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A HYBRID GROUP ACCEPTANCE SAMPLING PLANS FOR LIFETIMES BASED ON EXPONENTIATED WEIBULL DISTRIBUTION

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ABSTRACT

In this paper we have developed a hybrid group acceptance sampling plan for a truncated life test when the lifetime of an item follows exponentiated Weibull distribution. The minimum number of testers and acceptance number are determined when the consumer's risk and the test termination time and group size are specified. The operating characteristic values according to various quality levels are also obtained.

Keywords: *Exponentiated Weibull distribution, group acceptance sampling plan, consumer's risk, producer's risk, operating characteristic, truncated life tests.*

INTRODUCTION

Acceptance sampling is an important field of statistical quality control that was popularized by Dodge and Romig and originally applied by the U.S. military to the testing of bullets during World War II. If every bullet was tested in advance, no bullets would be left to ship. If, on the other hand, none were tested, malfunctions might occur in the field of battle, with potentially disastrous results. Dodge reasoned that a sample should be picked at random from the lot, and on the basis of information that was yielded by the sample, a decision should be made regarding the disposition of the lot. In general, the decision is either to accept or reject the lot. This process is called Lot Acceptance Sampling or just Acceptance Sampling. In most acceptance sampling plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. It is implicitly assumed in the usual sampling plan that only a single item is put in a tester. However, testers accommodating a multiple number of items at a time are used in practice because testing time and cost can be saved by testing those items simultaneously. Sudden death testing is frequently adopted by using this type of testers (Pascual and Meeker, 1998; Vlcek et al. 2003; Jun et al. 2006). For this type of testers the number of items to be equipped in a tester is given by the specification. The acceptance sampling plan under this type of testers will be called a group acceptance sampling plan. When designing a group sampling plan, determining the sample size is equivalent to determining the number of groups as the group size is already given. The items in a group are tested independently, identically and simultaneously on the different testers for a pre-assigned time. The experiment is truncated if more than the acceptable number of failures occurred in any group during the experiment time. The method of determining the minimum number of testers for a predetermined number of groups is called as Hybrid Group Acceptance Sampling Plan (HGASP). If the HGASP is used in conjunction with truncated life tests, it is called a HGASP based on truncated life test assuming that the lifetime of product follows a certain probability distribution.

Acceptance sampling based on truncated life tests were discussed by many authors. Aslam, M. and C.H., have studied, a group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log-logistic distributions. Ali M.M., Pal M., Woo J.in 2006, have studied Exponentiated Weibull distribution. Gupta R.D., and Kunda D., 2003 have studied, Discriminating between the Weibull and GE distributions. Gupta, S. S. and Groll, P.A., 1961, Gamma distribution in acceptance sampling based on life tests. Muhammad Aslam, Chi-Hyuck Jun, Munir Ahmad in 2009 studied a group acceptance plan based on truncated life test for Gamma distribution. Again Muhammad Aslam, Chi-Hyuck Jun, Munir Ahmad along with Mujahid Rasool in 2011 have studied Improved group sampling plans based on time – truncated life tests. Srinivasa Rao, in 2009, have studied a group acceptance sampling plans for lifetimes following a generalized exponential distribution. Srinivasa Rao, G., in 2010 have studied a group acceptance sampling plans for truncated life tests for Marshall-Olkin extended Lomax distribution. Also Srinivasa Rao, G., in 2011 have studied a hybrid group acceptance sampling plans for lifetimes based on generalized exponential distribution.

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The purpose of this study is to propose a HGASP based on truncated life tests when the lifetime of a product follows the exponentiated Weibull distribution.

EXPONENTIATED WEIBULL DISTRIBUTION

The cumulative distribution function (cdf) of the exponentiated Weibull distribution is given by

$$F(t,\sigma) = \left(1 - e^{-(\frac{t}{\sigma})^m}\right)^{\alpha} \tag{1}$$

where σ is a scale parameter. If some other parameters are involved, then they are assumed to be known, for an example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/ σ.

The failure probability of an item by time t_0 is given by

$$p = F\,(t_{0:}\,\sigma) \tag{2}$$

The quality of an item is usually represented by its true mean lifetime although some other options such as median lifetime or B_{10} life are sometimes used. Let us assume that the true mean μ can be represented by the scale parameter. Also, it is convenient to specify the test time as a multiple of the specified life so thataμ_0 and the quality of an item as a ratio of the true mean to the specified life (μ/μ_0).

