Chapter II

CHAPTER II

Supra Generalized Closed (Open) Soft Sets in Supra Soft Topological Spaces

Section 2.1

Supra generalized closed soft sets

Definition: 2.1.1

A soft set (F, E) is called a **supra generalized closed soft set (supra g-closed soft)** in a supra soft topological space (X, μ, E) if $cl^s(F, E) \widetilde{\subseteq} (G, E)$ whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G, E) is supra open soft in X.

Example: 2.1.2

Suppose that there are three cars in the universe X given by $X = \{h_1, h_2, h_3\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "expensive" and "beautiful" respectively.

Let $(F_1, E), (F_2, E)$ be two soft sets over the common universe X, which describe the composition of the cars, where

$F_1(e_1) = \{h_2, h_3\}$, $F_1(e_2) = \{h_1, h_2\}$,

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_1, h_3\}$.

Then $\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E)\}$ is the supra soft topology over X. Hence the soft sets in (X, μ, E), but the set (G, E) where

$G(e_1) = \{h_2\}$ $G(e_2) = \{h_2\}$ is not supra g-closed soft in (X, μ, E).

Remark: 2.1.3

The soft intersection (resp. soft union) of any two supra g-closed soft sets is not supra g-closed soft in general and is shown in the following examples.

Example: 2.1.4

1. In example 2.1.2, $(F_1, E), (F_2, E)$ are supra g-closed soft in (X, μ, E), but their soft intersection $(F_1, E) \tilde{\cap} (F_1, E) = (M, E)$ where

$M(e_1) = \{h_2\}$, $M(e_1) = \{h_1\}$ is not supra g-closed soft.

2. Suppose that there are four alternatives in the universe of houses $X = \{h_1, h_2, h_3, h_4\}$ and consider $E = \{e\}$ be the single parameter "quality of houses" to be the a linguistic variable. Let $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)$, (F_7, E), (F_8, E) be eight soft sets over the common universe X which describe the goodness of the houses, where

$F_1(e) = \{h_1\}$, $F_2(e) = \{h_4\}$, $F_3(e) = \{h_1, h_4\}$,

$F_4(e) = \{h_1, h_2\}$, $F_5(e) = \{h_2, h_4\}$, $F_6(e) = \{h_1, h_2, h_3\}$,

$F_6(e) = \{h_2, h_3, h_4\}$, $F_6(e) = \{h_1, h_2, h_4\}$.

Then $\mu = \{\tilde{X}, \tilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E)\}$ is the supra soft topology over X. Hence the sets $(F_1, E), (F_2, E)$ are supra g-closed soft sets in (X, μ, E), but their soft union $(F_1, E) \tilde{\cup} (F_1, E) = (H, E)$ where $H(e) = \{h_1, h_4\}$ is not supra g-closed soft.

Remark: 2.1.5

Every supra closed soft set is supra g-closed soft. But the converse is not true in general as shown in the following example.

Example: 2.1.6

In example 2.1.4 $(F_1, E), (F_2, E)$ are supra g-closed soft in (X, μ, E), but not supra closed soft over X.

Theorem: 2.1.7

Let (X, μ, E) be a supra soft topological space and (F, E) be a supra g-closed soft in X. If $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} cl^s(G, E)$, then (H, E) is a supra g-closed soft.

Proof:

Let $(H,E) \tilde{\subseteq} (G,E)$ and $(G,E) \in \mu$. Since $(F,E) \tilde{\subseteq} (H,E) \tilde{\subseteq} (G,E)$ and (F,E) be a supra g-closed soft in X, then $cl^s(F,E) \tilde{\subseteq} (G,E)$. Hence $cl^s(H,E) \tilde{\subseteq} cl^s(F,E) \tilde{\subseteq} (G,E)$. Thus $cl^s(H,E) \tilde{\subseteq} (G,E)$. Therefore, (H,E) is a supra g-closed soft.

Theorem: 2.1.8

Let (X,μ,E) be a supra soft topological space. Then (H,E) is supra g-closed soft in X if and only if $cl^s(H,E)\backslash(H,E)$ contains only null supra closed soft set.

Proof:

Let (H,E) is supra g-closed soft set, (F,E) be non null supra closed soft set in X and $(F,E) \tilde{\subseteq} cl^s(H,E)\backslash(H,E)$. Then $(F,E)'$ is supra open soft, $(F,E) \tilde{\subseteq} cl^s(H,E)$ and $(F,E) \tilde{\subseteq} (H,E)'$. Hence $(H,E) \tilde{\subseteq} (F,E)'$. Since (H,E) is supra g-closed soft. Then $cl^s(H,E) \tilde{\subseteq} (F,E)'$. Hence $(F,E) \tilde{\subseteq} cl^s(H,E)'$. This means that $(F,E) \tilde{\subseteq} cl^s(H,E) \tilde{\cap} cl^s(H,E)' = \tilde{\emptyset}$. Thus $(F,E) = \tilde{\emptyset}$ which is a contradiction. Therefore, $cl^s(H,E)\backslash(H,E)$ contains only null supra closed soft set.

Conversely, assume that $cl^s(H,E)\backslash(H,E)$ contains only null supra closed oft set, $(H,E) \tilde{\subseteq} (G,E)$, (G,E) is supra open soft and suppose that $cl^s(H,E) \tilde{\nsubseteq} (G,E)$. Then $cl^s(H,E) \tilde{\cap} (G,E)'$ is non null supra closed soft subset of $cl^s(H,E)\backslash(H,E)$ which is a contradiction. Thus (H,E) is supra g-closed soft in X. This completes the Proof.

Corollary: 2.1.9

Let (F,E) be supra g-closed soft set. Then (F,E) is supra closed soft if and only if $cl^s(F,E)\backslash(F,E)$ is supra closed soft.

Proof:

If (F,E) is supra closed soft, then $cl^s(F,E)\backslash(F,E) = \emptyset$ is supra closed soft. Conversely, suppose that $cl^s(F,E)\backslash(F,E)$ is supra closed soft. Since (F,E) be supra g-closed soft set. Then $cl^s(F,E)\backslash(F,E) = \emptyset$ from Theorem 2.1.8. Hence $cl^s(F,E) = (F,E)$. Thus (F,E) is supra closed soft.

Definition: 2.1.10

A soft set (F, E) is called a **supra generalized open soft set (supra g-open soft)** in a supra soft topological space (X, μ, E) if its relative complement $(F, E)'$ is supra g-closed soft in X.

Theorem 2.1.11

Let (X, μ, E) be supra soft topological space. Then supra soft set (F, E) is supra g-open soft if and only if $(F, E) \tilde{\subseteq} int^s(G, E)$ whenever $(F, E) \tilde{\subseteq} (G, E)$ and (F, E) is supra closed soft in X.

Proof:

Let (F, E) be a supra g-open soft in X, $(F, E) \tilde{\subseteq} (G, E)$ and (F, E) is supra closed soft in X. Then $(F, E)'$ is supra g-closed soft from Definition 2.1.10 and $(G, E)' \tilde{\subseteq} (F, E)'$. Since (F, E) is supra g-open soft in X. Then $cl^s(G, E)' \tilde{\subseteq} (F, E)'$. Hence $(F, E) \tilde{\subseteq} [\, cl^s(G, E)']' = int^s(G, E)$. Conversely, let $(F, E)' \tilde{\subseteq} (H, E)$ and (H, E) is supra open soft in X. Then $(H, E)' \tilde{\subseteq} (F, E)$ and $(H, E)'$ is supra closed soft in X. Hence $(H, E)' \tilde{\subseteq} int^s(F, E)$ from the necessary condition. Thus $[int^s(F, E)]' = cl^s[(F, E)'] \tilde{\subseteq} (H, E)$ and (H, E) is supra open soft in X. This means that $(F, E)'$ is supra g-closed soft in X. Therefore, (F, E) is supra g-open soft set from Definition 2.1.10. This completes the proof.

Example: 2.1.12

In example 2.1.2, $(F_1, E)', (F_2, E)'$ are supra g-open soft in (X, μ, E).

Remark: 2.1.13

Every supra open soft set is supra g-open soft. But the converse is not true in general as shown in the following example.

Example: 2.1.14

In Example 2.1.2, $(F_1, E)', (F_2, E)'$ are supra g-open soft in (X, μ, E), but not supra open soft over X.

Theorem: 2.1.15

Let (X, μ, E) be supra soft topological space and (F, E) be a supra g-open soft in X. If $\text{int}^s(F, E) \widetilde{\subseteq} (H, E) \widetilde{\subseteq} (F, E)$, then (H, E) is a supra g-open soft.

Proof:

Let $(G, E) \widetilde{\subseteq} (H, E)$ and $(G, E) \in \mu$. Since $(G, E) \widetilde{\subseteq} (H, E) \widetilde{\subseteq} (F, E)$ and (F, E) is supra g-open soft in X, then $(G, E) \widetilde{\subseteq} \text{int}^s(F, E)$. Hence $(G, E) \widetilde{\subseteq} \text{int}^s(F, E) \widetilde{\subseteq} \text{int}^s(H, E)$. Thus $(G, E) \widetilde{\subseteq} \text{int}^s(H, E)$. Therefore, (H, E) is supra g-open soft.

Section 2.2

Supra generalized closed soft sets with respect to a soft ideal

Definition: 2.2.1

Let I be non-null collection of soft sets over a universe X with the same set of parameters E. Then $\tilde{I} \subseteq SS(X)_E$ is called a **soft ideal** on X with the same set E if

1. $(F, E) \in \tilde{I}$ and $(G, E) \in \tilde{I} \Rightarrow (F, E) \tilde{\cup} (G, E) \in \tilde{I}$,

2. $(F, E) \in \tilde{I}$ and $(G, E) \widetilde{\subseteq} (F, E) \Rightarrow (G, E) \in \tilde{I}$,

i. e. $\tilde{I}$ is closed under finite soft unions and soft subsets.

Theorem: 2.2.2

Let (X_1, τ_1, A, I) be a soft topological space with soft ideal, (X_2, τ_2, B) be soft topological space and $f_{pu}: (X_1, \tau_1, A, I) \to (X_2, \tau_2, B)$ be a soft function. Then $f_{pu}(\tilde{I}) = \{f_{pu}((F, A)): (F, A) \in \tilde{I}\}$ is a soft ideal on X_2.

Definition: 2.2.3

A soft set $F_E \in SS(X, E)$ is called **supra generalized closed soft with respect to a soft ideal $\tilde{I}$ (supra-$\tilde{I}$g-closed soft)** in a supra soft topological space $(X. \mu, E)$ if $cl^s F_E \backslash G_E \in \tilde{I}$ whenever $F_E \widetilde{\subseteq} G_E$ and $G_E \in \mu$.

Example: 2.2.4

Let $X = \{h_1, h_2, h_3\}$ be the set of three houses under consideration and $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "wooden" and "green surroundings" respectively.

Let (F_1, E). (F_2, E) be two soft sets representing the attractiveness of the houses which Mr. A and Mr. B are going to buy. where

$F_1(e_1) = \{h_2, h_3\}$, $\quad F_1(e_2) = \{h_1, h_2\}$,

$F_2(e_1) = \{h_1, h_2\}$, $\quad F_2(e_2) = \{h_1, h_3\}$.

Then $\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E)\}$ is the supra soft topology over X. Let $\tilde{I} = \{\widetilde{\emptyset}, (I_1, E), (I_2, E), (I_3, E)\}$ be a soft ideal over X. where $(I_1, E), (I_2, E), (I_3, E)$ are soft sets over X defined by

$I_1(e_1) = \{h_1\}$, $\quad I_1(e_2) = \emptyset$,

$I_2(e_1) = \emptyset$, $\quad I_2(e_2) = \{h_3\}$, and

$I_3(e_1) = \{h_1\}$, $\quad I_3(e_2) = \{h_3\}$.

So (F_1, E) is a supra-$\tilde{I}$g-closed soft.

Theorem: 2.2.5

Every supra g-closed soft set is supra-$\tilde{I}$g-closed soft.

Proof:

Let F_E be supra g-closed soft set in a supra soft topological space (X, μ, E) and $F_E \widetilde{\subseteq} G_E$ such that $G_E \in \mu$. Since F_E is supra g-closed soft, then $cl^s(F, E) \widetilde{\subseteq} G_E$ and hence $cl^s F_E \setminus G_E = \emptyset \in \tilde{I}$. Consequently F_E is a supra-$\tilde{I}$g-closed soft set.

Example: 2.2.6

Suppose that there are three cars in the universe X given by $X = \{h_1, h_2, h_3\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "expensive" and "beautiful" respectively.

Let $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ be four soft sets over a common universe X, which describe the composition of the cars, where

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = X$,

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = X$,

$F_3(e_1) = \{h_3\}$, $F_3(e_2) = X$, and

$F_4(e_1) = \{h_1, h_3\}$, $F_4(e_2) = X$.

Then $\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ is the supra soft topology over X. Let $\tilde{I} = \{\widetilde{\emptyset}, (I_1, E), (I_2, E), (I_3, E)\}$ be a soft ideal over X, where $(I_1, E), (I_2, E), (I_3, E)$ are soft sets over X defined by

$I_1(e_1) = \{h_2\}$, $I_1(e_2) = \emptyset$,

$I_2(e_1) = \{h_3\}$, $I_2(e_2) = \emptyset$, and

$I_3(e_1) = \{h_2, h_3\}$, $I_3(e_2) = \{\emptyset\}$.

So (F_1, E) is a supra-Ĩg-closed soft but it is not supra g-closed soft.

Theorem: 2.2.7

A supra soft set (G, E) is a supra-Ĩg-closed soft set in a supra soft topological space (X, μ, E) if and only if there exist a supra closed soft set (F, E) such that $(F, E) \widetilde{\subseteq} cl^s(G, E) \setminus (G, E)$ implies $(F, E) \in \tilde{I}$.

Proof:

Necessity: Suppose that (G, E) is a supra-Ĩg-closed soft set and (F, E) be a supra closed soft set such that $(F, E) \widetilde{\subseteq} cl^s(G, E) \setminus (G, E)$. Then $(G, E) \widetilde{\subseteq} (F, E)'$. By our assumption, $cl^s(G, E) \setminus (F, E)' \in \tilde{I}$. But $(F, E) \widetilde{\subseteq} cl^s(G, E) \widetilde{\cap} (F, E) = cl^s(G, E) \setminus (F, E)'$. Thus $(F, E) \in \tilde{I}$ from Definition 2.2.1.

Sufficiency: Conversely, assume that $(G, E) \widetilde{\subseteq} (H, E)$ and $(H, E) \in \mu$. Then $cl^s(G, E) \setminus (H, E) = cl^s(G, E) \widetilde{\cap} (H, E)' = cl^s(G, E) \widetilde{\cap} cl^s((H, E)')$ is a supra closed soft set in (X, μ, E) and $cl^s(G, E) \setminus (H, E) \widetilde{\subseteq} cl^s(G, E) \setminus (H, E)$. By assumption $cl^s(G, E) \setminus (H, E) \in \tilde{I}$. This implies that (G, E) is a supra-Ĩg-closed soft.

Theorem: 2.2.8

If (F, E) and (G, E) are supra-$\tilde{I}$g-closed soft set in a supra soft topological space (X, μ, E), then $(F, E) \tilde{\cup} (G, E)$ is also supra-$\tilde{I}$g-closed soft in (X, μ, E).

Proof:

Suppose that F_E and G_E are supra-$\tilde{I}$g-closed soft in (X, μ, E). Let $F_E \tilde{\cup} G_E \tilde{\subseteq} H_E$ and $H_E \in \tau$, then $F_E \tilde{\subseteq} H_E$ and $G_E \tilde{\subseteq} H_E$. By assumption $cl^s F_E \backslash H_E \in \tilde{I}$ and $cl^s G_E \backslash H_E \in \tilde{I}$. It follows that $[cl^s F_E \backslash H_E] \tilde{\cup} [cl^s G_E \backslash H_E = cl^s[(F_E \tilde{\cup} G_E) \backslash H_E] \in \tilde{I}$. Thus $F_E \tilde{\cup} G_E$ is a supra-$\tilde{I}$g-closed soft.

Theorem: 2.2.9

If F_E is supra-$\tilde{I}$g-closed soft in a supra soft topological space (X, μ, E) and $F_E \tilde{\subseteq} G_E \tilde{\subseteq} cl^s F_E$, then G_B is supra-$\tilde{I}$g-closed soft in (X, μ, E).

Proof:

Let F_E is supra-$\tilde{I}$g-closed soft, $F_E \tilde{\subseteq} G_E \tilde{\subseteq} cl^s F_E$ in (X, μ, E) and $G_E \tilde{\subseteq} H_E$ such that $H_E \in \mu$. Then $F_E \tilde{\subseteq} H_E$. Since F_E is a supra-$\tilde{I}$g-closed soft, then $cl^s F_E \backslash H_E \in \tilde{I}$. Now $G_E \tilde{\subseteq} cl^s F_E$ implies that $cl^s G_E \tilde{\subseteq} cl^s F_E$. So $cl^s G_E \backslash H_E \tilde{\subseteq} cl^s F_E \backslash H_E$. Thus $cl^s G_E \backslash H_E \in \tilde{I}$. Consequently, G_E is a supra-$\tilde{I}$g-closed soft in (X, μ, E). This completes the proof.

Remark: 2.2.10

The soft intersection of two supra-$\tilde{I}$g-closed soft sets need not be a supra-$\tilde{I}$g-closed soft set and it is shown in the following example.

Example: 2.2.11

Suppose that there are three dresses in the universe X given b $X = \{h_1, h_2, h_3\}$. Let $E = \{e_1(\text{cotton}), e_2(\text{woolen})\}$ be the set of parameters showing the material of the dresses.

Let $(F_1, E), (F_2, E), (F_3, E)$ be three soft sets over a common universe X, which describe the composition of the dresses, where

$F_1(e_1) = \{h_2\}$, $F_1(e_2) = \{h_1\}$.

$F_2(e_1) = \{h_2\}$, $F_2(e_2) = \{h_2\}$ and

$F_3(e_1) = \{h_2\}$, $F_3(e_2) = \{h_1, h_2\}$.

Then $\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E)\}$ is the supra soft topology over X. Let $\tilde{I} = \{\widetilde{\emptyset}, (I_1, E), (I_2, E), (I_3, E)\}$ be a soft ideal over X, where $(I_1, E), (I_2, E), (I_3, E)$ are soft sets over X defined by

$I_1(e_1) = \{h_1\}$, $I_1(e_2) = \emptyset$,

$I_2(e_1) = \emptyset$, $I_2(e_2) = \{h_1\}$, and

$I_3(e_1) = \{h_1\}$, $I_3(e_2) = \{h_1\}$.

So the soft sets $(G, E), (H, E)$ which defined by

$G(e_1) = \{h_1, h_2\}$ $G(e_2) = \emptyset$ and

$H(e_1) = \{h_2, h_3\}$ $H(e_2) = \emptyset$ are supra-Ĩg-closed soft sets.

But their soft intersection $(G, E) \tilde{\cap} (H, E) = (K, E)$, where

$K(e_1) = \{h_2\}$ $K(e_2) = \emptyset$ is not supra-Ĩg-closed soft.

Theorem: 2.2.12

If H_E is supra-Ĩg-closed soft set and F_E is a supra closed soft in a supra soft topological space (X, μ, E). Then $H_E \tilde{\cap} F_E$ is a supra-Ĩg-closed soft in (X, μ, E).

Proof:

Assume $H_E \tilde{\cap} F_E \tilde{\subseteq} G_E$ and $G_E \in \mu$. Then $H_E \tilde{\subseteq} G_E \tilde{\cup} F_E'$. Since H_E is a supra-Ĩg-closed soft set. It follows that $cl^s H_E[G_E \tilde{\cup} F_E'] \in \tilde{I}$. Now, $cl^s[H_E \tilde{\cap} F_E] \tilde{\subseteq} cl^s H_E \tilde{\cap} cl^s F_E = cl^s H_E \tilde{\cap} F_E = cl^s H_E \tilde{\cap} F_E \backslash F_E'$.

Thus $cl^s[H_E \tilde{\cap} F_E] \backslash G_E \tilde{\subseteq} cl^s[H_E \tilde{\cap} F_E] \backslash [G_E \tilde{\cup} F_E'] \tilde{\subseteq} cl^s H_E \backslash [G_E \tilde{\cup} F_E'] \in \tilde{I}$. Hence $H_E \tilde{\cap} F_E$ is a supra-Ĩg-closed soft set.

Section 2.3

Supra generalized open soft sets with respect to a soft ideal

Definition: 2.3.1

A soft set $F_E \in SS(X, E)$ is called supra generalized open soft set with respect to a soft ideal $\tilde{I}$ (supra-$\tilde{I}$g-open soft) in a supra soft topological space (X, μ, E) if and only if its relative complement F'_E is a supra-$\tilde{I}$g-closed soft in (X, μ, E).

Theorem: 2.3.2

A supra soft set (F, E) is a supra-$\tilde{I}$g-open soft in a supra soft topological space (X, μ, E) if and only if $G_E \backslash I_E \tilde{\subseteq} int^s F_E$ for some $I_E \in \tilde{I}$, whenever $G_E \tilde{\subseteq} F_E$ and G_E is supra closed soft in (X, μ, E).

Proof:

Necessity: Suppose that F_E is a supra -$\tilde{I}$g-open soft set. Let $G_E \tilde{\subseteq} F_E$ such that G_E is supra closed soft. We have $F'_E \tilde{\subseteq} G'_E$, F'_E is a supra-$\tilde{I}$g-closed soft and $G'_E \in \mu$. It follows that $cl^s F'_E \backslash G'_E \in \tilde{I}$ from Definition 2.3.1. This implies that $cl^s F'_E \backslash G'_E = I_E \in \tilde{I}$, and then $cl^s F'_E \backslash G'_E = cl^s F'_E \tilde{\cap} G'_E = I_E \in \tilde{I}$, so $[cl^s F'_E \tilde{\cap} G'_E] \tilde{\cup} G'_E = I_E \tilde{\cup} G'_E$. This implies that $cl^s F'_E \tilde{\subseteq} cl^s F'_E \tilde{\cup} G'_E = I_E \tilde{\cup} G'_E$. Hence $cl^s F'_E \tilde{\subseteq} G'_E \tilde{\cup} I_E$ for some $I_E \in \tilde{I}$. So $(G'_E \tilde{\cup} I_E) \tilde{\subseteq} [cl^s F'_E]' = int^s F_E$. Therefore, $G_E \backslash I_E = (G'_E \tilde{\cup} I_E) \tilde{\subseteq} int^s F_E$.

Sufficiency: Conversely, assume that F_E be a supra soft set. We want to prove that F_E is a supra-$\tilde{I}$g-open soft set. It is sufficient to prove that F'_E is a supra-$\tilde{I}$g-closed soft set. So, let $F'_E \tilde{\subseteq} G_E$ such that $G_E \in \mu$. Hence $G'_E \tilde{\subseteq} F_E$. By assumption, $G'_E \backslash I_E \tilde{\subseteq} int^s F_E$ for some $I_E \in \tilde{I}$. Hence $cl^s F'_E \tilde{\subseteq} [G'_E \backslash I_E]' = [G_E \tilde{\cup} I_E]$. Thus $cl^s F'_E \backslash G_E \tilde{\subseteq} [G'_E \backslash I_E]' = [G_E \tilde{\cup} I_E] \backslash G_E = [G_E \tilde{\cup} I_E] \tilde{\cap} G'_E = I_E \tilde{\cap} G'_E \tilde{\subseteq} I_E \in \tilde{I}$. This shows that $cl^s F'_E \backslash G_E \in \tilde{I}$. Therefore F'_E is a supra-$\tilde{I}$g-closed soft set and hence F_E is a supra-$\tilde{I}$g-open soft set. This completes the proof.

Definition: 2.3.3

Two soft sets F_E and G_E are said to be **supra soft separated sets** in a supra soft topological space (X, μ, E) if $cl^s F_E \tilde{\cap} G_E = \tilde{\emptyset}$ and $F_E \tilde{\cap} cl^s G_E = \tilde{\emptyset}$.

Theorem: 2.3.4

If A_E and B_E are supra soft separated and supra-$\tilde{I}$g-open soft sets in a supra soft topological space (X, τ, E), then $A_E \tilde{\cup} B_E$ is a supra-$\tilde{I}$g-open soft in (X, μ, E).

Proof:

Suppose that A_E and B_E are supra soft separated and supra-$\tilde{I}$g-open soft sets in a supra soft topological space (X, τ, E) and F_E be a supra closed set such that $F_E \tilde{\subseteq} A_E \tilde{\cup} B_E$. Then $F_E \tilde{\cap} cl^s A_E \tilde{\subseteq} [A_E \tilde{\cup} B_E] \tilde{\cap} cl^s A_E = A_E$ and $F_E \tilde{\cap} cl^s B_E \tilde{\subseteq} B_E$ from Definition 2.3.3. $[F_E \tilde{\cap} cl^s A_E] \backslash D_E \tilde{\subseteq} int^s A_E$ and $[F_E \tilde{\cap} cl^s B_E] \backslash C_E \tilde{\subseteq} int^s B_E$ for some $D_E, C_E \in \tilde{I}$ from Theorem 2.3.2. This means that $[F_E \tilde{\cap} cl^s A_E] \backslash int^s A_E \in I$ and $[F_E \tilde{\cap} cl^s B_E] \backslash int^s B_E \in I$. Then $[(F_E \tilde{\cap} cl^s A_E) \backslash int^s A_E \in I] \tilde{\cup} [(F_E \tilde{\cap} cl^s B_E) \backslash int^s B_E \in I] \in \tilde{I}$.

Hence $[F_E \tilde{\cap} (cl^s A_E \tilde{\cup} cl^s B_E)] \backslash [int^s A_E \tilde{\cup} int^s B_E] \in \tilde{I}$.

But $F_E = F_E \tilde{\cap} (A_E \tilde{\cup} B_E) \tilde{\subseteq} F_E \tilde{\cap} [cl^s(A_E \tilde{\cup} B_E)]$,

and we have,

$$F_E \backslash int^s(A_E \tilde{\cup} B_E) \tilde{\subseteq} (F_E \tilde{\cap} (cl^s[A_E \tilde{\cup} B_E])) \backslash int^s(A_E \tilde{\cup} B_E)$$
$$\tilde{\subseteq} (F_E \tilde{\cap} (cl^s[A_E \tilde{\cup} B_E]) \backslash int^s A_E \tilde{\cup} int^s B_E) \in \tilde{I}.$$

Now, take $G_E = F_E \backslash int^s(A_E \tilde{\cup} B_E) \in \tilde{I}$.

Then $F_E \backslash G_E = F_E \backslash [F_E \backslash int^s(A_E \tilde{\cup} B_E)] \tilde{\subseteq} int^s(A_E \tilde{\cup} B_E) \in \tilde{I}$. Therefore, $A_E \tilde{\cup} B_E$ is a supra-$\tilde{I}$g-open soft in (X, μ, E) from Theorem 2.3.2.

Corollary: 2.3.5

If A_E and B_E are supra-$\tilde{I}$g-closed soft sets in a supra soft topological space (X, μ, E) such that A'_E and B'_E are supra soft separated sets, then $A_E \tilde{\cap} B_E$ is a supra-$\tilde{I}$g-closed soft in (X, μ, E).

Theorem 2.3.6

If A_E and B_E are supra-$\tilde{I}$g-closed soft sets in a supra soft topological space (X, μ, E), then $A_E \tilde{\cap} B_E$ is a supra-$\tilde{I}$g-open soft in (X, μ, E).

Proof:

Let A_E and B_E be a supra-Ĩg-open soft in a soft topological space (X, μ, E), then A'_E and B'_E are supra-$\tilde{I}$g-closed soft sets. Hence $(A_E \tilde{\cap} B_E)' = A'_E \tilde{\cup} B'_E$ is a supra-$\tilde{I}$g-closed soft sets from Theorem 2.2.8. Therefore, $A_E \tilde{\cup} B_E$ is a supra-$\tilde{I}$g-closed soft.

Theorem: 2.3.7

Let A_E be a supra-$\tilde{I}$g-open soft in a soft topological space (X, μ, E) such that $int^s A_E \tilde{\subseteq} B_E \tilde{\subseteq}$ for some $B_E \in SS(X)_E$. Then B_E is a supra-$\tilde{I}$g-open soft in (X, μ, E).

Proof:

Let A_E be a supra-$\tilde{I}$g-open soft in a soft topological space (X, μ, E) such that $int^s A_E \tilde{\subseteq} B_E \tilde{\subseteq}$ for some $B_E \in SS(X)_E$. Then $A'_E \tilde{\subseteq} B'_E \tilde{\subseteq} (int^s A_E)' = cl^s(A'_E)$ and A'_E is a supra-$\tilde{I}$g-closed soft. Hence B'_E is a supra-$\tilde{I}$g-closed soft from Theorem 2.2.9. Therefore, B_E is a supra-$\tilde{I}$g-open soft in (X, μ, E).

Theorem: 2.3.8

A soft set A_E is a supra-$\tilde{I}$g-closed soft in a supra soft topological space (X, μ, E) if and only if $cl^s A_E \backslash A_E$ is a supra-$\tilde{I}$g-open soft.

Proof:

($\Rightarrow$)Let $F_E \tilde{\subseteq} cl^s A_E \backslash A_E$ and F_E is a supra closed soft set. Then $F_E \in \tilde{I}$ from Theorem 2.2.7. Hence, there exists $I_E \in \tilde{I}$ such that $F_E \backslash I_E = \tilde{\emptyset}$. Thus $F_E \backslash I_E = \tilde{\emptyset} \tilde{\subseteq} int^s[cl^s A_E \backslash A_E]$. Therefore, $cl^s A_E \backslash A_E$ is a supra-$\tilde{I}$g-open soft from Theorem 2.3.2

($\Leftarrow$) Let $A_E \tilde{\subseteq} G_E$ such that $G_E \in \mu$. Then $cl^s A_E \tilde{\cap} G'_E \tilde{\subseteq} cl^s A_E \tilde{\cap} A'_E = cl^s A_E \backslash A_E$. By hypothesis, $[cl^s A_E \tilde{\cap} G'_E] \backslash int^s[cl^s A_E \backslash A_E] = \tilde{\emptyset}$, for some $I_E \in \tilde{I}$ from Theorem 2.3.2. This implies that $cl^s A_E \tilde{\cap} G'_E \tilde{\subseteq} I_E \in \tilde{I}$. Therefore, $cl^s A_E \backslash G_E \in \tilde{I}$. Thus A_E is a supra-$\tilde{I}$g-closed soft.

Theorem 2.3.9

Let $(X_1,\mu_1,A),(X_2,\mu_2,B)$ be supra soft topological spaces. Let $f_{pu}:SS(X_1)_A \to SS(X_2)_B$ be closed and continuous soft function. If $A_E \in SS(X,E)$ is a supra-$\tilde{I}$g-closed soft in (X_1,μ_1,A), then $f_{pu}(A_E)$ is a supra-$f_{pu}(\tilde{I})$g-closed soft in (X_2,μ_2,B), where $f_{pu}(\tilde{I}) = \{f_{pu}(I_E): I_E \in \tilde{I}\}$.

Proof:

Let $A_E \in SS(X)_A$ be a supra-$\tilde{I}$g-closed soft in (X_1,μ_1,A) and $f_{pu}(A_E) \tilde{\subseteq} G_E$ for some $G_E \in \mu_2$. Then $A_E \tilde{\subseteq} f_{pu}^{-1}(G_E)$. It follow that $cl^s A_E \backslash f_{pu}^{-1}(G_E) \in \tilde{I}$ from Definition 2.1.1. Hence $f_{pu}(cl^s A_E)\backslash G_E \in f_{pu}(\tilde{I})$. Since f_{pu} is a closed soft function, then $f_{pu}\, cl^s(A_E)$ is a supra closed soft in μ_2. Thus $cl^s(f_{pu}(A_E)) \tilde{\subseteq} cl^s(f_{pu}\, cl^s(A_E)) = f_{pu}\, cl^s(A_E)$. This implies that $cl^s(f_{pu}(A_E))\backslash G_E \tilde{\subseteq} f_{pu}\, cl^s(A_E)\backslash G_E \in f_{pu}(\tilde{I})$. Therefore, $f_{pu}(A_E)$ is a supra-$f_{pu}(\tilde{I})$g-closed soft in (X_2,μ_2,B). This completes the proof.