Review of Literature

REVIEW OF LITERATURE

To describe situations mathematically which are vague or fuzzy in nature, Zadeh [55] introduced the concept of fuzzy set theory. Since then this concept has been used in the development of almost all branches of Mathematics, Computer science, Control engineering, Decision theory, expert system, logic managements, science where the concepts of uncertainity, information and complexity are explained by means of fuzzy sets.

Fuzziness can be represented in different ways. One of the most useful representation is membership function. Also, depending the nature or shape of membership function a fuzzy number can be classified in different ways, such as Triangular Fuzzy Number (TFN), Trapezoidal Fuzzy Number (TRFN), Interval Fuzzy Number (IFN), etc.

It is well known that the matrix formulation of a mathematical formula gives extra facility to study the problem. Due to the presence of uncertainity in many mathematical formulation in different branches of science and technology, fuzzy matrices were introduced in 1977 by Thomason [48]. Fuzzy matrices play an important role in scientific development. In 2007, Shyamal, A.K. and Pal, M. introduced Triangular Fuzzy Number Matrices [4]. Circulant matrices appear in many mathematical problems. A detailed account of circulant matrices with examples can be found in the beautiful work of Diaconis [13]. Kim and Karbill introduced the concepts of circulant Boolean matrices [26]. In [25], Kim introduced the concept of circulant fuzzy matrices and characterized all idempotent circulant fuzzy matrices. Pal, M. and Bhowmik [36] introduced the concept of circulant triangular fuzzy number matrices and studied their properties such as adjoint, determinant etc. In 2010, Vijayalakshmi, V. and Sattanathan, R. [52] introduced trapezoidal fuzzy number matrices and circulant trapezoidal fuzzy matrices. Circulant matrices and their generalization have important applications in Physics, Image processing, Probability, Number Theory, Geometry and Numerical analysis.

Interval Arithmetic was first suggested by Dwyer [16] in 1951. Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by Moore [37, 38]. Interval Arithmetic has attracted the attention of many researchers – Alefeld and Herzberger [1], Hansen [18, 19, 20], Luc. Jaulin et al. [27], Lodwick and Jamison [29], Neumaier [42] and many others.

There are many research articles published regarding various operators on fuzzy numbers.

Some of them are

1. The four operations of Arithmetic on Fuzzy Number's, Mizumoto, M., and Tanaka, K., 1977 [34].
2. Operations of fuzzy number's, Dubois, D., and Prade, H., 1978 [15].
3. Operations on fuzzy numbers with function principle, Chen, S.H., 1985 [9].
4. Multiplication Operation on Fuzzy Numbers, Shang Gao, Zaiyue Zhang and Cungen Cao, 2009 [47].
5. A computational method for Fuzzy Arithmetic Operations, Thowhida Akther and Sanwar Uddin Ahmad, 2009 [50].
6. The Generalized Triangular Fuzzy Sets, Yong Sik Yun, Sang Uk Ryu and Jin Won Park, 2009 [54].
7. Some non linear operations on triangular fuzzy number (m, α, β), Bansal, A., 2010 [6].
8. Addition of two Generalized Fuzzy Numbers, Chakraborty, D. and Guha, D., 2010 [8].
9. Arithmetic of Triangular Fuzzy Variable from Credibility Theory, Rituparna Chutia, Supahi Mahanta and Datta, D., 2011 [44].

The fuzzy numbers and fuzzy number matrices are widely used in many real life applications because of their suitability for representing uncertain information.

There is much literature on the study of fuzzy numbers and fuzzy number matrices.

In this Review of Literature, a brief survey of some of the articles published on fuzzy numbers, fuzzy number matrices and their applications are given.

1. "Fuzzy distance of trapezoidal fuzzy numbers"
Shan-Huo Chen and Chien-Chung Wang (2006) [46]

Fuzzy distance is applied on data analysis, classification, and product positioning analysis widely. In this paper, a fuzzy distance is introduced by using graded mean integration representation of generalized fuzzy number and the span of fuzzy number.

2. "Ordering generalized trapezoidal fuzzy numbers"
Y.L.P.Thorani, P.Phani Bushan Rao and N.Ravi Shankar (2012)[49]

This paper describes a ranking method for ordering fuzzy numbers based on area, mode, spreads and weights of generalized trapezoidal fuzzy numbers. The area used in this method is obtained from the generalized trapezoidal fuzzy number, first by splitting the generalized trapezoidal fuzzy numbers into three plane figures and then calculating the centroids of each plane figure followed by the incentre of these centroids and then finding the area of this in centre from the original point. In this paper, we also apply mode and spreads in those cases where the discrimination is not possible. This method is simple in evaluation and rank various types of trapezoidal fuzzy numbers and also crisp number which are considered to be a special case of fuzzy numbers.

3. "Linear system of equations with trapezoidal fuzzy numbers"
S.H.Nasseri and M.Gholami, (2011) [41]

A general fuzzy linear system of equations is investigated using embedding approach. In the literature of fuzzy systems two types of linear are more important : 1) Fuzzy Linear Systems, 2) Fully Fuzzy Linear Systems. In both class of these systems usually the authors considered triangular type of fuzzy numbers. In this paper, we introduce a linear system of equations with trapezoidal fuzzy number as an extension of the fuzzy linear systems which was first introduced by Friedman et al. [17]. Conditions for the existence of a trapezoidal fuzzy solution to n x n linear system are derived and a numerical procedure for calculating the solution is illustrated with some examples.

