Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – June / July 2021
II Semester

Class : I UG **Time : 3 Hours**
Major : Mathematics **Max. Marks : 100**

18BMAC04 Integral and Vector Calculus

Part A **10 x 1 = 10**
Choose the Correct Answer

1. $\int_0^a \left(\int_0^b x^2 + y^2\right) dxdy$= CO1 K1
   a. $\left({ab}/{3}\right)(a^2 + b^2)$ b. $\left({ab}/{3}\right)(a^2 - b^2)$
   c. $\left({ab}/{3}\right)$ d. $(a^2 - b^2)$

2. A definite integral is defined as the limit of the -------------------- of series. CO1 K1
   a. difference b. sum c. Product d. derivative

3. If the region of integration is a rectangle between the lines x=a ,x=b ,y=c, y=d then $\int_R f(x,y)\, dA =$ CO2 K1
   a. $\int_c^b \int_a^d f(x,y)\, d\,ydx$ b. $\int_c^d \int_a^b d\,xdy$
   c. $\int_c^d \int_a^b f(x,y)\, d\,xdy$ d. $\int_c^d \int_a^b d\,ydx$

4. The centre of gravity can be expressed as CO2 K2
   a. $\bar{x}=\frac{\iint y\,dx\,dy}{\iint dxdy}$; $\bar{y}=\frac{\iint x\,dx\,dy}{\iint dxdy}$ b. $\bar{x}=\frac{\iint dx\,dy}{\iint xdxdy}$; $\bar{y}=\frac{\iint y\,dx\,dy}{\iint dxdy}$
   c. $\bar{x}=\frac{\iint x\,dx\,dy}{\iint ydxdy}$; $\bar{y}=\frac{\iint y\,dx\,dy}{\iint xdxdy}$ d. $\bar{x}=\frac{\iint x\,dx\,dy}{\iint dxdy}$; $\bar{y}=\frac{\iint y\,dx\,dy}{\iint dxdy}$

5. $\frac{\partial(u,v)}{\partial(x,y)}$. $\frac{\partial(x,y)}{\partial(u,v)}$ = CO2 K2
   a. c b. .cpp c. .h d. .txt

6. $\nabla\emptyset$ = CO3 K1
   a. $\frac{\partial\emptyset}{\partial x} - j\frac{\partial\emptyset}{\partial y}$ + k $\frac{\partial\emptyset}{\partial z}$ b. i $\frac{\partial\emptyset}{\partial x} + j\frac{\partial\emptyset}{\partial y}$ + k $\frac{\partial\emptyset}{\partial z}$
   c. i $\frac{\partial\emptyset}{\partial x} + j\frac{\partial\emptyset}{\partial y}$ - k $\frac{\partial\emptyset}{\partial z}$ d. i $\frac{\partial\emptyset}{\partial x} - j\frac{\partial\emptyset}{\partial y}$ - k $\frac{\partial\emptyset}{\partial z}$

7. If $\nabla\emptyset.\frac{d\bar{r}}{ds} = 0$,then $\nabla\emptyset$ acts in a direction ------------ to the direction of $\frac{d\bar{r}}{ds}$ CO3 K1
   a. parellel b. coincide
   c. perpendicular d. None of above

8. A vector is said to be solinoidal if its divergence is CO3 K2
   a. ∞ b. -1 c. 1 d. 0

9. $\iiint_v \nabla.\bar{F}\text{-} - dv =$ CO4 K1

a. $\iint \bar{n}.\bar{F}\, ds$ b. $\iint \quad n$

c. $\iint \quad \bar{F}\,.ds$ d. $\iint \quad ds$

CO4 K1

10. $\int_c \quad \bar{F}\,.d\bar{r} =$

a. $\iint$. n ds b. $\iint curl\bar{F}.\bar{n}\, ds$

c. $\iint$ ds d. $\iint \quad \bar{F}.ds$

Part B **5 x 6 = 30**

Answer ALL questions

Each answer should not exceed 400 words or two pages

11.a. Evaluate the integral $\int_0^3 \int_1^2 xy(x+y)dy\, dx$. CO1 K3

(or)