Then we can rewrite (2) as a function of 'a' (termination time) and the ratio μ/μ_0

$$p = F(a\,\mu_0 : \mu/\mu_0) \tag{3}$$

Here the underlying distribution is the Weibull distribution having known shape parameter and unknown scale parameter σ.

$$p = \left(1 - e^{-(\frac{ba}{\mu/\mu_0})^m}\right)^{\alpha} \tag{4}$$

DESIGN OF THE PROPOSED SAMPLING PLAN

We are interested in designing a group sampling plan in order to assure that the mean life of an item in a lot (μ, say) is greater than the specified life μ_0 with known shape parameter. A lot of products or items are considered to be "good" if the true average life μ is greater than the specified life μ_0. We will accept the lot if $\mu \geq \mu_0$ at a certain level of consumer's risk. Otherwise, we have to reject the lot. The following hybrid group acceptance sampling plan based on the truncated life test is proposed:

1. Select the number of testers, *r* and assign the *r* items to each predefined groups *g* so that the sample size for a lot will be $n = gr$.
2. Pre-fix the acceptance number, c for each group and the experiment time t_0.
3. Accept the lot if at most c failures occur in each of all groups.
4. Terminate the experiment if more than *c* failures occur in any group and reject the lot.

The proposed sampling plan is an extension of the ordinary sampling plan available in literature such as in *Kantam et al.* (2001) and *Rosaiah* and *Kantam* (2005), for which r =1. We are interested in determining the number of groups *g* required for each of two distributions under study, whereas the various values of acceptance number c and the termination time t_0 are assumed to be specified. Since it is convenient to set the termination time as a multiple of the specified life μ_0, we will consider $t_0 = a\mu_0$ for a specified constant *a* (termination ratio).

The probability of rejecting a good lot is called the producer's risk, whereas the probability of accepting a bad lot is known as the consumer's risk. When determining the parameters of the proposed sampling plan, we will use the consumer's risk. Often, the consumer's risk is expressed by the consumer's confidence level. If the confidence level is p*, then the consumer's risk will be β =1-p*. We will determine the number of groups in the proposed sampling plan so that the consumer's risk does not exceed β. According to the HGASP, the lot of products is accepted only if there are at most c failures observed in each of the g groups.

Table: 1

Consumer's risk (β), truncated time (a), group size (g) and acceptance number (c)

β	a	g	c
0.25	0.7	2	0
0.10	0.8	3	1
0.05	1.0	4	2
0.01	1.2	5	3
	1.5	6	4
	2.0	7	5
		8	6
		9	7
		10	8

The HGASP is characterized by the three parameters. So, the lot acceptance probability will be

$$L(p) = \left(\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-1} \right)^g \tag{5}$$

where p is the probability that an item in a group fails before the termination time $t_0 = a\mu_0$.

The probability p for the exponentiated Weibull distribution is given by

$$p = \left(1 - e^{-(\frac{ba}{\mu/\mu_0})^m} \right)^\alpha \tag{6}$$

where b = $(\Gamma(1/m)/m)^m$

The minimum number of testers required can be determined by considering the consumer's risk when the true median life equals the specified median life ($\mu = \mu_0$) (worst case) by means of the following inequality:

$$L(p) \le \beta \tag{7}$$

where p_0 is the failure probability at $\mu = \mu_0$, and it is given by

$$p_0 = \left(1 - e^{-(ba)^m}\right)^\alpha \tag{8}$$

OPERATING CHARACTERISTIC FUNCTIONS

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the median from its true value μ. This function is called Operating Charateristic (OC) function of the sampling plan. Once the minimum sample size is obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is sufficiently good. As mentioned earlier, the product is considered to be good if $\mu \ge \mu_0$. For c = 2 the probabilities of acceptance are displayed in Table 3 for various values of the median ratios μ/μ_0, producer's risks β and time multiplier a.

Notation:

g	-	Number of groups
r	-	Number of items in a group
n	-	Sample size
d	-	Number of defectives
c	-	Acceptance number
t_0	-	Termination time
a	-	Test termination time multiplier
m, α	-	Shape parameters
σ	-	Scale parameter
β	-	Consumer's risk
p	-	Failure probability
L(p)	-	Probability of acceptance
p^*	-	Minimum probability
μ	-	Mean life
μ_0	-	Specified life

DESCRIPTION OF TABLES AND EXAMPLES

The design parameters of HGASP are found at the various values of the consumer's risk and the test termination time multiplier in Table 2. It should be noted that if one needs the minimum sample size, it can be obtained by n = rg. Table 2 indicates that, as the test termination time multiplier a increases, the number of testers r decrease, i.e., a smaller number of testers is needed, if the test termination time multiplier increases at a fixed number of groups. For an example, from Table 2, if $\beta = 0.10$, g = 4, c = 2 and a changes from 0.7 to 0.8, the required values of design parameters of HGASP changes from r = 10 to r = 8. However, this trend is not monotonic since it depends on the acceptance number as well. The probability of acceptance for the lot at the median ratio corresponding to the producer's risk is also given in Table 3.

Suppose that the lifetime of a product follows the weibull distributions with b=1 and m=1. It is desired to design a HGASP to test if the median is greater than 1,000 hrs based on a testing time of 700 hrs and using 4 groups. It is assumed that c = 2 and $\beta = 0.10$. This leads to the termination multiplier $a = 0.700$. From Table 2, the minimum number of testers required is r = 10. Thus, we will draw a random sample of size 40 items and allocate 10 items to each of 4 groups to put on test for 700 hrs. This indicates that a total of 40 products are needed and that 10 items are allocated to each of 4 groups. We will accept the lot if no more than 2 failure occurs before 700 hrs in each of 4 groups. We truncate the experiment as soon as the 3rd failure occurs before the 700th hr. For this proposed sampling plan the probability of acceptance is 0.992847 when the true mean is 4,000 hrs. This shows that, if the true median life is 4 times of 1000 hrs, the producer's risk is 0.00713.