4. "Method for solving fully fuzzy assignment problems using triangular fuzzy numbers"

Amit Kumar, Anila Gupta and Amarpreet Kaur, (2009) [2]

In this paper, a new method is proposed to find the fuzzy optimal solution of fuzzy assignment problems by representing all the parameters as triangular fuzzy numbers. The advantages of the proposed method are also discussed. To illustrate the proposed method a fuzzy assignment problem is solved by using the proposed method and the obtained results are discussed. The proposed method is easy to understand and to apply for finding the fuzzy optimal solution of fuzzy assignment problems in real life situations.

5. "The determinant and adjoint of a square fuzzy matrix"

M.Z. Ragab and E.G. Emam, (1995) [43]

The concept of the determinant theory and the adjoint of a square fuzzy matrix is studied. Also the circulant fuzzy matrices are defined and it is proved that some properties of a square fuzzy matrix (such as reflexivity, transitivity and circularity) are carried out to the adjoint of the matrix. Finally, how to construct a transitive fuzzy matrix from a given one through the adjoint matrix of it is shown.

6. "Regular interval valued fuzzy matrices"

A.R.Meenakshi and M.Aliraja, (2010) [33]

In this paper, the concept of regular Interval Valued Fuzzy Matrices (IVFM) are introduced as a generalization of regular fuzzy matrices. The structure of row space and column space of an IVFM are obtained.

7. "An application of interval valued fuzzy matrices in medical diagnosis"
A.R.Meenakshi and M.Kaliraja (2011) [32]

In this paper, the authors extend Sanchez's approach for medical diagnosis using the representation of an interval valued fuzzy matrix as an interval matrix of two fuzzy matrices. The authors introduce arithmetic mean of an interval valued fuzzy matrix as the arithmetic mean of its lower and upper limit matrices and propose a method to study Sanchez's approach of medical diagnosis through the arithmetic mean of an interval valued fuzzy matrix, which is a simple technique than that of using intuitionistic fuzzy sets available in the literature.

8. "Two new operators on fuzzy matrices"
Amiya K. Shyamal, Madhumangal Pal, (2004) [3]

In this paper, two new binary fuzzy operators $\oplus$ and $\odot$ are introduced for fuzzy matrices. Several properties on $\oplus$ and $\odot$ are presented here.

9. "Circulant matrices"
Davis Philip J., (1980) [12]

Chapter I of this book motivates the study of circulants with a geometric example of nested polygons, the vertices of successive polygons being related by a circulant transformation.

Chapter II is packed full of useful information on general matrix theory and is intended to provide background for the more specific study of circulant to follow in chapter III.

In chapter III, a central result is the representation of all circulants as polynomials in the special circulant π = circ (0, 1, 0, ..., 0), from which many interesting properties of circulants may be obtained.

Generalizations of circulants (e.g. skew-circulants, block circulants) appears in the fifth chapter.

Finally the study of centralizers in chapter VI is used by Davis to encompass and unify a number of results previously obtained as well as to point us in several new directions.

10. "Some results on random circulant matrices"
Mark, W. Meckes (2009) [31]

This paper, considers random (non-Hermitian) circulant matrices, and proves several results analogous to recent theorems on non-Hermitian random matrices with independent entries. In particular, the limiting spectral distribution of a random circulant matrix is shown to be complex normal, and bounds are given for the probability that a circulant sign matrix is singular.

11. "Polynomials equations and circulant matrices"
Dankalman and James, E. White (2001) [11]

In this paper, a unified approach based on circulant matrices is presented. The idea is to construct a circulant matrix with a specified characteristic polynomial. The roots of a polynomial thus become eigen values, which are trivially found for ciruculant matrices.

12. "Cost efficiency with triangular fuzzy number input prices : An application of DEA"
H.Bagherzadeh Valami (2009) [5]

The cost efficiency model (CE) has been considered by researchers as a Data Envelopment Analysis (DEA) model for evaluating the efficiency of DMUs. In this model, the possibility of producing the outputs of a target DMU is evaluated by the input prices of the DMU. This provides a criterian for evaluating the CE of DMUs. The main contribution of this paper is to provide an approach for generalizing the CE of DMUs when the input prices are triangular fuzzy numbers, where preliminary concepts of fuzzy theory and CE, are directly used.

13. "Some arithmetic aggregation operators within intuitionistic trapezoidal fuzzy setting and their application to group decision making"
Lingling Shen, Hai Wang, Xiangqian Feng (2011) [28]

As intuitionistic trapezoidal fuzzy number can reveal the degree of an attribute satisfy or dissatisfy a trapezoidal fuzzy number rather than a certain fuzzy concept, it is interesting to introduce it to resolve multi-criteria decision making problems. This paper reviews some existed intuitionistic trapezoidal aggregation operators and presets a new arithmetic aggregation operator, i.e., induced intuitionistic trapezoidal fuzzy ordered weighting aggregation operator which includes a general reordering step. Some operational laws about the operator are studied, as well as relationships between it and other existed aggregation operators. Then a multi-criteria group decision making process is developed utilizing the proposed operator and the intuitionistic trapezoidal fuzzy weighting arithmetic average operator, the validity and practicality of which illustrated by a numerical example about the evaluation of security of information systems from related literature.

14. "Confidence analysis of fuzzy multi criteria decision making using trapezoidal fuzzy numbers"
G.Uthra and R.Sattanathan (2009) [51]

In this paper, a general fuzzy multi-criteria decision making problem (FMCDM) is introduced. Then confidence analysis of FMCDM is performed using trapezoidal fuzzy numbers by linguistic approach to model the decision maker's attitude. The approach is illustrated by a numerical example.