11.b. Evaluate the integral $\int_0^a \int_0^b xy(x-y)dy\, dx$. CO1 K3

12.a Find the volume of the paraboloid of revolution x^2+ y^2=4z cut off by the plane z=4. CO2 K3

(or)

12.b. Find the volume of a segment h of a sphere of radius a . CO2 K3

13.a. Evaluate $\iint xydxdy$ taken over the positive quadrant of the circle x^2 + y^2 =a^2. CO3 K4

(or)

13.b. Find the area of the curvilinear quadrilateral bounded by the four parabolas. CO3 K4

14.a. Find the directional derivative of Φ(x,y,z)=xy^2 + yz^3 at the point (2,-1,1) in the direction of the vector $\bar{\imath}$ + 2$\bar{\jmath}$ + 2$\bar{k}$. CO4 K5

(or)

14.b. Compute the divergence and curl of the vector $F = xyz\,\bar{\imath} + 3x^2\,y\bar{\jmath} + (xz^2 - y^2z)\bar{k}$ at (1,2,-1). CO4 K5

15.a Evaluate $\iiint_v$ ∇.F dv where $\bar{F}$= $x^2\,\bar{\imath}$ + $y^2\,\bar{\jmath}$ +z$^2\bar{k}$ and V is the volume enclosed by the cube 0≤ x,y,z ≤1. CO5 K5

(or)

15.b. Show that if $\bar{F}$ = $x^3\,\bar{\imath}$ + $y^3\bar{\jmath}$ + z$^3\bar{k}$, then $\iint_s$ $\bar{F}.\bar{n}$ ds = $\frac{12}{5}\pi$ a$^{5.}$ CO5 K5

Part C **5 x 12 = 60**

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. Evaluate $\iint \quad (x^2 + y^2)$ dxdy over the region for which x,y ≥ 0 and x + y ≤ 1 . CO1 K5

(or)

16.b. Evaluate $\iiint xyz$ dx dydz taken through the positive octant of the sphere x^2 + y^2 + z^2=a^2. CO1 K5

17.a. Find the volume and the position of the centre of gravity of the tetrahedron bounded by the plane $\frac{x}{a}$ $+\frac{y}{b}$ $+\frac{z}{c}$ = 1 and the coordinates planes CO2 K5

(or)

17.b. Find the area of the surface of the sphere of radius r CO2 K5

18.a. Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy \ dxdy$ and evaluate it CO3 K5

(or)

18.b. Evaluate $\iint y\,dxdy$ over the region between x^2 =y and the line x +y = 2 CO3 K5

19.a. Prove that $\bar{v}$= $r^n\bar{r}$ is irrotational . Find n when it is also solenoidal. CO4 K6

(or)

19.b. The temperature at any point (x,y,z) at time t is ϕ (x,y,z,t) = $2x^2$ y + yz^2 t – cos(xt). Find the rate of change of temperature with respect to time encountered by a particle passing through the point (3,-2,1) with velocity $\bar{v}$ = 2 $\bar{\imath}$ - $\bar{\jmath}$ + $\bar{k}$ at time t=0 CO4 K6

20.a. Evaluate $\iint_s \overline{F} \cdot \bar{n}$ ds where $\bar{F}$= $x \ \bar{\imath} + y \ \bar{\jmath}$ - 2 z $\bar{k}$ and S is the surface of the plane $x^2 + y^2 + z^2$=a^2 above the XOY plane. CO5 K6

(or)

20.b. (i)Find by Green's Theorem the value $\int_C (x^2y \ dx + ydy)$along the Closed curve c formed by y^2=x and y=x between (0,0) and (1,1)

(ii) Find the area between the curves y^2 = 4x and$x^2 = 4y$ CO5 K6

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