Table: 2

Minimum number of testers (r) for the proposed plan for the exponentiated Weibull distribution

p*	g	c	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.75	2	0	3	2	2	2	1	1
	3	1	5	5	4	3	3	2
	4	2	8	7	5	4	4	3
	5	3	11	9	7	6	5	4
	6	4	13	11	9	7	6	5
	7	5	16	14	11	9	8	7
	8	6	19	16	13	11	9	8
	9	7	22	18	14	12	10	9
	10	8	24	21	16	14	12	10
0.90	2	0	4	4	3	3	2	1
	3	1	7	6	5	4	3	2
	4	2	10	8	6	5	4	4
	5	3	13	11	8	7	6	5
	6	4	15	13	10	8	7	6
	7	5	18	15	12	10	8	7
	8	6	21	18	14	12	10	8
	9	7	24	20	16	13	11	9
	10	8	27	23	18	15	12	10
0.95	2	0	6	5	3	3	2	2
	3	1	8	7	5	4	3	3
	4	2	11	9	7	6	5	4
	5	3	14	12	9	7	6	5
	6	4	17	14	11	9	7	6
	7	5	20	16	13	10	9	7
	8	6	22	19	15	12	10	8
	9	7	25	21	17	14	11	9
	10	8	28	24	18	15	13	11
0.99	2	0	8	7	5	4	3	2
	3	1	11	9	7	5	4	3
	4	2	14	11	9	7	5	4
	5	3	16	14	10	8	7	5
	6	4	19	16	12	10	8	7
	7	5	22	19	14	12	9	8
	8	6	25	21	16	13	11	9
	9	7	28	24	18	15	12	10
	10	8	31	26	20	17	14	11

Table: 3

Operating characteristics values of the hybrid group sampling plan with g = 4 and c = 2 for exponentiated Weibull distribution

p*	r	a	μ/μ₀					
			2	4	6	8	10	12
0.75	8	0.7	0.897874	0.996525	0.999618	0.999925	0.999979	0.999993
	7	0.8	0.877708	0.995511	0.999495	0.999899	0.999972	0.999990
	5	1.0	0.888861	0.995659	0.999495	0.999897	0.999971	0.999990
	4	1.2	0.890441	0.995402	0.999446	0.999885	0.999967	0.999988
	4	1.5	0.753577	0.986208	0.998196	0.999609	0.999885	0.999958
	3	2.0	0.768211	0.985239	0.997926	0.999532	0.999858	0.999948
0.90	10	0.7	0.814781	0.992847	0.999197	0.999840	0.999955	0.999984
	8	0.8	0.823971	0.993001	0.999201	0.999839	0.999955	0.999984
	6	1.0	0.809622	0.991647	0.999008	0.999796	0.999942	0.999979
	5	1.2	0.778428	0.989109	0.998651	0.999716	0.999918	0.999971
	4	1.5	0.753577	0.986208	0.998196	0.999609	0.999885	0.999958
	4	2.0	0.454968	0.948545	0.992222	0.998196	0.999446	0.999794
0.95	11	0.7	0.766549	0.990361	0.998906	0.999781	0.999939	0.999979
	9	0.8	0.763362	0.989770	0.998816	0.999760	0.999932	0.999976
	7	1.0	0.716922	0.985945	0.998294	0.999647	0.999899	0.999964
	6	1.2	0.646482	0.979383	0.997369	0.999441	0.999838	0.999942
	5	1.5	0.558030	0.968253	0.995659	0.999045	0.999716	0.999897
	4	2.0	0.454968	0.948545	0.992222	0.998196	0.999446	0.999794
0.99	14	0.7	0.608222	0.980010	0.997653	0.999525	0.999866	0.999953
	11	0.8	0.630391	0.980937	0.997729	0.999536	0.999868	0.999954
	9	1.0	0.518795	0.968874	0.996052	0.999169	0.999760	0.999915
	7	1.2	0.511694	0.965895	0.995511	0.999036	0.999718	0.999899
	5	1.5	0.558030	0.968253	0.995659	0.999045	0.999716	0.999897
	4	2.0	0.454968	0.948545	0.992222	0.998196	0.999446	0.999794

CONCLUSION

We here proposed a hybrid group acceptance sampling plan from the truncated life test, the number of testers and the acceptance number was determined for exponentiated Weibull distributions when the consumer's risk (β) and the other plan parameters are specified. It can be observed that the minimum number of testers required decreases as test termination time multiplier increases and also the operating characteristics values increases more rapidly as the quality improves. This HGASP can be used when a multiple number of items at a time are adopted for a life test and it would be beneficial in terms of test time and cost because a group of items will be tested simultaneously.

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