consider the map $f: X \to Y$ is defined by $f(a) = a = f(c)$, $f(b) = b$. Then f is sg-continuous but not wg-continuous as the inverse image of an the open set {a} in Y is {a, c} is not wg-open in X.

Theorem: 1.3.25

If a map $f: X \to Y$ is α-irresolute then it is wg-continuous.

Remark: 1.3.26

The converse of the above theorem [1.3.25] need not be true.

Example: 1.3.27

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, b}} and σ ={Y, ϕ, {a, c}} and f be the identity map, then f is wg-continuous, but not α-irresolute, since for the α-open set {c}, the inverse image {c} is not α-open in X.

Remark: 1.3.28

From the above discussion we obtain the following diagram: