INTRODUCTION

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The concept of fuzzy sets and fuzzy set operations were introduced by Zadeh [83] in the year 1965. Since then several authors have worked on various basic concepts from general topology using fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology introduced by Chang [20]. Levin [44] introduced the concept of generalized closed sets in general topological spaces in the year 1970. Fukutake Saraf, Caldas and Mishra [33] introduced generalized pre-closed fuzzy sets in fuzzy topological spaces. In 2002, g·p-closed sets, g·p-continuous, g·p-irresolute, g·p-closed, g·p-open maps and $T_p^*, {}^*T_P$ -spaces were introduced and studied by Veerakumar [80] for general topology. Pu and Liu [59] introduced the concept of quasi-coincidence and q-neighbourhoods by which the extension of functions in fuzzy setting can very interestingly and effectively be carried out.

Azad [9] introduced the concept of fuzzy semi continuity and studied the concepts of fuzzy almost continuity and fuzzy weakly continuity. Further he introduced the concepts of Fuzzy Hausdorff spaces and fuzzy perfect mappings. Warren [81, 82] introduced the concepts of neighbourhoods, bases and continuity in fuzzy topological spaces. Many authors such as Goguen [36], Malghan and Benchalli [47,48], Mukherjee and Ghosh [54] helped in the development of fuzzy topological spaces. Thakur et al. [76] defined fuzzy semi-preopen sets. Saraf et al. [66] generalized the concept of fuzzy semi-pre open sets and introduced fuzzy semi – $T_{1/2}$-spaces, Fgsp-continuity and Fgsp-irresoluteness. As an extension of fuzzy topological spaces Atanassov [6] introduced the notion of Intuitionistic fuzzy sets in 1986. Using the notion of Intuitionistic fuzzy sets, Coker [22] defined the notion of Intuitionistic fuzzy topological spaces in 1997. The properties of Intuitionistic fuzzy sets were introduced by Gurcay [37] in 1997. Shanthi and Arun Prakash [61] introduced the concepts of intuitionistic fuzzy semi-generalized closed sets, intuitionistic fuzzy semi-generalized closed mappings, intuitionistic fuzzy almost semi-generalized continuous mappings and intuitionistic fuzzy quasi semi-generalized closed mappings and studied their properties and applications.

This thesis is devoted to the study of fuzzy semi-pre-generalized closed sets, fuzzy rw-open sets, fuzzy rw-closed sets, fuzzy g· pre-continuous maps in fuzzy topological spaces and Intuitionistic fuzzy almost semi-generalized closed sets and Intuitionistic fuzzy almost semi-generalized closed mappings in intuitionistic fuzzy topological spaces.

The following papers are chosen for discussion:

- "Fuzzy topological spaces", by Chang [20].
- "Fuzzy rw-closed sets and fuzzy rw-open sets in fuzzy topological spaces", by Benchalli et al. [16].
- "Fuzzy semi-pre-generalized closed sets" by Saraf et al. [67]
- "Fuzzy g·-pre-continuous maps in fuzzy topological spaces" by Benchalli and Siddapur [15].
- "Introduction to Intuitionistic fuzzy topological spaces", by Coker [22].
- "Intuitionistic fuzzy almost semi-generalized closed mappings", by Santhi et al. [62].

In **chapter 1**, preliminary definitions and results on fuzzy sets, fuzzy topological spaces, fuzzy neighbourhood of a fuzzy set, fuzzy continuous functions and fuzzy compact spaces due to Chang [20] are discussed.

In **chapter 2**, preliminary definitions and results on Intuitionistic fuzzy sets, intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuous mappings, intuitionistic fuzzy compactness spaces, intuitionistic fuzzy c_5- connectedness spaces and intuitionistic fuzzy Hausdorff spaces due to Coker [25] are discussed.

In **chapter 3**, fuzzy rw-closed sets and fuzzy rw-open sets due to Benchalli et al. [16] are studied.

Sec 3.1, deals with the Preliminary definitions and results of fuzzy semi closed sets and fuzzy semi open sets.

Some of the results discussed are as follows:

(i) The following are equivalent:

(a) λ is a fuzzy semiclosed set,

(b) λ^c is a fuzzy semiopen set

(c) int(cl(λ)) ≤ λ

(d) cl(int(λ))≥ λ^c

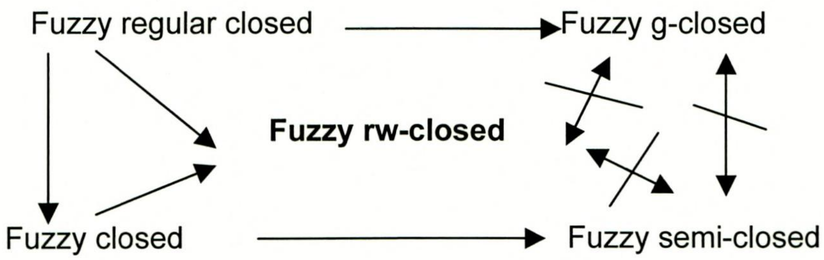
(ii) Every fuzzy open set is fuzzy semiopen but not conversely.

(iii) The closure of a fuzzy open set is a fuzzy semiopen set.

Sec 3.2, deals with fuzzy rw-closed sets and its properties.

"Let (X, T) be a fuzzy topological space. A fuzzy set α of X is called fuzzy regular w-closed (briefly, rw-closed) if cl (α) ≤σ whenever α≤σ and σ is fuzzy regular semiopen in fts X".

1) Every fuzzy closed set is a fuzzy rw-closed set in a fts X
2) Fuzzy generalized closed sets and fuzzy rw-closed sets are independent.
3) Union of fuzzy rw-closed sets in fts X is a fuzzy rw-closed set.
4) If a fuzzy set α of fts X is both fuzzy regular open and fuzzy rw- closed, then α is a fuzzy regular closed set in fts X.



Sec 3.3, deals with fuzzy rw-open sets. Some of the results discussed here are:

1) If a fuzzy set α of a fuzzy topological space X is fuzzy open, then it is fuzzy rw-open but not conversely true.
2) The intersection of fuzzy rw-open sets in fts X is a fuzzy rw- open set.
3) The union of two fuzzy rw-open sets in a fts X is generally not a fuzzy rw-open set in fts X.
4) Let α and β be two fuzzy subsets of a fts X. If β is a fuzzy rw-open set and $\alpha \geq int(\beta)$, then $\alpha \cap \beta$ is a fuzzy rw-open set in fts X.

In **Chapter 4**, fuzzy semi pre-generalized closed sets and fuzzy generalized closed mappings due to Saraf et al. [67] are studied.

"A fuzzy set A of (X,τ) is called fuzzy semi-pre-generalized closed (fspg-closed) if $Spcl(A) \leq H$, whenever $A \leq H$ and H is fs-open in X".

The relationship between fuzzy semi pre-generalized closed sets and other generalized closed sets are discussed. Properties, characterizations and applications of fuzzy semi pre-generalized continuous mappings are also studied.

In sec 4.1, preliminary definitions and results of fuzzy semi pre-generalized closed sets are studied.

"A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy generalized Semi pre-continuous (fgsp-continuous) if $f^{-1}(V)$ is fgsp-closed in X, for every fuzzy closed set V in Y".

Some of the results discussed here are:

1) Every fp-closed set is fspg-closed.

2) Every gfs-closed set is fspg-closed.

3) Every fsp-closed set is fspg-closed.

4) Every fspg-closed set is fgsp-closed.

In sec 4.2, fuzzy semi-pre-generalized continuous and fuzzy semi-pre-generalized-irresolute mappings are studied.

"A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is called fuzzy semi-pre-generalized continuous (fspg-continuous) if $f^{-1}(V)$ is fspg-closed in (X,τ) for every fuzzy closed set V of (Y,σ)".

"A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is called fuzzy semi-pre-generalized irresolute (fspg-irresolute) if $f^{-1}(V)$ is fspg-closed in (X,τ) for every fspg-closed set V of (Y,σ)".

Some of the important results discussed are as follows:

1) Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be gfs-continuous. Then f is fspg-continuous.

2) Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be fspg-irresolute. Then f is fspg-continuous.

3) Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be fspg-continuous. Then f is fgsp-continuous but not conversely.

4) Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be onto fspg-irresolute and fuzzy M-semi-pre closed. If X is fsp $T_{½}$ -space then (Y,σ) is also fsp $T_{½}$ -space.

Sec 4.3 deals with the fuzzy semi-pre-generalized connected spaces.

A fts (X,τ) is said to be fuzzy semi-pre-generalized connected (fspg-connected) iff the only fuzzy sets which are both fspg-open and fspg-closed are 0_X and 1_X.

Some of the important results discussed are as follows:

1. Let (X,τ) be a fts. If X is an fspg-connected space, then it is fs-connected.
2. A fts (X, τ) is fspg-connected if X has no non-zero fspg-open sets A and B such that $A + B = 1_X$.
3. If f: $(X, \tau) \rightarrow (Y, \sigma)$ is fspg-continuous surjection and (X, τ) is fspg-connected, then (Y,σ) is fuzzy connected.

In **chapter 5**, properties and characterization of fuzzy g·p-continuous maps, fuzzy g·p-irresolute maps, fuzzy g·p-closed maps, fuzzy g·p- open maps and fuzzy g·p-homeomorphisms in fuzzy topological spaces due to Benchalli and Siddapur [15] are studied.

Sec 5.1, deals with the preliminary definitions and results of fuzzy g·pre- continuous mappings in fuzzy topological spaces.

Some of the characterizations and properties discussed are as follows:

1. A function f: X → Y is fg·p-continuous iff the inverse image of every closed fuzzy set in Y is a g·p-closed fuzzy set in Y.
2. Every f-continuous function is a fg·p-continuous function.
3. Every fg·p-continuous function is a fgp-continuous function.
4. Every fg·p-continuous function is a fgsp-continuous function

Sec 5.2 deals with the fuzzy g·pre-homeomorphisms in fuzzy topological spaces.

"A function f: X → Y is called fuzzy g·p-homeomorphism if f and f^{-1} are fg·p-continuous".

Characterizations of fuzzy g· pre-homeomorphisms are given as follows:

Let f: X → Y be a bijective function. Then the following are equivalent.

1. f is fg·p-homeomorphism.
2. f is fg·p-continuous and fg·p-open maps.
3. f is fg·p-continuous and fg·p-closed maps

In chapter 6, intuitionistic fuzzy almost semi-generalized closed sets, characterization of intuitioinistic fuzzy almost semi-generalized closed mappings and open mappings due to Shanthi et.al [62] are studied.

Sec 6.1 deals with preliminary definitions which are needed to define intuitionistic fuzzy almost semi-generalized closed sets.

Some of the definitions studied here are:

1) Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP), written as $\rho_{(\alpha,\beta)}$ is defined to be an IFS(X) given by

$$\rho_{(\alpha,\beta)} = \{(\alpha,\beta) \quad \text{if } x=\rho \\ (0, 1) \quad \text{otherwise}$$

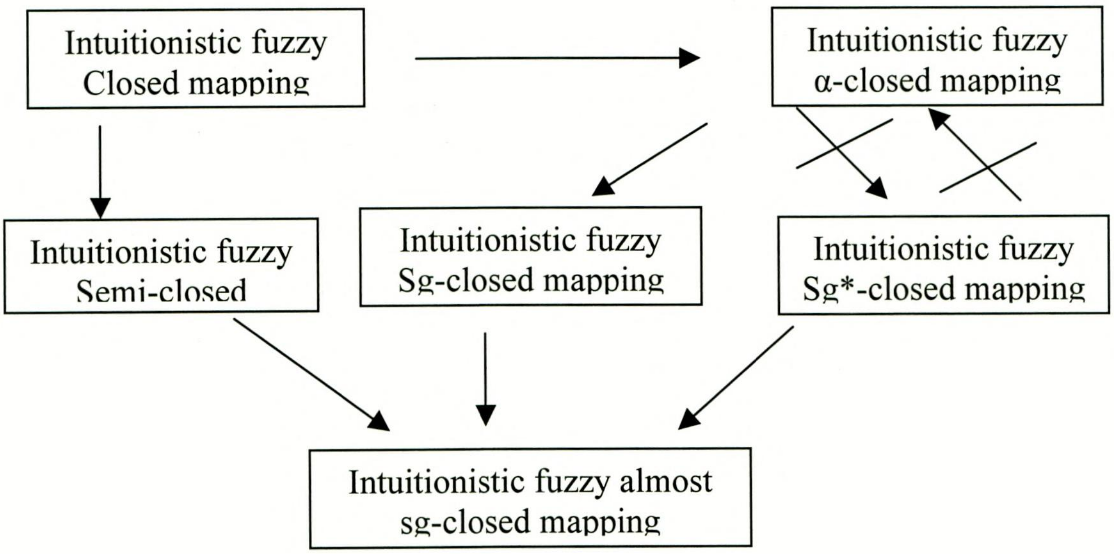
2) Let A= {(x, μ_A(x), γ_A (x)) / x∈X} be a IFS in an IFTS (X, T), then A is called an intuitionistic fuzzy semi open set (IFSOS) if A ⊆ cl(int(A)).
3) Let A= {(x, μ_A(x), γ_A (x)) / x∈X} be a IFS in an IFTS (X, T), then A is called an intuitionistic fuzzy α-open set (IFαOS) if A ⊆ int(cl(int(A))).

Sec 6.2 deals with intuitionistic fuzzy almost semi-generalized continuous mappings.

"A mapping f: X → Y is said to be an intuitionistic fuzzy almost semi-generalized closed (intuitionistic fuzzy almost sg-closed) mapping if f (A) is an IFSGCS in Y for every IFRCS A in X".

Some of the results discussed here are as follows:

1) Every intuitionistic fuzzy closed mapping is an intuitionistic fuzzy almost sg-closed mapping.
2) Every intuitionistic fuzzy semi-closed mapping is an intuitionistic fuzzy almost sg-closed mapping.
3) Every intutionistic fuzzy sg-closed mapping is an intutionistic fuzzy almost sg-closed mapping.
4) Every intuitionistic fuzzy quasi sg-closed mapping is an intuitionistic fuzzy almost sg-closed mapping.



Here A ⟶ B means A implies B

A ⟶̸ B means A does not implies B.