Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – May 2025
II Semester

Class : I P.G. **Time: 3 Hours**
Major : Mathematics **Max. Marks: 100**

23MMAC08 Real Analysis II

Course Outcomes:

CO1: Distinguish between the Lebesgue and Riemann integrals.
CO2: Apply the concept of Lebesgue integral to broader class of functions.
CO3: Test the convergence using Riemann's localization theorem.
CO4: Solve problems in a closed form using Fourier integrals.
CO5: Evaluate the multiple integrals using iterated integration.

Part A **10 x 1 = 10**
Choose the Correct Answer

1. In Lebesgue's approach the interval is subdivided into more general types of sets called CO1K1
   a. Lebesgue integral set b. bounded set c. measurable set d. closed set

2. A sequence of real – valued functions $\{f_n\}$ defined on a set S is said to be increasing on S if __ for all x in S and all n. CO1K1
   a. $f_n(x) \le f_{n+1}(x)$ b. $f_n(x) \ge f_{n+1}(x)$ c. $f_n(x) = f_{n+1}(x)$ d. $f_n(x) < f_{n+1}(x)$

3. If f is measurable and bounded on a bounded interval I, then CO2K1
   a. $|f| \in L(I)$ b. $f \in L(I)$ c. $\frac{1}{f} \in L(I)$ d. none of the above

4. If A and B are measurable and $A \subseteq B$, then CO2K1
   a. $\mu(A) \le \mu(B)$ b. $\mu(A) = \mu(B)$ c. $\mu(A) \ge \mu(B)$ d. $\mu(A) + \mu(B) = 0$

5. A function f is said to be ____ with period $p \ne 0$ if f is defined on R and if $f(x+p) = f(x)$ for all x. CO3K1
   a. integral b. Fourier c. periodic d. partial

6. The Parseval's formula is CO3K1
   a. $\sum_{n=0}^{\infty}|c_n|^2 = f$ b. $\sum_{n=0}^{\infty}|c_n|^2 \le f^2$ c. $\sum_{n=0}^{\infty}|c_n|^2 = ||f||^2$ d. $\sum_{n=0}^{\infty}|c_n|^2 \le ||f||^2$

7. $2\cos nx =$ ______ CO4K1
   a. $e^{inx} - e^{-inx}$ b. $e^{inx} + e^{-inx}$ c. $e^{nx} - e^{inx}$ d. $e^{nx} + e^{inx}$

8. $\int_0^{\infty} e^{-xy}\ f(x)dx$ is a ____ transform. CO4K1
   a. Mellin b. Fourier c. Laplace d. exponential Fourier

9. Let S be a bounded set in R^n having at most a finite number of accumulation points then CO5K1
   a. $c(S) = 1$ b. $c(S) = \infty$ c. $c(S) \ne 0$ d. $c(S) = 0$

10. Let Γ be a rectifiable curve in R^n. Then Γ has n-dimensional Jordan content CO5K1
    a. zero b. infinte c. finite d. constant

Part B **5 x 6 = 30**

Answer ALL questions

Each answer should not exceed 400 words or two pages

11.a. If $f \in U(I)$ and $g \in U(I)$, then prove that $\max(f, g) \in U(I)$ and $\min(f, g) \in U(I)$. CO1K3

(or)

11.b. State and prove Levi theorem for step functions. CO1K3

12.a. Let f be defined on I and assume that $\{f_n\}$ is a sequence of measurable functions on I such that $f_n(x) \to f(x)$ almost everywhere on I. Then prove that f is measurable on I. CO2K2

(or)

12.b. Derive the integral representation for the Riemann zeta function. CO2K2

13.a. State and Prove the Riesz-Fischer theorem. CO3K3

(or)

13.b. State and prove the Riemann-Lebesgue lemma. CO3K3

14.a. Let f be real-valued and continuous on a compact interval $[a, b]$. Then prove that for every $\varepsilon > 0$ there is a polynomial p such that $|f(x) - p(x)| < \varepsilon$ for every x in $[a, b]$. CO4K3

(or)

14.b. State and prove the exponential form of the Fourier integral theorem. CO4K3

15.a. State and Prove additive property of the Riemann integral. CO5K4

(or)

15.b. state and prove Mean-Value theorem for multiple integrals. CO5K4

Part C **5 x 12 = 60**

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. Let $\{S_n\}$ be a decreasing sequence of nonnegative step functions such that $S_n \searrow 0\ a.e.$ on an interval I. Then prove that $\lim_{n\to\infty} \int_I S_n = 0.$ CO1K3

(or)

16.b. State and prove Levi theorem for series of Lebesgue-integrable functions. CO1K3

17.a. State and prove Lebesgue dominated convergence theorem. CO2K4

(or)

17.b. Prove that $\lim_{b\to+\infty} \int_0^b \frac{\sin x}{x}\, dx = \frac{\pi}{2}$. CO2K4

18.a. State and prove properties of the Fourier coefficients. CO3K4

(or)

18.b. State and prove Jordan theorem. CO3K4

19.a. State and prove Fejér theorem. CO4K5

(or)

19.b. State and prove Fourier integral theorem. CO4K5

20.a. Let S be a compact Jordan-measurable set in R^n. Then prove that the integral $\int_S 1$ exists and $c(S) = \int_S 1$. CO5K5

(or)

20.b. State and prove theorem on evaluation of a multiple integral by iterated integration. CO5K5

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