CHAPTER - VI

INTUITIONISTIC FUZZY SOFT RELATIONS

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Definition : 6.1 [9]

Let U be an initial universal set and E be the set of parameters. Let A, B $\subseteq$ E and ($\mathcal{F}$, A), ($\mathcal{G}$, B) be two intuitionistic fuzzy soft sets over (U, E). Then the **cartesian product** of ($\mathcal{F}$, A) and ($\mathcal{G}$, B) is denoted by

($\mathcal{F}$, A) × ($\mathcal{G}$, B) = ($\mathcal{H}$, C) where C = A × B and $\mathcal{H}$: C $\rightarrow$ IF^U is defined as

$\mathcal{H}$(a, b) = ($\mathcal{F}$, A) $\cap$ ($\mathcal{G}$, B), (a, b) $\in$ C.

Example: 6.2

Suppose that U is the set of medicine under consideration and U = {m_1, m_2, m_3, m_4}. Let A = {fever, chest pain, cough } and B = { fever, cough}. i,e., A = {f, p, c} and B = {f, c}.The intuitionistic fuzzy soft set (F, A) be defined as follows:

$\mathcal{F}$(f) = {(m_1, 0.9, 0.05), (m_2, 0.25, 0.6), (m_3, 0.65, 0.2), (m_4, 0.8, 0.1)}

$\mathcal{F}$(p) = {(m_1, 0.3, 0.6), (m_2, 0.9, 0.1), (m_3, 0.4, 0.6), (m_4, 0.3, 0.65)}

$\mathcal{F}$(c) = {(m_1, 0.6, 0.2), (m_2, 0.3, 0.6), (m_3, 0.9, 0.05), (m_4, 0.85, 0.1)}.

The intuitionistic fuzzy soft set (G, B) be defined as follows:

$\mathcal{G}$(f) = {(m_1, 0.85, 0.1), (m_2, 0.2, 0.7), (m_3, 0.5, 0.4), (m_4, 0.8, 0.1)}

$\mathcal{G}$(c) = {(m_1, 0.65, 0.3), (m_2, 0.3, 0.65), (m_3, 0.9, 0.1), (m_4, 0.7, 0.2)}.

Let ($\mathcal{F}$, A) × ($\mathcal{G}$, B) = ($\mathcal{H}$, C), where C = A × B. Consider the t-norm function $*$ and t-conorm function $\Diamond$ as $a * b = ab$ and $a \Diamond b = a + b - ab$.

Then the elements of $(\mathcal{H}, C)$ will be as follows:

$\mathcal{H}(f, f) = \{(m_1, 0.765, 0.145), (m_2, 0.05, 0.88), (m_3, 0.325, 0.52), (m_4, 0.64, 0.19)\}$

$\mathcal{H}(f, c) = \{(m_1, 0.585, 0.335), (m_2, 0.075, 0.86), (m_3, 0.585, 0.28), (m_4, 0.56, 0.28)\}$

$\mathcal{H}(p, f) = \{(m_1, 0.255, 0.64), (m_2, 0.18, 0.73), (m_3, 0.2, 0.76), (m_4, 0.24, 0.685)\}$

$\mathcal{H}(p, c) = \{(m_1, 0.195, 0.72), (m_2, 0.27, 0.685), (m_3, 0.36, 0.64), (m_4, 0.21, 0.72)\}$

$\mathcal{H}(c, f) = \{(m_1, 0.51, 0.28), (m_2, 0.06, 0.88), (m_3, 0.45, 0.43), (m_4, 0.68, 0.19)\}$

$\mathcal{H}(c, c) = \{(m_1, 0.39, 0.44), (m_2, 0.09, 0.86), (m_3, 0.81, 0.145), (m_4, 0.595, 0.28)\}$.

Definition : 6.3

Let $(\mathcal{F}, A)$, $(\mathcal{G}, B)$ be two intuitionistic fuzzy soft sets over (U, E). Then an **intuitionistic fuzzy soft relation** from $(\mathcal{F}, A)$ to $(\mathcal{G}, B)$ is an intuitionistic fuzzy soft subset of $(\mathcal{F}, A) \times (\mathcal{G}, B)$.

In other words, an intuitionistic fuzzy soft relation from $(\mathcal{F}, A)$ to $(\mathcal{G}, B)$ is of the form $(\mathcal{R}, C)$ where $C \subseteq A \times B$ and $\mathcal{R}(a, b) \subseteq (\mathcal{F}, A) \times (\mathcal{G}, B)$, $\forall (a, b) \in C$.

Example : 6.4

Consider the intuitionistic fuzzy soft sets $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ over U defined as follows:

Let U be the set of medicines under consideration and $U = \{m_1, m_2, m_3, m_4\}$. A and B describes the disease of the patients. A = {maleria, dengue} and B = {fileria, cough}. i.e., A = {m, d} and B = {f, c}.

Define an intuitionistic fuzzy soft relation ($\mathcal{R}$, C) from ($\mathcal{F}$, A) to ($\mathcal{G}$, B) as follows:

(a, b) $\in$ C $\subseteq$ A × B if and only if a and b are both from mosquito.

Then the intuitionistic fuzzy soft relation ($\mathcal{R}$, C) = {$\mathcal{F}$ (m) × $\mathcal{G}$ (f), $\mathcal{F}$ (d) × $\mathcal{G}$ (f)}.

Example : 6.5

Let U = {c_1, c_2, c_3, c_4, c_5, c_6, c_7} be the set of candidates attending for an interview of school service and A denotes the academic qualification of candidates. A = {B.Sc., M.Com., M.A., M.Sc} i.e., A = {b, c, a, s}.

Then the intuitionistic fuzzy soft set ($\mathcal{F}$, A) describes the B.Sc. passed students, M.Com. passed students and so on. A relation ($\mathcal{R}$, C) on ($\mathcal{F}$, A) be defined by

(a, b) $\in$ C $\subseteq$ A × A if and only if a and b both are of science.

Then ($\mathcal{R}$, C) = {$\mathcal{F}$ (b) × $\mathcal{F}$ (b), $\mathcal{F}$ (b) × $\mathcal{F}$ (s), $\mathcal{F}$ (s) × $\mathcal{F}$ (s), $\mathcal{F}$ (s) × $\mathcal{F}$ (b)}.

Definition : 6.6

Let $\mathcal{R}$ be an intuitionistic fuzzy soft relation from ($\mathcal{F}$, A) to ($\mathcal{G}$, B) then $\mathcal{R}^{-1}$ is defined as $\mathcal{R}^{-1}$ (a, b) = $\mathcal{R}$(b, a), $\forall$ (a, b) $\in$ C $\subseteq$ A × B

Theorem : 6.7

If $\mathcal{R}$ is an intuitionistic fuzzy soft relation from ($\mathcal{F}$, A) to ($\mathcal{G}$, B) then $\mathcal{R}^{-1}$ is a intuitionistic fuzzy soft relation from ($\mathcal{G}$, B) to ($\mathcal{F}$, A).

Proof :

$\mathcal{R}^{-1}$ (a, b) = $\mathcal{R}$(b, a) = $\mathcal{G}$ (b) $\cap$ $\mathcal{F}$ (a), $\forall$ (a, b) $\in$ C $\subseteq$ A × B.

Hence $\mathcal{R}^{-1}$ is an intuitionistic fuzzy soft relation from ($\mathcal{G}$, B) to ($\mathcal{F}$, A).

Theorem : 6.8

If $\mathcal{R}_1$ and $\mathcal{R}_2$ be two intuitionistic fuzzy soft relations from $(\mathcal{F}, A)$ to $(\mathcal{G}, B)$ then

1) $(\mathcal{R}_1^{-1})^{-1} = \mathcal{R}_1$ and
2) $\mathcal{R}_1 \subseteq \mathcal{R}_2 \Rightarrow \mathcal{R}_1^{-1} \subseteq \mathcal{R}_2^{-1}$

Proof:

1) $(\mathcal{R}_1^{-1})^{-1}(a, b) = (\mathcal{R}_1^{-1})(b, a)$.

   Hence $(\mathcal{R}_1^{-1})^{-1} = \mathcal{R}_1$.

2) $\mathcal{R}_1(a, b) \subseteq \mathcal{R}_2(a, b) \Rightarrow \mathcal{R}_1^{-1}(b, a) \subseteq \mathcal{R}_2^{-1}(b, a) \Rightarrow \mathcal{R}_1^{-1} \subseteq \mathcal{R}_2^{-1}$

Definition : 6.9

The **composition** $\circ$ of two intuitionistic fuzzy soft relations $\mathcal{R}_1$ and $\mathcal{R}_2$ is defined by $(\mathcal{R}_1 \circ \mathcal{R}_2)(a, c) = \mathcal{R}_1(a, b) \cap \mathcal{R}_2(b, c)$, where $\mathcal{R}_1$ is a intuitionistic fuzzy soft relation form $(\mathcal{F}, A)$ to $(\mathcal{G}, B)$ and $\mathcal{R}_2$ is a intuitionistic fuzzy soft relation from $(\mathcal{G}, B)$ to $(\mathcal{H}, C)$.

Theorem : 6.10

If $\mathcal{R}_1$ and $\mathcal{R}_2$ be two intuitionistic fuzzy soft relations from $(\mathcal{F}, A)$ to $(\mathcal{G}, B)$ then $(\mathcal{R}_1 \circ \mathcal{R}_2)^{-1} = \mathcal{R}_2^{-1} \circ \mathcal{R}_1^{-1}$

Proof :

$$(\mathcal{R}_1 \circ \mathcal{R}_2)^{-1}(a, c) = (\mathcal{R}_1 \circ \mathcal{R}_2)(c, a)$$

$$= \mathcal{R}_1(c, b) \cap \mathcal{R}_2(b, a)$$

$= \mathcal{R}_2(b, a) \cap \mathcal{R}_1(c, b)$

$= \mathcal{R}_2^{-1}(a, b) \cap \mathcal{R}_1^{-1}(b, c)$

$= (\mathcal{R}_2^{-1} \circ \mathcal{R}_1^{-1})(a, c).$

Hence $(\mathcal{R}_1 \circ \mathcal{R}_2)^{-1} = \mathcal{R}_2^{-1} \circ \mathcal{R}_1^{-1}$.

Theorem : 6.11

$\mathcal{R}_1$ is an intuitionistic fuzzy soft relation form $(\mathcal{F}, A)$ to $(\mathcal{G}, B)$ and $\mathcal{R}_2$ is a intuitionistic fuzzy soft relation from $(\mathcal{G}, B)$ to $(\mathcal{H}, C)$ both satisfying the conditions $a * a = a$ and $a \lozenge a = a$ then $\mathcal{R}_1 \circ \mathcal{R}_2$ is an intuitionistic fuzzy soft relation from $(\mathcal{F}, A)$ to $(\mathcal{H}, C)$.

Proof :

By definition

$\mathcal{R}_1(a, b) = \mathcal{F}(a) \cap \mathcal{G}(b)$

$= \{(x, \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x), \nu_{\mathcal{F}(a)}(x) * \nu_{\mathcal{G}(b)}(x) : x \in U\}, \forall (a, b) \subseteq A \times B.$

$\mathcal{R}_2(b, c) = \mathcal{G}(b) \cap \mathcal{H}(c)$

$= \{(x, \mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x), \nu_{\mathcal{H}(c)}(x) * \nu_{\mathcal{H}(c)}(x)) : x \in U\}, \forall (b, c) \subseteq B \times C.$

Therefore

$(\mathcal{R}_1 \circ \mathcal{R}_2)(a, c) = \mathcal{R}_1(a, b) \cap \mathcal{R}_2(b, c)$

$= \{(x, (\mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x)) * (\mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x)), (\nu_{\mathcal{F}(a)}(x) \lozenge \nu_{\mathcal{G}(b)}(x)) \lozenge (\nu_{\mathcal{G}(b)}(x) \lozenge \nu_{\mathcal{H}(c)}(x)) : x \in U\}, \forall (a, b, c) \subseteq A \times B \times C.$

Now

$$(\mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x)) * (\mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x)) = \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x)$$

$$\leq \mu_{\mathcal{F}(a)}(x) * 1 * \mu_{\mathcal{H}(c)}(x)$$

$$= \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{H}(c)}(x)$$

$$(\nu_{\mathcal{F}(a)}(x) \Diamond \nu_{\mathcal{G}(b)}(x)) \Diamond (\nu_{\mathcal{G}(b)}(x) \Diamond \nu_{\mathcal{H}(c)}(x)) = \nu_{\mathcal{F}(a)}(x) \Diamond \nu_{\mathcal{G}(b)}(x) \Diamond \nu_{\mathcal{H}(c)}(x)$$

$$\geq \nu_{\mathcal{F}(a)}(x) \Diamond 0 \Diamond \nu_{\mathcal{H}(c)}(x)$$

$$= \nu_{\mathcal{F}(a)}(x) \Diamond \nu_{\mathcal{H}(c)}(x).$$

Hence $\mathcal{R}_1(a, b) \cap \mathcal{R}_2(b, c) \subseteq \mathcal{F}(a) \cap \mathcal{H}(c)$.

Thus $\mathcal{R}_1 \circ \mathcal{R}_2$ is an intuitionistic fuzzy soft relation from $(\mathcal{F}, A)$ to $(\mathcal{H}, C)$.

Definition : 6.12

An intuitionistic fuzzy soft relation $\mathcal{R}$ on $(\mathcal{F}, A)$ is said to be **intuitionistic fuzzy soft symmetric relation** if $\mathcal{R}(a, b) = \mathcal{R}(b, a)$, $\forall (a, b) \in A$.

Definition : 6.13

An intuitionistic fuzzy soft relation $\mathcal{R}$ on $(\mathcal{F}, A)$ is said to be **intuitionistic fuzzy soft transitive relation** if $\mathcal{R} \circ \mathcal{R} \subseteq \mathcal{R}$.

Definition : 6.14

An intuitionistic fuzzy soft relation $\mathcal{R}$ on $(\mathcal{F}, A)$ is said to be **intuitionistic fuzzy soft reflexive relation** if $\mathcal{R}(a, b) \subseteq \mathcal{R}(a, a)$ and $\mathcal{R}(b, a) \subseteq \mathcal{R}(a, a)$, $\forall (a, b) \in A$.

Definition : 6.15

An intuitionistic fuzzy soft relation $\mathcal{R}$ on ($\mathcal{F}$, A) is said to be an **intuitionistic fuzzy soft equivalence relation** if it is symmetric, transitive and reflexive.

Theorem : 6.16

If $\mathcal{R}$ is symmetric if and only if $\mathcal{R}^{-1}$ is so.

Proof:

Let $\mathcal{R}$ is symmetric.

Then $\mathcal{R}^{-1}$(a, b) = $\mathcal{R}$(b, a) = $\mathcal{R}$(a, b) = $\mathcal{R}^{-1}$(b, a).

So, $\mathcal{R}^{-1}$ is symmetric.

Conversely, let $\mathcal{R}^{-1}$ is symmetric.

Then $\mathcal{R}$(a, b) = $(\mathcal{R}^{-1})^{-1}$ (a, b) = $\mathcal{R}^{-1}$(b, a) = $\mathcal{R}^{-1}$(a, b) = $\mathcal{R}$(b, a).

So, $\mathcal{R}$ is symmetric.

Theorem : 6.17

$\mathcal{R}$ is symmetric if and only if $\mathcal{R} = \mathcal{R}^{-1}$.

Proof:

Let $\mathcal{R}$ is symmetric.

Then $\mathcal{R}^{-1}$(a, b) = $\mathcal{R}$(b, a) = $\mathcal{R}$(a, b).

So, $\mathcal{R}^{-1} = \mathcal{R}$.

Conversely, let $\mathcal{R}^{-1} = \mathcal{R}$.

Then $\mathcal{R}$ (a, b) = $\mathcal{R}^{-1}$ (a, b) = $\mathcal{R}$ (b, a).

So, $\mathcal{R}$ is symmetric.

Theorem : 6.18

If $\mathcal{R}_1$ and $\mathcal{R}_2$ are symmetric relations on (F, A) then $\mathcal{R}_1 \circ \mathcal{R}_2$ is symmetric on ($\mathcal{F}$, A) if and only if $\mathcal{R}_1 \circ \mathcal{R}_2 = \mathcal{R}_2 \circ \mathcal{R}_1$.

Proof:

$\mathcal{R}_1$ and $\mathcal{R}_2$ are symmetric implies $\mathcal{R}_1^{-1} = \mathcal{R}_1$ and $\mathcal{R}_2^{-1} = \mathcal{R}_2$.

Now $(\mathcal{R}_1 \circ \mathcal{R}_2)^{-1} = \mathcal{R}_2^{-1} \circ \mathcal{R}_1^{-1}$

So, $\mathcal{R}_1 \circ \mathcal{R}_2$ is symmetric implies $\mathcal{R}_1 \circ \mathcal{R}_2 = (\mathcal{R}_1 \circ \mathcal{R}_2)^{-1} = \mathcal{R}_2^{-1} \circ \mathcal{R}_1^{-1} = \mathcal{R}_2 \circ \mathcal{R}_1$.

Conversely, $(\mathcal{R}_1 \circ \mathcal{R}_2)^{-1} = \mathcal{R}_2^{-1} \circ \mathcal{R}_1^{-1} = \mathcal{R}_2 \circ \mathcal{R}_1 = \mathcal{R}_1 \circ \mathcal{R}_2$.

So, $\mathcal{R}_1 \circ \mathcal{R}_2$ is symmetric.

Corollary : 6.19

If $\mathcal{R}$ is symmetric then $\mathcal{R}^n$ is symmetric for all positive integer n, where $\mathcal{R}^n$ is $\mathcal{R} \circ \mathcal{R} \circ \cdots\cdots \circ \mathcal{R}$ (n times).

Theorem : 6.20

If $\mathcal{R}$ is transitive then $\mathcal{R}^{-1}$ is also transitive.

Proof:

$\mathcal{R}^{-1}(a, b) = \mathcal{R}(b, a)$

$\supseteq (\mathcal{R} \circ \mathcal{R})(b, a)$

$= \mathcal{R}(b, c) \cap \mathcal{R}(c, a)$

$= \mathcal{R}(c, a) \cap \mathcal{R}(b, c)$

$= \mathcal{R}^{-1}(a, c) \cap \mathcal{R}^{-1}(c, b)$

$= (\mathcal{R}^{-1} \circ \mathcal{R}^{-1})(a, b).$

So, $\mathcal{R}^{-1} \circ \mathcal{R}^{-1} \subseteq \mathcal{R}^{-1}$.

Hence the result.

Theorem : 6.21

If $\mathcal{R}$ is transitive then $\mathcal{R} \circ \mathcal{R}$ is so.

Proof:

$(\mathcal{R} \circ \mathcal{R})(a, b) = \mathcal{R}(a, c) \cap \mathcal{R}(c, b)$

$\supseteq (\mathcal{R} \circ \mathcal{R})(a, c) \cap (\mathcal{R} \circ \mathcal{R})(c, b)$

$= (\mathcal{R} \circ \mathcal{R} \circ \mathcal{R} \circ \mathcal{R})(a, b).$

So, $\mathcal{R} \circ \mathcal{R} \circ \mathcal{R} \circ \mathcal{R} \subseteq \mathcal{R} \circ \mathcal{R}$.

Hence the result.

Theorem : 6.22

If $\mathcal{R}$ is reflexive then $\mathcal{R}^{-1}$ is so.

Proof:

$\mathcal{R}^{-1}$ (a, b) = $\mathcal{R}$ (b, a) $\subseteq$ $\mathcal{R}$ (a, a) = $\mathcal{R}^{-1}$ (a, a) and

$\mathcal{R}^{-1}$ (b, a) = $\mathcal{R}$ (a, b) $\subseteq$ $\mathcal{R}$(a, a) = $\mathcal{R}^{-1}$ (a, a).

Hence the proof.

Theorem : 6.23

If $\mathcal{R}$ is symmetric and transitive then $\mathcal{R}$ is reflexive.

Proof:

$\mathcal{R}$ (a, a) $\supseteq$ ($\mathcal{R} \circ \mathcal{R}$) (a, a), since $\mathcal{R}$ is transitive

= $\mathcal{R}$ (a, b) $\cap$ $\mathcal{R}$ (b, a)

= $\mathcal{R}$ (a, b) $\cap$ $\mathcal{R}$ (a, b), since $\mathcal{R}$ is symmetric

= $\mathcal{R}$ (a, b).

Similarly we can show that $\mathcal{R}$ (a, a) $\supseteq$ $\mathcal{R}$ (b, a).

Hence the proof.